Exploring high school teacher’s design of rich algebra tasks

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Abstract

The phenomenon in schools today is that teachers rarely change their tasks. However, the teacher's activities to change their tasks, build students' arguments, support solutions, and maintain arguments without long debates are important and exciting things to learn. This study explores the ideas of high school teachers about task design and practice in teaching mathematics. The authors surveyed twelve high school teachers who teach mathematics in East Java Province. First, the authors conducted preliminary observations to observe the design of mathematics teacher tasks for six months in eight schools. Second, the authors state that teachers are engaging and consistent in designing rich algebra tasks. Third, the authors examine the teacher's ideas through direct observation and unstructured interviews. The results show how teachers' ideas about task design enhance students' creative thinking by reforming tasks from textbooks into rich mathematics tasks. The design of the task carried out by the teacher is to create to stimulate creative thinking. The teachers also use their knowledge and understanding of the material and curriculum to modify mathematics tasks in students' mathematics books. The task given by the teacher is to improve students' reasoning, not just memorize formulas or properties.

Keywords: high schools teachers'; rich algebra tasks; tasks design


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Introduction

Teachers can choose math tasks already in the book to meet students' needs and interests or create new tasks. Teachers are also expected to select appropriate lessons for assessment, evaluate the effectiveness of their classroom tasks, and determine a sequence of functions to support their student's learning. It is in line with Levenson et al. (2018) that the teacher's mathematics tasks should be in order according to the student's abilities.

The International Council on Mathematics Instruction (ICMI) (2013) compiles and summarizes research relevant to task design and the difficulties encountered when designing and carrying out tasks. ICMI (2013) considers the role of teaching to include "selection, modification, design, sorting, observation, and task evaluation." Teachers believe that by practicing selecting and using appropriate tasks, textbooks need to reform tasks (Bakken & Andersson-Bakken, 2021). Despite attempts to divert teachers from using books as the sole provider of the mathematics curriculum, readers continue to be a mainstay in traditional classrooms. Li and Schoenfeld (2019) encourage teachers to select and perfect the tasks suggested in the textbook but must be able to refrain from using the whole mathematics task in them.

Facts that occur in schools, teachers always make tasks for students based on textbooks without being modified or adapted to students' abilities. It is in line with the research of van den Ham and Heinze (2018), textbooks can follow as long as following the curriculum, but teachers often use tasks presented in books. Still, they must present material covered in textbooks (van den Ham & Heinze, 2018). A study by Bademo and Tefera (2016) provides evidence linking task selection with teacher decisions. These decisions are based on how teachers perceive tasks based on their beliefs and how students react to lessons. Teacher confidence and student diversity play a role in teacher decisions to make task choices and adaptations. Therefore, it takes work to generalize how teachers make these decisions.

Teachers said they often gave their students the same tasks (Le et al., 2018). It will affect the task or other contributions to the task selection factors (Peteros et al., 2020). A study describing how teachers evaluate the importance of real-world-related tasks shows that most teachers focus on the significance of the mathematics skills underlying the task and how they are familiar to students (Darling-Hammond et al., 2020a; Kim et al., 2019). Johnson et al. (2017) urge teachers to provide challenging tasks for students. They argue that "effective tasks provide opportunities for students to investigate the structure of mathematics, generalize, and set an example."

To change how teachers perceive and create ideas about tasks, they must give sufficient opportunities to learn about the complexity of tasks and practice selecting and implementing appropriate lessons (Darling-Hammond et al., 2020a; Kim et al., 2019; Sullivan et al., 2013). Thus, teachers are encouraged to practice selecting tasks based on the thinking process (Albay & Eisma, 2021). The selection of tasks is an essential role for teachers. Still, many teachers have little instruction on selecting an appropriate task for their class if tasks bridge student learning with mathematical understanding (Álvarez et al., 2020). So, teachers need to pay attention to the design of mathematical tasks that can provoke students' reasoning.
In this study, the teacher selects a mathematics task based on the criteria of a rich mathematics task. Rich mathematics tasks are one approach that can be used to promote students' higher-order thinking skills that provoke students' reasoning (Lithner, 2017). They can provide students with learning opportunities to solve real-world problems requiring higher-order thinking skills, including problem-solving, reflective thinking, and reasoning.

Several studies on rich mathematics tasks have been conducted, such as Fitriati et al. (2020) and Mastuti et al. (2016), which show the role of rich tasks in developing student learning. Based on the results of research by Barrot et al. (2021), Bringula et al. (2021); Stehle and Peters-Burton (2019) have reported rich task potential, such as student challenges to solving problems that have links to the world outside the classroom. Encouraging them to think reflectively and offering various opportunities to meet the different needs of students (Darling-Hammond et al., 2020b). In connection with the rich mathematics task potential, it can be used to answer several problems in mathematics education in Indonesia (Fitriati et al., 2020b). Especially in improving students' high-order thinking skills in mathematics, which are still weak, as previously reported (Hamdi et al., 2018; Hilmi & Dewi, 2021). Also, learning about the application of rich tasks in mathematics classes in Indonesia is still limited; even the rich task approach has yet to be widely used in Indonesia.

Consequently, this research bridges the gap as additional knowledge in this approach. However, the difference between this research and the existing task design research lies in the type of task given by the teacher. The teacher used a type of rich mathematics task in this study, exclusively algebra.

In this condition, it is necessary to explore the design of teacher-rich algebra tasks, and several things will be done. Authors observe the teacher's ideas in providing task designs, observe the teacher's learning process, and explain the task concept to the teacher. Therefore, it is essential to conduct this study to assist professional mathematics teachers in designing their assignments. From the background above, this study explores teachers' ideas about task algebra design and practice in teaching mathematics.

**Methods**

This research was a qualitative descriptive study, which was a case study. Qualitative research explores teachers' ideas about task design and practice in mathematics teaching. This research was a case study, where the subject was considered very rare because it takes a consistent teacher to provide rich mathematics tasks.

The participants in this study initially consisted of 12 selected teachers from high schools who were members of the MGMP (*Musyawarah Guru Mata Pelajaran*) in four cities in East Java, specifically Jombang, Mojokerto, Malang, and Surabaya. The conditions for determining research participants were:

1. A teacher teaching experience for a minimum of five years or a teacher with a master's education,
2. A mathematics teacher with a characteristic in designing assignments during teaching - it was observed and proven by his/her portfolio,
3. A Mathematics teacher who is ready to be a research participant, and
4. Teachers are divided into four groups for initial observations in each school.

This research was carried out in stages, starting with initial observations on the twelve teachers while designing rich algebra tasks. From the results of these initial observations, it was found that only three teachers were consistent in designing and implementing rich algebra tasks in learning mathematics. Of the three teachers, researchers conducted field observations for six months. The result was only one teacher who consistently develops and carries out rich algebra tasks in mathematics learning. Then, the researcher clarified for the teacher to explore the design and implementation of rich algebra tasks in mathematics learning.

To find out the teacher's ideas in rich mathematics tasks, the researcher asked the teacher to do rich mathematics tasks based on learning. Therefore, this research instrument is the researcher's primary instrument and the interview sheet to explore the teacher's ideas about the task design. A team of experts has validated the researcher's interview sheet instrument.

This study explores teachers' ideas about rich mathematics tasks that cause students' reasoning to develop over time. Attend to the problems presented in mathematics tasks. This study focuses on tasks' characteristics (Mastuti et al., 2016; Mata-Pereira & da Ponte, 2017). These characteristics include: 1) multiple questions of varying levels of challenge, emphasizing exploratory problems and items; 2) generalization questions arise; 3) questions asking to justify answers and the resolution process; and 4) questions that allow various completion processes.

Data collection consisted of primary research data from classroom observations, teacher interview transcripts, and researcher notes, while documents such as teacher notes were secondary data. In conducting the research, researchers used several recording tools, such as Handycam, voice recorder, and notes, to record learning that involved tasks in class. After each lesson, the researcher immediately clarifies the subject. Researchers carried this research pattern several times for six months until the results were saturating. Before analyzing the data, the researcher triangulated the data.

Data analysis was carried out directly based on the actions of the mathematics teacher and the characteristics of the tasks developed. It was based on determining the teacher's ideas in designing rich algebra tasks and implementing them in the mathematics learning process. Researchers observe, collect data, and depth interview mathematics teachers in each process of learning mathematics. The interview was based on questions asked based on the subject's responses. Then the data were transcribed for analysis in order to obtain research results. Data from observations, designed algebra tasks, and interviews were analyzed based on the characteristics of the task.

**Results**

The twelve high school teachers selected according to the requirements set by the researchers and are members of the MGMP from four cities in East Java were then observed. The preliminary observations were made to identify the ability to design rich algebra tasks. The result showed that the researchers determined three teachers who were consistent in designing and implementing rich algebra tasks in learning mathematics. Furthermore, the researchers
conducted extension observations for six months to see the consistency and uniqueness of the three teachers in designing algebra tasks. These observations showed that only one teacher was unique in designing rich algebra tasks and then determining them as subjects.

Researchers have made observations ten times to the teacher who determined as a research subject. However, in this study, only one rich mathematics task was present because its lesson was consistent. One example of a mathematics task given by a teacher in class is algebra, as follows.

Consider the following system of equations:

\[
\begin{align*}
(a - 1)x^2 + \left(\frac{b}{2} - 3\right)x - 1 &= y \\
2x^2 - 2x + (3 - 2c) &= y
\end{align*}
\]

Does the following system of equations have multiple solutions or not? Explain your opinion!

The example of an algebra task above shows that the design of a task made by a mathematics teacher satisfies the four characteristics of a rich algebra task. It can be seen in the characteristics of the task (1) students are involved in various questions with various levels of challenge, emphasizing problem and exploratory questions. The task above directs students to generalize the procedure according to the characteristics of the system of equations; (2) Students are asked to choose a solution, then asked to express their opinion, resulting from generalizing several solutions they have chosen. Tasks also ask for justification, and implicit reasons are asked indirectly on the students' views; (3) students are not expected to indicate whether the task has many solutions or none. The teacher's task also has various completion processes; (4) students are asked to think of multiple solutions. Each student may have a different solution for logical reasons. Design principles for teacher action regarding student work and whole-class discussion are considered in the lesson plans used throughout the lessons.

During the task, the teacher tries to dig up the students' conceptual knowledge about the Quadratic and Quadratic Equation Systems. It can be seen from the universe whether the Quadratic and Quadratic Equation System requirements have the right solution from the coefficients or use discriminant requirements to see whether the curves intersect or coincide. Teachers' instructional skills and strategies are needed, as how they try to identify students' prejudices and learning difficulties and what teachers do to correct misconceptions, if any.

After observing the learning using the Mathematical task of Quadratic and Quadratic Equation Systems for 40 minutes, the researcher interviewed the teacher about the mathematics task design ideas. The following is part of the interview transcript.

Researcher : Where did you get the initial idea to create problems in your task today?
Teacher : The idea of his task was from the two-variable system of linear equation task last week. It is a routine problem in the student book; I revised it a little.

Researcher : Oh, yes, what was revised, sir?
Teacher : If the problem in the book says, "Look at the following system of equations:

\[
\begin{align*}
(a - 1)x^2 + \left(\frac{b}{2} - 3\right)x - 1 &= y \\
2x^2 - 2x + (3 - 2c) &= y
\end{align*}
\]
If the system has many solutions, what is the value of $a^2 + b^2 - c^2$? The answer is easy, if $a, b, p, q \in R$ and the constant $c, r \in R$.

Has multiple solutions if $a = p, b = q$ and $c = r$, then $a^2 + b^2 - c^2 = 3^2 + 2^2 - 2^2 = 9$. The understanding is a little unattractive compared to my question earlier.

Researcher: Then, what are the main concepts that you want to show your students this time?

Teacher: The general form of a quadratic and quadratic linear equation system and the conditions for a quadratic and quadratic linear equation system have possible solutions based on their discriminants and coefficients.

Based on the interview above, the mathematics task idea came from the previous two-variable Linear Equation System task, which was revising from a textbook’s routine task. The teacher wants to show that the mathematics task can explore students' understanding and creativity compared to books. The teacher also explains the study's key concepts, namely the general form and requirements of the Quadratic and Quadratic Equation System, having possible solutions based on the discriminants and their coefficients. The teacher also explains his notes about possible answers that students might answer when giving the task (Figure 1).

The teacher's possible answers are made by the teacher based on the students' level of thinking in the class. In Figure 1, the teacher explains to the researcher that their students can answer three possibilities. One student may think if $a, b, p, q \in R$, then the quadratic and quadratic linear equations can have one solution, two solutions, not many solutions, or no solution. Two students may think if $a, b, p, q \in \text{imaginr}$, then the quadratic and quadratic linear equations system must have no solution. Three students may believe that if $a, b, c, p, q, r$ are not numbers $a, b, p, q \in \{ \}$, then the system of quadratic and quadratic linear equations also has no solution.

\[
(a - 1)x^2 + \left(\frac{b}{2} - 3\right)x - 1 = y \\
2x^2 - 2x + (3 - 2c) = y
\]

Does the system of equations have many solutions or not?

Solutions:
If you look at the universe of the conversation
There are several possibilities, namely
\[
ax^2 + bx + c = y \\
px^2 + qx + r = y
\]
1. If $a, b, c, p, q, r \in R$
   - 1 Solution ($D = 0$)
   - 2 Solutions ($a = p, b \neq q$) or $D > 0$
   - Many solutions ($a = p, b = q, c = r$)
   - No solutions ($a = p, b = q, c \neq r$) or $D < 0$
When designing these mathematics tasks, the mathematics teacher has several considerations. The most important consideration is the student's prior knowledge and understanding of what the student might have, as shown in the following quote from the teacher's statement. The student's initial knowledge is the basic concepts that students have based on the existing schemata.

Teacher: My consideration is the student's understanding of quadratic and quadratic equations and the possible solutions or solutions that the quadratic and quadratic equation systems have using either discriminants or coefficients. Students' initial knowledge also becomes my consideration. They memorize or understand the concept well.

The following interview explains the importance of teacher-designed mathematics tasks for students.

Researcher: Please, looking at your task, what is the importance of such a task for your students?
Teacher: It is essential for the relationship between the procedures and concepts they have. The reason is that this task does not require students to make concrete examples. It is a task that requires higher-order thinking, but students of average ability can be provoked to answer logically.

Researcher: If it concerns the relationship between procedures and concepts, what are the challenges in this task for students?
Teacher: This time, the task question is, "Does the system of quadratic and quadratic equations have many solutions or not?". I hope that students think of many possibilities, not getting stuck in an equation that looks complicated but memorizing formulas or properties rather than understanding them.

Based on the interview above, the teacher wants to show that the quadratic and quadratic equation sub-system material task seeks to establish the relationship between the procedure and the students' concepts. The teacher hopes that their students' thoughts can develop through this task. The teacher wants to improve students' reasoning, not just memorize formulas or traits. The teacher's lessons also explain that students who initially have lower abilities are motivated to answer questions logically.

Researcher: When generating this task idea, did it occur to differentiate the questions for students with low and high criteria for this case?
Teacher: For intelligent students, I want them to sketch their parabolas with the possible answers, while for ordinary students, it is enough with their initial understanding. It may continue when we discuss it again.

Researcher: How do you know the students' initial knowledge? It could be that every student is different, right, sir?

Teacher: From the way they responded to my initial question before the task, they answered the task weeks ago.

Researcher: You said earlier that you thought about asking a different question according to cognition level. Did you have any other goals when generating the task idea this time?

Teacher: Yes, a student who stands out in the class in terms of high cognitive ability can become peer tutors for their friends and understand the quadratic and quadratic equation tasks this time.

Researcher: Since when did you have the idea to design a task like this?

Teacher: Since they first taught in high school, since they know that the character of teenagers does not want to be cornered or demeaned, they always want to be respected and heard so that their creativity can emerge.

The teacher understands the student's initial knowledge based on the interview above. The teacher understands students' conceptual knowledge, so he aims to explore the potential of students with high mathematical abilities with more demands than other students with mediocre mathematical skills. The teacher's discussion forum was formed to aim that students with a high level of understanding can become peer tutors for their friends. The teacher also explained that his task design idea began when he knew the characteristics of his students. This explanation shows that the teacher makes the task design understandable for the students.

The teacher trains students to see the completion of mathematics tasks from various perspectives. In this case, prior knowledge is needed, but the relationship between ideas and schemes in students' minds must also be trained. The teacher explains to the researchers that problems such as quadratic and quadratic equations often appear in the Olympics because issues like these often link several existing concepts. Therefore the teacher tries to get students to think flexibly and critically.

"I want them to look at it from the discriminant. Where the general form of a quadratic and quadratic system of equations is substituting for \((a - p)x^2 + (b - q)x + (c - r) = 0\). So, 
\[D = (b - q)^2 - 4(a - p)(c - r),\] with probability \(D > 0\) if the system of quadratic and quadratic equations has two solutions or the curve intersects at two points. For \(D = 0\) if the quadratic and quadratic equations have one solution or the curves intersect at one point or \(D < 0\) if the system of quadratic and quadratic equations has no solutions or the curves drawn, will not intersect (Figure 2). Also, because this task often appears at the Olympics and I lead a lot of my students there, I want them to practice more of this kind of problem ".

The teacher explained that he wanted his students to solve quadratic and quadratic equations when viewed from the discrimination. As the teacher explains in Figure 2, the teacher
explains if there is a possibility that the students think about a possible solution when viewed from the position of the curve that they will draw later.

\[
ax^2 + bx + c = px^2 + qx + r \\
(a - p)x^2 + (b - q)x + (c - r) = 0 \\
D = (b - q)^2 - 4(a - p)(c - r)
\]

\[K1: D > 0 \rightarrow \text{have two points of intersection or solution} \]
\[D = 0 \rightarrow \text{have one point of intersection or solution} \]
\[D < 0 \rightarrow \text{have no solution} \]

**Figure 2.** An example of another alternative to task 2

Indirectly, the teacher shows the material knowledge needed to produce mathematics tasks in Figure 2. This content knowledge is in line with the teacher's explanation. The general form of a system of quadratic and quadratic equations

\[
\begin{align*}
\{ y &= ax^2 + bx + c \\
\{ y &= px^2 + qx + r
\end{align*}
\]

where the coefficients \(a, b, p, q \in R\) and the constant \(c, r \in R\). It has many solutions \(a = p, b = q,\) and \(c = r,\) when we look at the coefficients of each equation. Moreover, there is no wrong answer because someone answered that the equations have no solution or only has one or two different solutions with logical reasons and correct conditions. It can also use the discriminant requirements that have been previously explaining. The teacher also explained to the researcher that he wanted to allow students to develop many solutions and strategies, invite unusual responses, and bring up mathematical ideas.

The examples of tasks designed by mathematics teacher and reinforced by the results of interviews show that the teacher challenges students to solve them. The challenge given to students is whether the given equations system has multiple solutions. In this situation, students are required to explore their abilities in finding solutions and generalizing them. It is in line with the characteristics of the first and second tasks, questions from algebra tasks that provide various levels of challenge, exploration, and raise generalizations. The generalization expected from students is to form general equations from quadratic and linear equation systems.

In addition, the expected conditions of the task are that students can determine possible solutions based on the discriminant and its coefficients. The focus of this task is that the initial knowledge possessed by students is based on the procedures carried out and supported by a good understanding of the concept. Rich algebra tasks also require students to justify the answers given and the process involved in determining solutions. This situation is in line with the third and fourth characteristics, the justification of the problem-solving process and the possibility of various relevant solutions.
Discussion

In this study, the researcher found that the teacher needed much information support to revise the tasks in the textbook in designing mathematics tasks. Teachers also use their knowledge and understanding of the material and curriculum to modify mathematics tasks in students' math books. The teacher also uses the ability of the content of the material to explain each note and alternative answers that he makes. It means that the teacher always writes down alternative solutions that students can answer before being given to students. It follows Murtafiah and Lukitasari's (2019) research, where teachers also have unique characteristics in recognizing the abilities of their students it affects their learning. The teacher's knowledge of recognizing students' abilities is called pedagogical content. In addition, teachers must also have content knowledge, namely knowledge of teacher material, one of which is designing tasks (Mustafa & Derya, 2016).

The teacher's understanding of the concept makes the teacher consider the students' previous experiences. The teacher wants to hone the students' initial knowledge using investigative questions such as tasks. The teacher also explained that he was not the type of teacher who gave spontaneous mathematics tasks because it was pre-planned. However, the teacher felt that he often used random questions in the middle of his task to practice his students' understanding, reasoning, and critical thinking skills. It is in line with Keiler (2018) and Kim et al. (2019), which state that a good teacher is a teacher who gives tasks that can lead students to think. The teacher has more knowledge about the students' understanding, this can be seen when there is a new task that he feels is challenging, but he does not give it to students because the time required will be longer. This task has great difficulty and is replacing a mathematics task that he has considered great care of. Teachers know to recognize student difficulties. Student difficulties are part of the teacher's understanding of their students (Blazar & Kraft, 2017; Keiler, 2018; Tiani et al., 2019).

In designing mathematics task ideas, the teacher always knows the fundamental concepts and explains them in detail. In creating the task, the teacher explains if the task results from a modification or reform of the student's mathematics book situation. It is one of the stages of decision-making, namely building task ideas by teachers (Murtafiah et al., 2019). A study conducted by Hadar and Tirosh (2019) provides evidence linking task selection with teacher decisions. These decisions are based on how the teacher views the task, their beliefs, and how students react to the study. A good teacher is a teacher who knows the characteristics of their students well. The teacher's confidence in the task and the student's knowledge identify if the teacher knows about the curriculum and applies it to every math task.

Teacher belief is essential to students' learning (Norton, 2017; Pratiwi et al., 2022). Based on the teacher's explanation in each interview and observations on the task, the teacher knows what the students need. The teacher understands his students' initial or previous knowledge to become the teacher's consideration in developing his task. Goethe and Colina (2018), Gravemeijer et al. (2017), and Messiou et al. (2022) highlighted the importance of considering the diverse needs of students. For example, a mathematical task that the teacher makes must meet the student's individual needs. In particular, teachers choose their tasks and trust students...
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if they will be able to solve them. They recognize that rich mathematics tasks can change from year to year based on changing student interests but must always involve teacher experience. Teachers with a solid knowledge base in this domain know math concepts that are difficult to understand, concepts that students usually have, including misconceptions, and know the sources of student error (Lee et al., 2018; Siregar & Siagian, 2019). Also, they realize how to get rid of difficulties, errors, and misconceptions.

Conclusion

The teacher's design of tasks can stimulate students' creative thinking. The teacher also uses their knowledge and understanding of algebra material and curriculum to modify mathematics tasks in students' math books into rich algebra tasks. The teacher also uses the ability of the content of the material to explain each note and alternative solutions that he makes. Besides that, the teacher has unique characteristics in recognizing the abilities of their students. The most important consideration is their prior knowledge and understanding of what their students might have. The concept of the task given by the teacher aims to improve students' reasoning rather than just memorizing formulas or traits. The teacher pays attention to students to develop many solutions and strategies, invites unusual responses, and expresses their mathematical ideas. Then, the teacher carries out mathematical tasks by combining several concepts in solving problems, with the aim that students are trained to think flexibly and critically in solving them.

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Conflicts of Interest

The authors declare no conflict of interest regarding the publication of this manuscript. In addition, the ethical issues, including plagiarism, misconduct, data fabrication and/or falsification, double publication and/or submission, and redundancies, have been completed by the authors.

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**Author Contributions**

**Ajeng Gelora Mastuti:** conceptualization and design, acquisition of data, analysis and interpretation of data, drafting the article, review, and editing; **Lydia Lia Prayitno:** design, acquisition of data, analysis and interpretation of data, drafting the article, review, and editing.

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