Trivium curriculum in Ethno-RME approach: An impactful insight from ethnomathematics and realistic mathematics education

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Abstract

In implementing Ethnomathematics-Realistic Mathematics Education (Ethno-RME), teaching mathematics correctly is needed through learning practices in teaching and learning activities. This pedagogical activity requires guidance in the form of a curriculum. So, the teacher can determine the ethnomathematics context as a starting point in teaching mathematics using Ethno-RME. Therefore, this paper focuses on constructing an Ethno-RME curriculum to guide teachers to apply Ethno-RME in their learning process. This research was collected data from a few relevant kinds of literature to build the Ethno-RME curriculum, such as literature about the D’Ambrosio trivium curriculum, the principles, and character of RME, and literature about the implementation of RME and Ethnomathematics in school. Then, the data were reviewed, criticized, and synthesized, which was done by integrating the ideas in the literature with the new ideas to form the new concept and formula for Ethno-RME curriculum. This study comprehensively explains the goal of Ethno-RME learning, Ethno-RME competencies, and the procedure of Ethno-RME learning. Its process consists of several steps: determining, exploring, processing, and finding mathematics in the ethnomathematics context. Furthermore, this procedure continues with conducting self-development models and critical reflection as an assessment.

Keywords: Ethno-RME; ethnomathematics; realistic mathematics education; trivium curriculum


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Introduction

Since the initiation of Ethnomathematics by D’Ambrosio in 1985 until now, research on the exploration and implementation of ethnomathematics in mathematics learning in schools has grown rapidly in various countries (Long & Chik, 2020; Orey, 2000; Pradhan, 2017; Risdiyanti & Prahmana, 2018; Rubio, 2016; Zhang et al., 2021). D’Ambrosio’s idea of restoring mathematics to be rooted in the culture and daily life of students and the internalization of socio-cultural values that can form good character in students are in great demand by education and teachers in various countries (Acharya et al., 2021; Brandt & Chernoff, 2015; François, 2012). Many researchers and educators have realized that ethnomathematics is part of a powerful pedagogical strategy to understand mathematical concepts based on students’ experiences and culture (Pradhan & Sharma, 2021).

Regarding implementing ethnomathematics in mathematics learning in schools, D’Ambrosio has proposed a trivium curriculum that emphasizes the achievement of literacy, matheracy, and technoracy competencies (D’Ambrosio, 1999; Rosa & Orey, 2015). However, some researchers and educators still have had difficulty implementing ethnomathematics in schools over the past three decades (Pradhan & Sharma, 2021). Teachers are less able to explore and implement in schools because they have not been integrated with the mathematics education curriculum from the government, even though teachers have been eager to apply ethnomathematics in learning in schools (Tesfamicael et al., 2021).

In addition, D’Ambrosio’s trivium curriculum, which emphasizes more on competence, makes educators confused about carrying out pedagogical actions in schools, thus finally creating their own curriculum (Adam, 2004). Some teachers also often find it difficult to determine activities in learning design using the context of ethnomathematics (Ergene et al., 2020). There are also those who feel that the implementation of the context of ethnomathematics in mathematics learning is less deep because there is no systematic procedure (Tutak et al., 2011). Many teachers have applied ethnomathematics in mathematics learning at school, such as Putra and Mahmudah (2021), Widada et al. (2018), Ergene et al. (2020), Fouze and Amit (2018), Massarwe et al. (2013), Tlonaen and Deda (2021), and Dias et al. (2015). However, teachers often have difficulty finding methods or learning activity steps to apply ethnomathematics in school to make students discover mathematics from their culture. So, ethnomathematics is often used just as context to introduce the concepts of mathematics, not to discover mathematics and find its social–culture value that can be learned and used by students in daily life. Often, ethnomathematics is also applied without regard to the competency that students must achieve and the process of mathematics in students thinking. That’s all because teachers did not have rules or curriculum which could guide teachers to implement ethnomathematics in learning mathematics at school.

Therefore, regarding how ethnomathematics is implemented in mathematics learning in schools, a special formula is needed to make it easier for teachers to implement. A mathematics learning approach that uses real context in real life is Realistic Mathematics Education (RME) which emphasizes the mathematical process and the level of thinking of students in understanding mathematical concepts (Prahmana, 2022; Zulkardi, 2002). The real context in RME can be filled with ethnomathematics contexts, which contain mathematical concepts and sociocultural values that can form good character in students (D’Ambrosio, 2007, 2016; Rosa & Orey, 2015). In addition, RME and Ethnomathematics have the same history and purpose, to make students easier to understand mathematical concepts by finding their own mathematics based on their own experiences and being able to use mathematics in solving problems in the reality of student life (D’Ambrosio, 2007; Gravemeijer & Terwel, 2000). Both theories rise from the same criticism from Freudenthal and D’Ambrosio about the new
mathematics that is mechanistic, anti-didactical, and attention less to socio-cultural problems which exist in the human life reality (D'Ambrosio, 2016; Streefland, 1994; Treffers, 1993; van den Heuvel-Panhuizen & Drijvers, 2020). Furthermore, both theories also have the same goal, which is students can find their own knowledge from their own experiences and from things that are around them (D'Ambrosio, 2007; Gravemeijer & Terwel, 2000). So, students can be able to understand mathematics, and can learn and use the socio-cultural values which are contained in the culture around the students.

Like Ethnomathematics, since its inception, until now the research and implementation of RME in mathematics learning have also grown rapidly (Papadakis et al., 2021; Paroqi et al., 2020; Risdiyanti & Prahmana, 2021; Sitorus & Masrayati, 2016). However, so far, the real context used in RME only functions as a starting point in learning, even though in these real contexts, there are many good socio-cultural values to be internalized by students in daily life. Meanwhile, in ethnomathematics, the real context is used not only as a starting point but also impregnated with its values to form good character and ethics in students. Therefore, the combining of Ethnomathematics and RME or known as Ethno-RME is a solution to clarify and systematize learning procedures with the context of the ethnomathematics context and internalize the socio-cultural values that exist in the context used so that students can easily understand mathematical concepts and have good character and ethics (Prahmana, 2022).

It is necessary to create a clear and systematic curriculum to facilitate teachers’ enthusiasm in applying ethnomathematics in mathematics learning in schools and to make teachers easier to implement it. Curriculums in some countries have not been integrated with Ethno-RME, so it makes teachers difficult to implement Ethno-RME in learning mathematics at school. This curriculum will not change the whole education curriculum, but it is to integrate Ethno-RME in learning mathematics at school using the curriculum which is constructed in this study. Hopefully, the Ethno-RME curriculum that we construct can make mathematics more relevant and meaningful for students to increase the quality of mathematics education. Besides that, the Ethno-RME curriculum can answer the gaps or teacher problems in applying ethnomathematics in mathematics learning and make RME livelier by internalizing socio-cultural values in the context used as a starting point in learning. In addition, this research is expected to be a reference for teachers and researchers who will explore and develop Ethno-RME further.

**Methods**

This study employs an Integrative Literature Review as its research approach because it aims to design an Ethno-RME curriculum that combines the D'Ambrosio trivium curriculum with the principles and characteristics of RME. Integrative Literature Review lays the groundwork for building new conceptual theories by reviewing, criticizing, and synthesizing representative literature on a given issue in an integrated manner so that theoretical frameworks and good views can be produced (Torraco, 2005). Therefore, it was found appropriate and helpful to employ the Integrative Literature Review in this study to develop the Ethno-RME curriculum.

This investigation has three phases. In the reviewing phase, we select a topic, confirm the reasoning and objectives, formulate the scope and particular research questions, and collect the relevant literature. The issue focuses on the trivium curriculum announced by Ubiratan D'Ambrosio and Hans Freudenthal’s RME principles and characteristics. Furthermore, we critique the existing literature through critical analysis, which thoroughly examines the primary ideas and their relationship to a
problem and criticism of the current literature. Lastly, we synthesize the review by combining old and new ideas into a discussion subject. In this research, synthesis forms a new curriculum to support the Ethno-RME approach and a new way of thinking about the concerns addressed in the integrative literature review. This curriculum must be directly developed from analysis and synthesis.

We base our arguments and explanations on logic and clear conceptual reasoning in this research method. These are the two most significant elements used to build the proposed curriculum. They enable the reader to understand the relationship between research difficulties, literary criticism, and academic outcomes in this curriculum framework. Finally, we expressed the review results directly, addressed the rationale and necessity of the study, and straightforwardly presented the review process, beginning with how the literature is located, evaluated, synthesized, and published. This study is not analyzed and assessed with the same rigor as empirical studies. However, the quality of a given field or topic is determined by its depth, thoroughness, and essential, valuable, and novel contributions (Prahmana, 2022; Prahmana & Istiandaru, 2021).

Results

Theoretical review of Ethno-RME

The idea of Ethnomathematics D’Ambrosio and Freudenthal’s Realistic Mathematics Education (RME), which have a common purpose and substance, was successfully combined into an idea of Ethno-RME. Ethnomathematics and RME aim to allow students to gain mathematical knowledge from their daily experiences and culture. Combining RME, which has a highly didactic approach and pays great attention to the thinking level and mathematization processes of students, with Ethnomathematics which emphasizes ideas, methods, mathematical techniques, and socio-cultural values in student culture, form an idea of Ethno-RME theory which is not only very didactic and pays attention to the thinking level and mathematization processes of students but also has socio-cultural values that can make a good character and ethics of students (Prahmana, 2022; Treffers, 1993; Wubbels et al., 1997). So that the combination of Ethnomathematics and RME can make students more able to understand mathematical concepts efficiently and correctly and absorb the socio-cultural values contained in the experiences and cultures around students (Prahmana, 2022; Prahmana & Istiandaru, 2021).

In Ethno-RME, students’ thought processes are directed to advance gradually from one level to the next so that it can be ensured that no mathematical process is interrupted. So, when students understand mathematics with Ethno-RME, students will be able to truly understand mathematical concepts from their roots, not just memorize mathematical formulas. Therefore, in Ethno-RME learning, the teacher must design situations or phenomena that can trigger the mathematical process in the student’s thinking. This will make it easier for students to do mathematics, understand concepts and internalize the socio-cultural values in the student’s culture.

The steps of student thinking in Ethno-RME are achieved from the process of mathematical self-modeling carried out by students of phenomenological situations designed by the teacher. The modeling process starts with “model of”, “model for”, and “formal model” such as iceberg modeling in RME. On “model of”, students model situations close to the student, such as the student's culture or daily activities. Then, in the “model for”, students will homogenize and formalize the modeling done before to form “formal model” or formal mathematical knowledge. In addition to modeling, the teacher also directs students to reflect on the contexts and situations used in learning to be able to critically reflect on socio-cultural values that can be internalized in students' daily lives.
Theoretical review of the *Trivium* D'Ambrosio curriculum

Regarding Ethnomathematics in mathematics education, D’Ambrosio ever developed a curriculum called the *trivium* curriculum, which consists of the achievement of three abilities such as *literacy*, *matheracy*, and *technoracy*, which allows being developed in school activities and are needed in 21st-century life. The first ability is *literacy*, the ability to process information by reading, writing, representing, and calculating in various diverse contexts (D'Ambrosio, 1999; Rosa & Orey, 2015). From the ethnomathematics perspective, *literacy* can be interpreted as integrating the school’s cultural context and society’s cultural context through cultural dynamics that allow students to exchange academic knowledge and local knowledge. This *literacy* can be understood as a competence related to excavating, processing, and creating mathematical information (D'Ambrosio & D'Ambrosio, 2013).

In the pedagogical practice, teachers can start with the history and context close to student culture to be able to identify and explore mathematical ideas, methods, and techniques that have existed and developed in their culture (D'Ambrosio, 1999, 2007). Teachers can guide students to find topics and explore mathematics creatively through brainstorming, dialogue, and discussion (Rosa & Orey, 2015). Then, mathematical modeling is carried out by investigating mathematical ideas, methods, and techniques in a knowledge system of a cultural community related to academic mathematics in school (Ferreira, 1997; Fonseca, 2010). In the investigation process, students can utilize and get information from various sources such as books, the internet, field research, and others. This can help students develop their *literacy* ability (Rosa & Orey, 2015).

The second ability is *matheracy* which can be interpreted as ability, strategies, and competencies that trigger students to realize how they explain their culture, interpret, recognize, and manipulate signs, symbols, and codes, and use of mathematical models in daily life (D'Ambrosio, 1999; Rosa & Orey, 2015). It can help students develop their creativity and analytical skills to understand and solve problems (D'Ambrosio & D'Ambrosio, 2013). In addition, it allows students to be able to interact with the outside world, so they can understand and find solutions to face the problem in daily life through modeling that represents reality (Rosa & Orey, 2015).

*Matheracy* also focuses on deep and critical reflections on how mathematics plays a role in society’s culture. So, *matheracy* refers not only to the meaning of mathematics ability but also to the competencies necessary to interpret and act in the social, cultural, political, and economic situations formed by mathematics (Skovsmose, 2019). It means that mathematical concepts are not only about basic mathematical skills but also about developing more complex mathematical reasoning abilities that include numeracy and *literacy* (D'Ambrosio, 1999).

Therefore, it needed the teachers’ ability to guide students to be able to use, translate and model the artifact culture from the phenomena found in daily life into code and symbols. The development of mathematical models can allow students to understand the phenomena and problems faced by society. The results of mathematical modeling can be used as the basis for making decision processes that are social, cultural, political, and economic situations in the realities of life (Skovsmose, 2019). At its core, *matheracy* allows an understanding of the facts and phenomena that focus on deep reflection on society through modeling (D'Ambrosio & D'Ambrosio, 2013).

The third ability is *technoracy*, which is the ability of individuals to critically use and combine technologies and their ability to evaluate possibilities and limitations in diverse everyday situations to make appropriate decisions for themselves and others (D'Ambrosio, 1999; Rosa & Orey, 2015). *Technoracy* is an important figure of knowledge to translate and interpret matters related to the natural, social, cultural, political, and economic environment. In ethnomathematics, cultural community ideas,
methods, and mathematical techniques can be interpreted analytically, geometrically, graphically, and others using appropriate mathematical techniques and technologies (Rosa & Orey, 2015). Technoracy is an important figure of knowledge to translate and interpret matters related to the natural, social, cultural, political, and economic environment. As technology advances, mathematics develops the power to project a reality for the future. The use of technology is inevitable to improve the quality of life (D'Ambrosio, 1999, 2007). This technology can help individuals process modeling by representing, integrating, and generalizing using technology (Rosa & Orey, 2010).

The trivium curriculum provides education in a critical method with instruments of literacy, matheracy, and technoracy necessary to develop 21st-century education. Combining its curriculum in the classroom would imply a conception in which Ethnomathematics and mathematical modeling are tools for pedagogical action. In addition, ethnomathematics with a trivium curriculum also provides awareness that educators need to care about humanity as a whole and respect each culture that students have. It’s important to consider students as individuals with future expectations that reflect themselves and their cultural history (Rosa & Orey, 2015). Educators also need to care about the condition of students who are improving their abilities, and at the same time, students are learning about the realities of their lives in a cultural community (D'Ambrosio, 1999). Students increase creativity, and at the same time, students learn values in society (Rosa & Orey, 2015). In education, these two things must go hand in hand and be equal parts to avoid irresponsible creativity, such as utilizing mathematical science to create technology that can harm human civilization (D'Ambrosio, 1999). Ethnomathematics with a trivium curriculum strives to balance creativity and values so that students who study mathematics have creativity and remain responsible for upholding values and ethics in living.

**Theoretical review of the principles and characteristics of RME**

In mathematics education, Freudenthal emphasizes that mathematics should be seen as a human activity and, at the same time, should produce mathematics as a product (Gravemeijer & Terwel, 2000; Treffers, 1993). The Realistic Mathematics Education (RME) approach is designed to be able to integrate these two goals, that are creating a learning environment that allows students to rediscover in a guided manner (guide reinvention) and produce mathematics from students' daily realities and mathematization processes occur (van den Heuvel-Panhuizen & Drijvers, 2020). The principle of "guided reinvention" is that students can find mathematics by themselves. The way is by starting a thought experiment, imagining a step where with that step students can find mathematics (Gravemeijer & Terwel, 2000). Guided reinvention allows learners to assume the knowledge they acquire is their own knowledge for which they are responsible (Gravemeijer & Terwel, 2000; Treffers, 1993). In that process, the history of mathematics can be used to ignite and inspire students’ discoveries.

Guided reinvention must be experienced by students, and students must start from the student daily reality, then they must switch to analyzing their own activities. From the process of guided reinvention occurs the process of mathematization, which is to gain experience from reality and generate mathematical knowledge from contextual problems of the student's everyday reality (Gravemeijer & Terwel, 2000). What can be seen as guided reinvention from the educator’s point of view must be experienced by students as progressive mathematization moves from real problems to formal mathematics (Treffers, 2012). According to this author, there are two mathematical nations, horizontal mathematization and vertical mathematization. Horizontal mathematization is the process of converting contextual problems into mathematical problems, and vertical mathematization is the process of reorganization in the mathematical system itself, such as finding relationships between concepts and
strategies and the application of inventions (Michelsen, 2006; Treffers, 2012). The difference between horizontal mathematization and vertical mathematization can be seen more clearly in Figure 1.

![Mathematization process (Doorman, 2016)](image)

**Figure 1.** Mathematization process (Doorman, 2016)

The mathematization process will lead students to progress in thinking from one level to the next level (Treffers, 1987). Referring to Van Hiele’s thinking in de Lange (1996) regarding the level of thinking, in the mathematical thinking process, there are three levels; first level, a student reaches the first level of thinking when he can manipulate the known characteristics of the patterns he knows; the second level, a student reaches the second level of thinking when able to manipulate the interrelationships of those characteristics; the third level, a student reaches the third level of thinking when he begins to manipulate the intrinsic characteristics of those relations.

In learning with the Realistic Mathematics Education approach, phenomenologically rich situations are used, as in didactic phenomena, situations must be selected in such a method that they can be organized by mathematical objects built or constructed by students (Freudenthal, 1986; Gravemeijer & Terwel, 2000; Larsen, 2018). The goal is to describe how thoughts are described, and phenomena are analyzed. Such phenomenological analysis laid the foundation for a didactic phenomenology that also discusses what is meant by phenomenological analysis from an educational perspective (Freudenthal, 1986).

In the framework of didactic phenomenology, situations to which a particular mathematical topic is applied are then investigated to assess its suitability as an impact point for progressive mathematical processes (Freudenthal, 1986; Gravemeijer & Terwel, 2000; Larsen, 2018). The objective of phenomenological investigations is to find problem situations from which the specific situation approach can be generalized and to find situations that can trigger paradigmatic solution steps as the basis of mathematics (Freudenthal, 1986). To be able to find phenomena that can be mathematized, it can be done by trying to understand how these phenomena are discovered (Gravemeijer & Terwel, 2000).

In the mathematical process of a phenomenological process, there is a process of self-development models or the development of models independently. Students can model independently from concrete situations to formal mathematical situations (Freudenthal, 2006). This means that students develop their own model of solving problems by modeling a situation close to the student's world, then generalizing and formalizing the model will turn into “model of” which will then shift to “model for” similar problems and finally, the model becomes formal in mathematics (Freudenthal, 2006). In the modeling level RME approach, there are four first levels, the situational level, namely the level where specific situational knowledge and strategies are used in the context of the situation; second, the referential level or “model of”. The level at which the model and strategy refer to the situation
described in the problem; the general level or model is the level at which the mathematical focus on the
strategy dominates the reference to the context; the formal mathematics level is the level at which a
person works with conventional procedures and notation (Gravemeijer, 1994).

A combination of Van Hiele’s three levels of thinking, the didactic phenomenology of Freudenthal (1986) and the progressive mathematics of Treffers (1993) produced five characteristics of RME. That is the use of context in phenomenological exploration, the use of models or bridging by vertical instruments, the use of pupil’s own creation and contributions, the interactive character of the teaching process or interactivity, dan the intertwining of various mathematics strands or units. A more detailed description of the RME characters is as follows (de Lange, 1987; Gravemeijer, 1994; Zulkardi, 2002):

The use of context in phenomenological exploration

In RME, the starting point must be based on real experiences or phenomena, and students can engage in contextual situations. Thus, in mathematics learning, students should not start from a formal system (Gravemeijer, 1994; Larsen, 2018; Zulkardi, 2002). Teachers should allow students to think from informal to formal reasoning and representation (Prahmana, 2012; Zulkardi, 2002). Students’ formal mathematical activity must be obtained from abstract things (Cobb, 1994; Zulkardi, 2002). The process of conceptual mathematization seeks to allow students to explore situations, find and identify relevant mathematical elements, create schemes, and visualize sequentially to find patterns and develop models that generate mathematical concepts (Gravemeijer, 1994; Larsen, 2018; Zulkardi, 1999). Students then carry out a process of reflection and generalization and will then develop a more complete understanding of the concept (Gravemeijer, 1994; Larsen, 2018; Zulkardi, 1999). Furthermore, students are expected to be able to apply mathematical concepts to other aspects of daily life (applied mathematics) (Gravemeijer, 1994; Gravemeijer & Terwel, 2000; Larsen, 2018; Zulkardi, 2002).

The use of models or bridging by vertical instruments

The model developed by the student consists of a situational model and a mathematical model. The situational model developed by the student is then generalized and formalized, and finally, the model can become an entity for mathematical reasoning. There are four levels of models in RME learning and teaching (Gravemeijer, 1994, 2000; Streefland, 1985; van den Heuvel-Panhuizen, 2003):
1. Situational Level, the level at which specific situational knowledge and strategies are used in the context of the situation
2. Referential Level or ‘model of’, the level at which the model and strategy refer to the situation described in the problem
3. General level or ‘model for’, the level at which mathematical focus on strategy dominates the reference to the context
4. Formal mathematical level, the level at which a person works with conventional procedures and notation.

Webb et al. (2011) illustrate the RME learning and teaching model level using an iceberg. Iceberg is a metaphor that communicates the role of context, models, representations, and strategies in developing students' mathematical understanding (Webb et al., 2008). Iceberg is used to convey various ways of mathematical representation to support formal mathematical understanding (Webb et al., 2011). The iceberg tip represents the formal mathematics to be addressed or understood. In this
section, the student's reasoning level reaches the level of formal mathematics. Then at the bottom of the end of the iceberg or below the surface of the water, there is a much larger volume of ice. This section contains various informal and pre-formal mathematical representations related to formal mathematics to be understood.

Basically, the ice part of the bottom of the water surface is the substance of learning or is the basis for understanding the mathematics represented at the end of the iceberg. This section covers reasoning with a related context (informal) and using models, representation tools, and strategies (pre-formal). The ice of the lower part of the water surface is divided into several parts; the most basic part contains the context that supports the informal reasoning of the student; in this part, the level of reasoning of the student is at the situational level. Then the middle part between the base and the ice tip above the surface, representation models, and pre-formal strategies related to formal mathematics are to be understood. In this section, the level reaches the referential level or 'model of,' and one level above it again reaches the general level (Gravemeijer, 1994). The purpose of the pre-formal section is to provide a larger mathematical structure. This section provides a way for students to make connections between the context of the problem and the abstraction of symbolic mathematics and more formal procedures (Webb & Abels, 2011).

**The use of pupil’s own creation and contributions**

Students use their creations and contributions to do concrete things. Then, students reflect on their learning process (Gravemeijer & Terwel, 2000). Students will demonstrate greater initiative when encouraged to build and develop their own solutions (Streefland, 1991).

**The interactive character of the teaching process or interactivity**

In the instructional process of RME learning, to reach a formal level, the interaction between students and teachers or between students and students is an important part (Gravemeijer & Terwel, 2000; Zulkardi, 1999, 2002). The process of questioning, answering, discussion, reflection, cooperation, and evaluation can give rise to thinking strategies that arise from the students themselves. These informal processes can usher in students' thinking strategies to be able to reach the formal level (Zulkardi, 2002). This can minimize students' dependence on the teacher to tell whether their answers or thoughts are right or wrong so that students' independence and confidence in mathematics will also develop (Gravemeijer & Terwel, 2000; Zulkardi, 1999).

**The intertwining of various mathematics strands or units**

A holistic approach that combines different mathematical units is essential in learning. This will make it easier for students to apply mathematics to solve real-life problems (van den Heuvel-Panhuizen & Drijvers, 2020). It is because real-life problems often cannot be solved only with one mathematical unit but with the integration or combination of various units (Zulkardi, 2002).
Discussion

Construction of Ethno-RME curriculum

To determine the pedagogical actions of teachers in mathematics learning using Ethno-RME, a guide is needed that can guide teachers to be able to determine the context of ethnomathematics to be used in learning, determine student achievement competencies, and determine the methods used to achieve these competencies and assessments that can measure the achievement of these competencies. Before discussing the learning procedures, it is necessary to emphasize first the goals or objectives and competencies of learning Ethno-RME.

Goal of Ethno-RME

The beginning of Ethno-RME was initiated to answer problems regarding applying ethnomathematics in mathematics learning in schools. Ethno-RME was initiated by combining two theories, Ethnomathematics and Realistic Mathematics Education, which have similar goals and substance (Prahmana, 2022). Ethno-RME and RME have the same goal, that students can gain mathematical knowledge from their daily experiences and culture, but the emphasized methods are different (D'Ambrosio, 2007; Gravemeijer & Terwel, 2000). Ethnomathematics emphasizes the use of culture as a learning context because, in essence, mathematical science has existed and is developed by culture itself to face problems and phenomena in the reality of life and emphasizes the internalization of values in the culture so that it can form a good and ethical character of mathematician in living (Prahmana, 2022). In comparison, Realistic Mathematics Education emphasizes the use of the real context around students, the process of mathematization, and the student’s level of thinking. Thus, the combination of Ethnomathematics and Realistic Mathematics Education or referred to as Ethno-RME, has a big goal, building an efficient and correct understanding of mathematical concepts by the mathematical process and students’ level of thinking and internalizing socio-cultural values contained in the experience and culture around students so that they can form a good and ethical character of mathematics users in living (Prahmana, 2022; Prahmana & Istiandaru, 2021).

Ethno-RME competences

To achieve this goal, there are three competencies that students will achieve according to the trivium D'Ambrosio curriculum ideas about ethnomathematics: literacy, matheracy, and technoracy.

1. Literacy

The first competency is literacy which is the ability of students to be able to integrate the cultural context in the school with the cultural context in society through cultural dynamics that allow students to exchange academic knowledge and local knowledge (D'Ambrosio, 1999; Rosa & Orey, 2015). To build this ability, the teacher can determine the topic or context of ethnomathematics first that will be explored by the student. Furthermore, the teacher guides students to obtain information about the context and process and integrates the cultural context obtained with the cultural context in the school to exchange academic and local knowledge. It is in this process that students' literacy abilities are developed. For example, in a study conducted by Risdiyanti and Prahmana (2021), teachers chose the topic or context of Javanese puppets, which became the culture of the Javanese society in Indonesia. In learning, the teacher guides students to get information about Javanese puppets contained in student books and from stories presented by teachers. Then guide students to be able to process and express
information by inviting students to have a dialogue about the information obtained. In this process, students' literacy abilities are awakened and will continue in subsequent learning activities that are triggered by questions or dialogues between teachers and students.

2. *Matheracy*

Then the second competency is *matheracy* which is a skill, strategy, and competence that triggers students to realize how they explain their culture, as well as interpret, recognize, and manipulate signs, symbols, codes, and the use of mathematical models in daily life (D'Ambrosio, 1999; Rosa & Orey, 2015). This can help students develop their creativity and analytical skills to understand and solve problems (D'Ambrosio & D'Ambrosio, 2013). To develop mathematical abilities, the teacher needs to guide students to be able to use, translate and model cultural artifacts from events and phenomena found in students’ daily lives into codes or symbols. For example, in the research of Risdiyanti and Prahmana (2021), the teacher guides students to be able to translate and model the arrangement of Javanese puppets to form a model that can be symbolized in the form of mathematical symbols. In the Javanese puppet arrangement on *kelir*, there are groups of Javanese puppets with good characters and puppets with evil characters. The good character puppet group consists of the Pandhawa five puppet array and other good puppet figures, while the evil character puppet group consists of the Kurawa puppet array and other evil figures. Students then model the puppet arrangement into a set of good puppet figures, a set of evil puppet figures, a set of *Wayang Pandhawa lima*, and a set of *wayang* Kurawa figures. In this process, students' mathematical abilities can be developed. Students can understand and explain their Javanese puppet culture and interpret and model in the form of sets.

3. *Technoracy*

The third competence is *technoracy* which is the ability of individuals to critically use and combine technologies and their ability to evaluate possibilities and limitations in diverse everyday situations to make appropriate decisions for themselves and others (D'Ambrosio, 1999; Rosa & Orey, 2015). Technoracy is an important figure of knowledge to translate and interpret matters related to the natural, social, cultural, political, and economic environment. In ethnomathematics, a cultural group's ideas, ways, and mathematical techniques can be interpreted analytically, geometrically, graphically, and others using appropriate mathematical techniques and technologies (Rosa & Orey, 2015). As technology advances, mathematics develops the power to project a reality for the future.

The use of technology is inevitable to improve the quality of life (D'Ambrosio, 1999, 2007). This technology can help individuals process modeling by representing, integrating, and generalizing using technology (Rosa & Orey, 2010). For example, in the Risdiyanti and Prahmana (2021) research, after students explore and process information about *wayang* culture and model it in the form of a set, the teacher then guides students to be able to critically realize that Javanese people interpret their ideas, methods, and techniques in conveying stories and messages of life using technology in the form of Javanese puppets. Students are then also guided by the teacher to be able to interpret and interpret their findings using miniature Javanese puppet technology arranged on miniature *Kelir* based on mathematical modeling that students have made before.

There is no significant difference between literacy, matheracy, and technoracy in the D’Ambrosio Trivium Curriculum with the competencies in the OECD because D’Ambrosio participated in constructing mathematical literacy in the OECD 2003 (François, 2016). At that time, D’Ambrosio proposed a political proportion that mathematics education should be accessible to all students and not just to people who have privileged (François, 2016). Then OECD gave a report that interpreted the
D’Ambrosio proposed in the form of literacy competencies, defined as the capacity to identify, understand, and engage in mathematics and make reasoned judgments about the role of mathematics, as necessary for an individual’s current and future personal life, work life, social life with peers and relatives, and life as a constructive, caring and reflective citizen (OECD, 2003). It reflects a form of literacy which is the human right of every child to have the opportunity to participate in the world fully, constructively, relevantly, and wisely. Then, D’Ambrosio emphasized the proposal by creating a trivium curriculum that contains literacy, matheracy, and technoracy in the ethnomathematics perspective. Of the three forms of basic competence, D’Ambrosio wants to fulfill the Declaration of Human Rights, which relates to the right of education and the right of scientific knowledge benefit (François, 2016). The ethno-RME curriculum adopts the three basic competencies in the trivium curriculum which are in line with Human Right Declaration and OECD. In the trivium curriculum and Ethno-RME curriculum, the definition of literacy, matheracy, and technoracy have been adapted to the ethnomathematics perspective, but basically, they are the same definition as in the OECD (Prahmana, 2022).

**Ethno-RME learning procedure**

The Ethno-RME learning procedure consists of several steps that are: *first*, determining the context of ethnomathematics; *second*, exploring and processing information about the context of ethnomathematics; *third*, finding mathematics in the context of ethnomathematics; *fourth*, conducting self-development models; *fifth*, conduct critical reflection as an assessment, as follows:

1. **Determining the context of ethnomathematics**

In learning mathematics using Ethno-RME, the first step that must be done by teachers and students is to determine the context of ethnomathematics, which is a context that is a student culture close to students' daily lives and can develop students’ literacy, material, and technocratic abilities. The goal is that the learning can depart from the student's life experience so that students will find it easier to understand and strengthen their minds. This follows the principle in RME that the starting point of mathematics learning must be based on real experiences or phenomena and that students can engage in contextual situations (Gravemeijer, 1994; Larsen, 2018).

Students’ formal mathematical activity must be obtained from abstract things (Cobb, 1994; Zulkardi, 2002). In Ethno-RME, abstract things are in the form of cultural forms such as ideas, activities, or cultural artifacts. The process of conceptual mathematization seeks to allow students to explore situations, find and identify relevant mathematical elements, create schemes, and visualize sequentially to find patterns and develop models that generate mathematical concepts (Gravemeijer, 1994; Larsen, 2018; Zulkardi, 1999).

2. **Exploring and processing information about the ethnomathematical context**

Furthermore, students are guided by the teacher to extract and process information about the context. In this process, literacy abilities are developed, and teachers guide students to obtain information about the context and process and integrate the cultural context obtained with the cultural context in the school to exchange academic and local knowledge (Prahmana, 2022). The processing information can be triggered by the teacher by asking questions to the student or conducting a dialogue related to the context of ethnomathematics that has been explored by the student. The process of asking, answering, discussing, reflecting, collaborating, and evaluating can bring up thinking strategies that emerge from the students (Gravemeijer & Terwel, 2000). These informal processes can deliver students thinking strategies to achieve the formal level (Zulkardi, 2002). The process of literacy development does not
only stop here but continues in subsequent activities, and literacy abilities will be developed continuously through dialogues or student presentations.

3. Finding mathematics in the context of ethnomathematics

Then students are guided by the teacher to be able to understand how the ideas, methods, and techniques of culture are interpreted in the form of culture or the context of ethnomathematics that the student explores. Students are guided to be able to explain their culture, interpret, recognize and manipulate signs, symbols, codes, and the use of mathematical models used in their culture or cultural forms explored by these students. In this process, mathematical abilities are developed, and students can use, translate, and model cultural artifacts from events and phenomena in their daily lives into codes or symbols. In other words, students find their mathematics in their own culture. This follows the principle of RME, which is guided reinvention. Students can find mathematics by themselves by starting a thought experiment, imagining a step in which they can find mathematics (Gravemeijer & Terwel, 2000). Guided reinvention allows learners to assume the knowledge they acquire is their own knowledge for which they are responsible (Gravemeijer & Terwel, 2000; Treffers, 1993). In this process, the process of progressive mathematization begins, and the student's thinking moves from real problems to formal mathematics. The mathematical process will lead students to progress in thinking from one level to the next (Treffers, 1987). The process of developing mathematical skills and this mathematization process will continue and be encouraged by teachers through dialogues.

4. Conducting self-development models

In the mathematical process, students are guided to be able to do self-development models or model development independently. Students can model independently from concrete situations to situations or to formal mathematical forms (Freudenthal, 2006). So, from the context of ethnomathematics that students have explored, they are then identified as related to mathematical concepts and developed by students themselves into a model or formal mathematical form. Students develop their own model in the first method, modeling a situation close to the student's world, then generalizing and formalizing the model will turn into a model which will then shift to a model for similar problems, and finally, the model for that becomes formal knowledge of mathematics (Freudenthal, 2006).

Similar to RME, the modeling level in Ethno-RME is the first four levels, the situational level, which is the level at which specific situational knowledge and strategies are used in the context of the situation; second, the referential level or “model of”, the level at which the model and strategy refer to the situation described in the problem; the general level or model for is the level at which the mathematical focus on the strategy dominates the reference to the context; the formal mathematics level is the level at which a person works with conventional procedures and notation (Gravemeijer, 1994). The level can be illustrated as an iceberg or iceberg to support formal mathematical understanding. The iceberg tip represents the formal mathematics to be addressed or understood. The student's reasoning level in this section reaches the formal mathematics level. Then at the bottom of the end of the iceberg or below the surface of the water, there is a much larger volume of ice. This section contains various informal and preformal mathematical representations related to formal mathematics to be understood. In this modeling process, not only mathematical capabilities are developed but also technocratic capabilities. In ethnomathematics, a cultural group's mathematical ideas, methods, and techniques can be interpreted analytically, geometrically, graphically, and others using appropriate mathematical techniques and technologies. In Ethno-RME, technocracy is seen as the ability of students to understand
the technoracy resulting from modeling developed by a cultural group or the ability of students to interpret the modeling, they do use technology.

5. Conducting critical reflection as an assessment

Furthermore, students are guided by the teacher to do a critical reflection on the mathematical activities they do. Critical reflection is defined as awareness of the reasons behind emotions, actions, and insights, including the conscious and unconscious motives of the individual behavior (Ghanizadeh et al., 2020). Humans must move from routine action to reflective action which is characterized by continuous self-assessment and development (Dewey, 1993). In the reflection process, doubt is the starting point for reflection and the key to learning from the contemplated problem (Dewey, 1993; Ghanizadeh, 2017). According to Farrell (2012), being able to think reflectively is an essential goal in education because it involves intelligent thoughts and actions. In the Ethno-RME, critical reflection is part of the learning assessment to find out the extent to which students can understand the mathematical context and internalize the values present in that context. This critical reflection can be done through dialogue between teachers to students or through the activity of writing reflections on student activity sheets or student books. Critical reflection has two points that are reflected. The first is a reflection on the mathematical concepts found, and the second is a reflection on cultural values impregnated from the context of ethnomathematics used. In the first critical reflection, the teacher can assess how to understand mathematical concepts and the achievement of literacy, material, and technocratic competencies in students. Then on the second reflection, the teacher can assess the extent to which the sociocultural values of the context of ethnomathematics used are impregnated and internalized by the students. The two reflections also assess whether the learning objectives using Ethno-RME are achieved.

This Ethno-RME curriculum contributes to facilitating teachers in implementing Ethno-RME in mathematics learning in schools. Thus, Ethno-RME can contribute to learning to make it easier for students to understand mathematical concepts that follow the mathematical process and students' level of thinking and can form mathematics users who have good character and ethics in living. However, further research needs to be carried out regarding implementing the Ethno-RME curriculum in mathematics learning in schools.

Conclusion

RME was initiated to answer problems regarding applying ethnomathematics in mathematics learning in schools. Ethno-RME was initiated by combining two theories, namely Ethnomathematics and Realistic Mathematics Education which have similar goals and substance. Ethno-RME aims to build an efficient and correct understanding of mathematical concepts by the mathematical process and students' level of thinking and internalize the socio-cultural values contained in the experiences and cultures around students to form a good and ethical character of mathematicians in living. The Ethno-RME learning procedure consists of several steps that are first, determining the context of ethnomathematics; second, exploring and processing information about the context of ethnomathematics; third, finding mathematics in the context of ethnomathematics; fourth, conducting self-development models; fifth, conducting critical reflection as an assessment. This Ethno-RME curriculum makes it easier for teachers to implement Ethno-RME in mathematics learning in schools. However, further research needs to be carried out regarding implementing the Ethno-RME curriculum in mathematics learning in schools.
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