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Features of teaching supplements designed to help primary teachers reduce student misconceptions in mathematics

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Abstract

The findings of teacher misconceptions of mathematics at various levels indicate a wide variation of mathematics taught to students. This research aims to produce valid, practical, and effective teaching supplementary materials for integers, fractions, two dimensional and three-dimensional shapes that can overcome misconceptions. This two-year development research applied Plomp's development phases: preliminary investigation, design, realization, test phase, evaluation, and revision. The data were analyzed from teacher misconceptions about mathematics from the first year of the study, the development of teaching material supplements, the results of observations on the implementation of teaching materials, the teacher's response questionnaire to the implementation of the teaching materials, and test results. The research subjects were primary school teachers in Sidoarjo who experienced mathematical misconceptions. The results showed that the teaching material supplements developed met valid, practical, and effective criteria based on expert validation, teacher responses, and teacher work results on the assessment sheets. The features of the supplements were developed based on cognitive conflict and the resolution of conflicting perspectives and the teachers' existing ideas and extend them, through, for example, the analogy to a new domain, where those are presented in either the materials or the assessments.

Keywords: student misconception; teacher misconception; teaching supplement

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Introduction

There is a consensus that misconceptions can act as barriers to learning mathematics. It is defined as ideas that are inconsistent with current scientific views, arising from students' previous learning, as students' conceptions that result in a pattern of systematic errors (Smith et al., 1993). Olivier (1989) reports that misconceptions arise from beliefs and principles in cognitive structures that underlie systematic conceptual errors. Swan (2001) states that misconceptions come from developing concepts or generalizations of concepts by students, rather than their wrong thinking. Furthermore, Swan (2001) argues that misconceptions may be a natural stage in conceptual development. Therefore, students' misconception should not be considered their failure. In addition, Zembat (2010) reported that misconceptions may also be triggered by teaching methods. In addition, student misconceptions can also occur because of the misconceptions experienced by the teacher so that when teaching, the teacher conveys his misconceptions about certain materials to students. In this case, the conceptual knowledge of teachers and the teaching environment they create is important.

The existence of conveying the incorrect teacher's conception during learning causes students to experience misconceptions about the topic being taught (Mutambara & Bansilal, 2022; Sbaragli & Santi, 2011). This is by believing that the conception possessed by a teacher is correct first. If the initial concept owned by the teacher itself is wrong, the concepts that will be taught to students will also be wrong and this results in new concepts that will be taught by the teacher being irrelevant. Furthermore, if someone experiences a mathematical misconception on a particular topic during the first lesson and does not immediately get a new experience that can correct the misconception, it will have an impact on misconceptions on related material at the time of subsequent mathematics learning (Flevares & Schiff, 2014).

Findings of teachers' misconceptions about mathematics material at various levels indicate a wide variation in terms of the mathematics material taught to students. Kusmaryono et al. (2019) found the misconceptions of primary school teachers in three types of misconceptions, namely preconception, undergeneralization, and modeling errors in numbers and fractions. For example, Iriyani (2019) showed that primary school teachers who were this study's samples experienced misconceptions on numbers and geometry. On the topic of numbers, misconceptions were found related to the competence of performing mixed operations on integers, adding, and subtracting integers, determining fractions between two fractions, and converting fractions from one form to another. On the topic of geometry, misconceptions were found to be related to the competence to understand the concepts of perimeter and area of flat shapes, to understand the elements of geometric shapes and webs of plane shapes, understand the relationship between surface area and volume of flat shapes, and to understand the relationship between surface area. and volume of curved side space. Astawa et al. (2020) and Palupi et al. (2022) also found some learners' misconceptions when using incorrect procedures of mathematical equations and inequality.

The results of the first-year research specifically provide information about the teacher's misconceptions on the topic of curved side geometry. For example, an angle can be defined as a measure of the rotation of a line segment from one starting point to another (Sbaragli & Santi, 2011). In addition, in a regular two-dimensional form, the angle can also be interpreted as the space between two intersecting straight-line segments. Meanwhile, the definition of a corner point in a three-dimensional shape is a meeting between several edges. The misconception found is that the teacher defines the vertex on the curved side of the cone as the angle formed by the curved side (Tsamir et al., 2015). The teacher teaches the students that a cone has 1 vertex, this is not in accordance with the definition of a vertex in a geometric figure. On the topic of numbers, the misconceptions found were related to the teacher's error in understanding the concept of the sequence of operations on numbers where the teacher did not pay attention to the power of operations on numbers and only saw the order of operations located in the questions (Jabal & Rosjanuardi, 2019).

The search for possible causes of misconceptions has been investigated to find the main problem as the basis for giving treatment both in the preparation of learning designs and teaching materials used. Khalid and Embong (2019) found that the cause of a person experiencing misconceptions is shallow understanding, which is most likely caused by teachers rushing to complete an extensive syllabus, and as a result, students using memorization rules. Kusmaryono et al. (2019) emphasize the possible causes of misconceptions, namely misinterpretation of new experiences through misunderstandings that interfere with the ability to understand new information correctly, and belief in the truth of a wrong understanding in students. In line with this, Permata and Wijayanti (2019) add that the cause of a person's misconceptions can also be caused by wrong intuition and incomplete reasoning in understanding certain concepts.

To answer the challenge of improving the understanding of primary school teachers on mathematical topics, relevant learning resources are needed that can facilitate teachers to understand mathematical material conceptually and procedurally well. This learning resource is expected to be able to minimize the causes of teacher misconceptions as described above. Among the learning resources in question are textbooks which are supplements given to teacher training in primary schools. This is necessary considering the limited availability of teaching materials that explicitly provide features that can teach teachers to avoid possible causes of misconceptions.

Cognitive conflict is a psychological state that involves differences between cognitive structures and experiences or between various cognitive structures (i.e., mental representations that govern knowledge, beliefs, values, motives, and needs). This discrepancy occurs when simultaneously, incompatible representations compete for a single response (Waxter & Morton, 2011). In education in general, (e.g., Swan (2001)), and in mathematics education in particular, cognitive conflict is seen as a drive to learn. Some studies explicitly use cognitive conflict as a strategy to find students' ideas related to various mathematical concepts, such as division (Tirosh & Graeber, 1990), average, and measurement (Shahbari & Peled, 2015; Surgandini et al., 2019; Watson et al., 2007).

The approaches used to compile teaching materials to reduce teacher misconceptions have been applied. For example, Wanner (2019) uses hands-on activity-based problems with the help of puzzle game media to understand the teacher about the concept of area and perimeter of flat shapes. Çavuş and Deniz (2022) use a technology-assisted realistic approach to build teacher understanding of the transformation geometry material. Lucariello and Naff (2013) suggest conducting learning activities that can measure and build learners' preconceptions, present new concepts, and theories for learners, bridge understanding through analogies, build model-based reasoning, build an understanding of students' metacognition, and provide cognitive conflict. While the existing learning resources for assisting teachers to reduce their potential misconceptions about mathematics (Kajander & Lovric, 2009; Yang & Sianturi, 2017), limited resources found to employ cognitive conflict strategies as the main tool to reduce those potentials. Previous research has shown the effectiveness of using cognitive conflict strategies to reduce students' and teachers' misconceptions about mathematics (Parwati & Suharta, 2020; Susilawati et al., 2017), but how those strategies are written up in a series of activities in the learning resources such as books or teaching supplements for teachers are not reported. Thus, this research aims to develop teaching supplements that present the features based on cognitive conflict strategies through activities that focus on minimizing the factors that cause misconceptions among primary school teachers in mathematics.

Methods

This is development research that aims to develop valid, practical, and effective teaching supplements for primary teachers to reduce students' misconceptions of mathematics. The four phases of Plomp's development model (Plomp, 1997), which is called the general model of solving educational problems, were used for the development processes: preliminary investigation, design, realization, test phase, evaluation, and revision. This development model was chosen as it incorporates cutting-edge knowledge from prior research into the design process and fine-tunes educational innovations based on field testing (Nieveen et al., 2006). The design principles that might inform future development and implementation decisions are produced by unpacking the design process of this teaching supplement so that it opens opportunities for future researchers regarding the findings of this development study.

Phases of developing teaching materials

Phase 1: Preliminary Investigation

The activities in the first phase of the model concern the execution of a preliminary investigation. It is sometimes called needs analysis. In this investigation, important elements are gathering and analyzing information, defining the problem, and planning the possible continuation of the project.

This phase was a needs analysis phase or problem analysis which includes (1) collecting problems found currently in the field, (2) identifying information, (3) analyzing information, (4) defining problems, and (5) planning follow-up activities.

Phase 2: Design

The activities in this phase have the objective of designing a solution for the problem as it has been defined in the preliminary investigation. The result of the design phase is a design document, a blueprint of the solution. Overall, the design development of the teaching supplements is a systematic process in which the complete problem is being divided into partial problems and in which the partial solution continuously related to the complete solution structure.

In this phase, a solution to the problem defined in the initial investigation is designed. The result of this phase is in the form of a design document, which is a grid of solutions. The design made is a systematic process, namely by dividing the big problem into small problems, with each problem-solving design. Furthermore, all forms of solutions were collected and linked into a complete solution structure.

Phase 3: Realization/Construction (Realization/Construction)

On the basis of the detailed design, a first version or prototype was produced. In this phase, the first version of the design called the draft, was made.

Phase 4: Testing, Evaluation, and Revision (Test, Evacuation, and Revision)

This phase is at judging the quality of the developed solution and taking continuation decisions based on the result of that judgment. Evaluation is the process of systematically collecting, processing, and analyzing to assess the value of the realized solution. Without evaluation, it cannot be determined whether a problem has been solved satisfactorily.

In this phase, the quality of the solution (design) developed was considered and then a sustainable decision was made based on the results of careful consideration. Evaluation is the process of assessing the solution, that is, determining whether the design specifications have been met. Without evaluation, it cannot be determined whether the problem has been satisfactorily resolved or not. Revision is a follow-up activity from the evaluation. Then back to the activities of design, realization, and so on. This cycle is called the feedback cycle.

Phase 5: Implementation

Only when it has been concluded, based on the evaluation that a satisfying solution for the problem has been developed, it can be implemented, in other words, carried through in the situation in which the problem actually occurs.

In this phase a solution has been generated which is based on the evaluation. This solution fits the problem at hand. Thus, this solution (design) can be implemented, in other words, it can be applied in situations where the problem occurs.

The five phases are described by Plomp (1997) as follows in Figure 1 below.

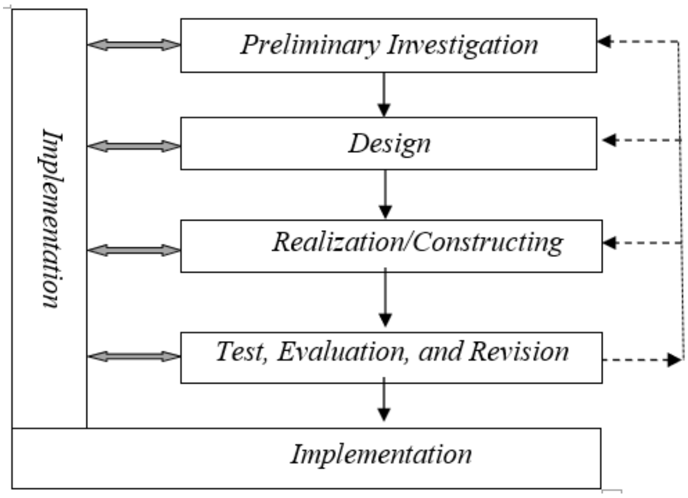


Figure 1. Phases in the developmental research

The criteria for the final draft of teaching materials in this study refer to criteria of Nieveen et al. (2006), namely validity, practicality, and effectiveness. Nieven's illustration of the relationship between the three quality aspects discussed above can be presented in Table 1.

Table 1. Nieven's illustration of the relationship between the three aspects of quality (validity, practicality, and effectiveness)

	Quality Aspects		
	Validity	Practicality	Effectiveness
Representation	Intended (Ideal + Formal): • state-of-the-art • internally consistent	Consistency between: • intended ↔ perceived • intended ↔ operational	Consistency between: • intended ↔ experiential • intended ↔ attained

The third quality indicator from Nieveen et al. (2006) in Table 1, is then used as a basis for determining indicators of the quality of teaching materials to overcome elementary teachers' misconceptions about mathematics topics. This is in line with Nieveen's opinion that the quality criteria of good learning materials can be identified from the extent to which such materials can provide theoretical background information resulting from the development process, demonstrate the practical meaning of the proposed changes, and stimulate discussion of educational change among teachers using the materials. Thus, we consider the three aspects of quality: validity, practicality, and effectiveness to measure the quality of the learning materials so that they are also applicable to a much wider range of educational products.

Regarding the assessment of the quality of textbooks, as the form of teaching supplements developed in this research, Giyatmi (2016) mentioned eleven criteria, i.e. books that could generate interest in reading, are designed and written considering students' prior knowledge and psychological points of view, present clear learning objectives, are used by lecturers and students in the learning processes, are flexible, systematic and structured based on the needs of students and the final competencies to be achieved, focus on providing opportunities for students to practice, present clear summaries, use communicative writing style, promote teachers' feedback, accommodating students' learning difficulties, and present obvious learning guide on how to study the learning materials. Those criteria became fundamental principles to develop teaching supplements. The experts were then asked to score the quality of the teaching supplements based on those criteria and give the score for each of the criteria from 1 to 5. The total score was then divided by all the criteria to get the average score of validity (AV) resulting from the experts' judgments.

$4 \leq AV \leq 5$: very valid

$3 \leq AV < 4$: Valid

$2 \leq AV < 3$: less valid

$1 \leq AV < 2$: not valid

The research instruments that were used in this study (second year) were expert validation sheets. These instruments were used to examine the validity of the teaching materials developed. Meanwhile, the observation sheets were used to examine the effectiveness of implementing the teaching materials as a strategy to overcome misconceptions in elementary school teachers. On the other hand, questionnaires were used to find out the teacher participants' responses about the teaching materials used to address the existing misconceptions, while the mathematics tests were used to identify the possible misconceptions about the topics of numbers, integer operations, and two-dimensional and three-dimensional figures at the preliminary phase.

Results

The process and results of the development of teaching material supplements

The development of teaching material supplements to overcome mathematical misconceptions among elementary school teachers in this study was carried out based on the Plomp development model. The process and results of the development of teaching material supplements are described as follows.

Preliminary investigation phase

At this stage, an analysis of the supporting theory of supplementary teaching materials was carried out as an analysis of the results of the first year's research related to the misconceptions of elementary school teachers on the topic of mathematics based on the mathematics test. In summary, these results are presented in Table 2.

1. Analysis of teacher misconceptions

Table 2. Teacher misconceptions from the first year's research

No	Topics	Findings
1	Integer operations	The teachers did not notice the importance of the nature of ordering operations on integers.
2	Application of integer operations	The teachers are weak in the concept of distance where distance calculations are presumed to be always positive.

No	Topics	Findings
3	Find a fraction between two fractions	The teachers added up the two fractions to determine a fraction between two fractions
4	Converting decimal (repeating) fractions to rational numbers	The teachers made a mistake on the concept of multiplying decimal fractions due to the understanding that all repeating fractions cannot be rational numbers.
5	Application of perimeter and area of flat shapes involving algebraic forms	The teachers made a mistake in the operation of the algebraic form used to determine the circumference and area of a flat shape
6	Root number operations	The teachers experienced a misconception about converting numbers in the form of roots to decimal numbers
7	Cube Nets	The teachers only know at most 4 cube nets out of 11 nets
8	Elements of solids	The teachers were wrong in determining the number of edges and vertices on cones and cylinders. Fooled by the image build space in R2
9	The relationship between surface area and volume of a flat side shape	When given the volume of a cube, the teachers were incorrectly determining the cube's surface area. The teacher made a mistake in determining the length of the edge, namely taking the cube root of a number
10	The relationship between surface area and volume of curved side shapes	The teacher made a mistake in solving the problem that the surface area of the cone that was worked on was equal to the area of the conical blanket.
11	Applications related to the surface of the flat side of the room	Determine the surface area of a geometric shape with trapezoidal walls. The teacher made a mistake in constructing a room that was always closed, so he immediately used the existing formula

The results of the first year's research presented in Table 2 revealed that many elementary school teachers in Sidoarjo still had misconceptions about the material in learning mathematics. Most misconceptions are made by teachers when determining the elements in cones and cylinders. In addition, the teacher also made many mistakes in determining the surface area of a cone. The teacher made a mistake in determining the area of a polyhedron that has bases in the shape of a trapezoid. The study results show that one of the most common misconceptions among elementary school teachers is the elements of curved-sided spatial construction (69.2%). The misconception is that the tube has no edges because the edges must be straight lines if drawn on R2. This shows that the definition of ribs (as an initial concept) has not been well understood.

2. Analysis of teaching materials

Analysis of teaching materials tailored to the needs to reduce student misconceptions. This adjustment is written in the form of learning objectives written in each textbook. The learning objectives set are as follows.

a. Understand the concept of integers and their operations
b. Design learning of integers and their operations
c. Design an integer assessment and its operations
d. Understand the concept of perimeter and area of flat shapes
e. Designing learning that can reduce student misconceptions regarding the circumference and area of flat shapes
f. Designing assessments related to the circumference and area of flat shapes

Figure 2 shows the concept map of the developed teaching material supplements.

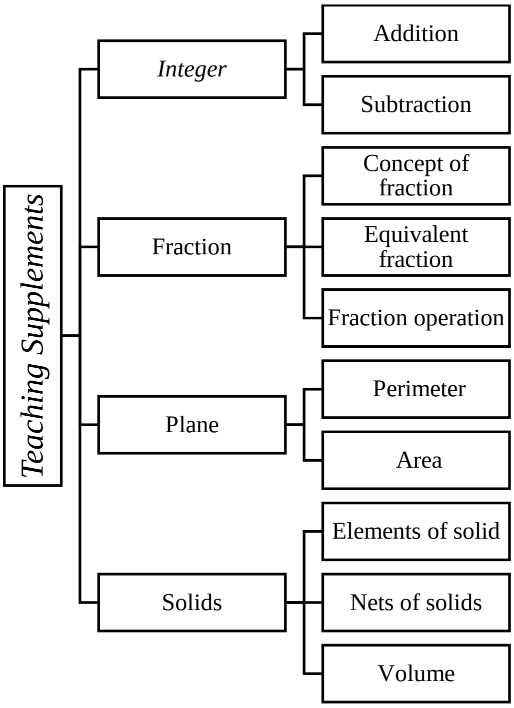


Figure 2. A concept map of teaching material supplements

Furthermore, based on the results of the analysis of this phase, the researcher obtained important elements in identifying problems that could be used as capital for compiling supplementary teacher teaching materials that support the implementation of a more optimal mathematics learning process.

Design phase

Based on the analysis of the preliminary investigation phase, the researcher prepared the design of teaching materials and research instruments.

1. Preparation of teaching material supplements

The student teaching materials developed in this study are teaching material supplements for elementary schools. The preparation of this book begins with studying the content of integers, fractions, plane shapes, and geometric shapes. The next step is to make an outline that is adjusted to the components of the book, namely the opening, content, and closing. The opening section consists of a book cover, preface, table of contents, and book cover. The content section consists of the learning objectives presented in the material for each sub-chapter which is equipped with a learning design for teaching the material presented, examples of assessment models for the material, discussion of problem models, correcting student misconceptions, examples, and practice questions.

2. Preparation of research instruments

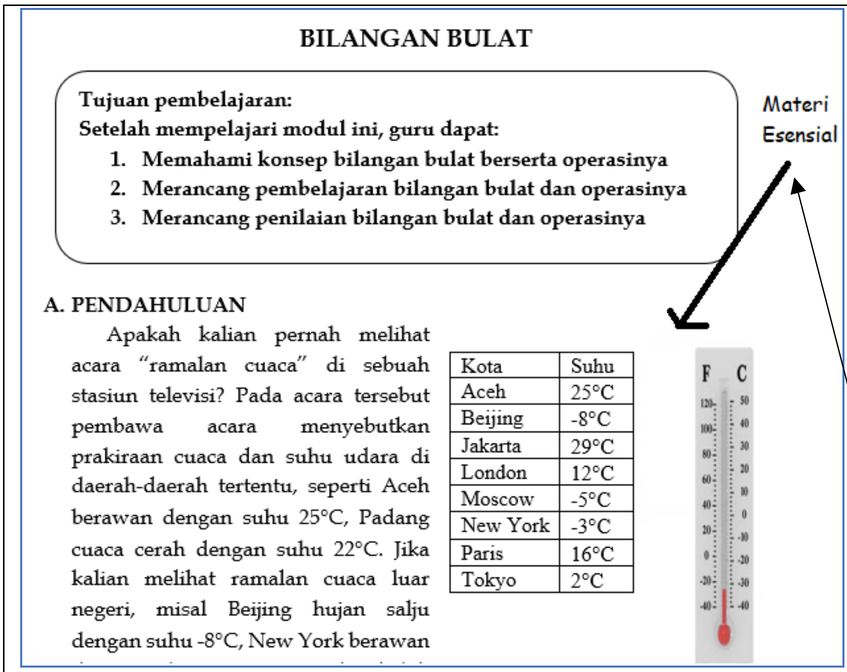
The research instruments developed in this study consisted of teaching material validation sheets and teacher knowledge test sheets. This validation sheet is used to obtain validity data for teaching material supplements.

Realization phase

This phase is a continuation of the design phase. Based on this phase, the design that has been made is used as the basis for making teaching material supplements and research instruments known as prototype 1. In this phase, the teaching material supplements that have been produced have not been validated by the validator, but before being validated, teaching material supplements and research instruments were consulted with a supervisor. The following is a description of prototype 1.

1. Teaching materials supplement

Teaching materials are made based on the format that has been prepared in the design phase. Some of the features related to this teaching material supplement are presented in Figure 3.



BILANGAN BULAT

Tujuan pembelajaran:
Setelah mempelajari modul ini, guru dapat:
1. Memahami konsep bilangan bulat berserta operasinya
2. Merancang pembelajaran bilangan bulat dan operasinya
3. Merancang penilaian bilangan bulat dan operasinya

Materi Esensial

A. PENDAHULUAN

Apakah kalian pernah melihat acara "ramalan cuaca" di sebuah stasiun televisi? Pada acara tersebut pembawa acara menyebutkan prakiraan cuaca dan suhu udara di daerah-daerah tertentu, seperti Aceh berawan dengan suhu 25°C, Padang cuaca cerah dengan suhu 22°C. Jika kalian melihat ramalan cuaca luar negeri, misal Beijing hujan salju dengan suhu -8°C, New York berawan

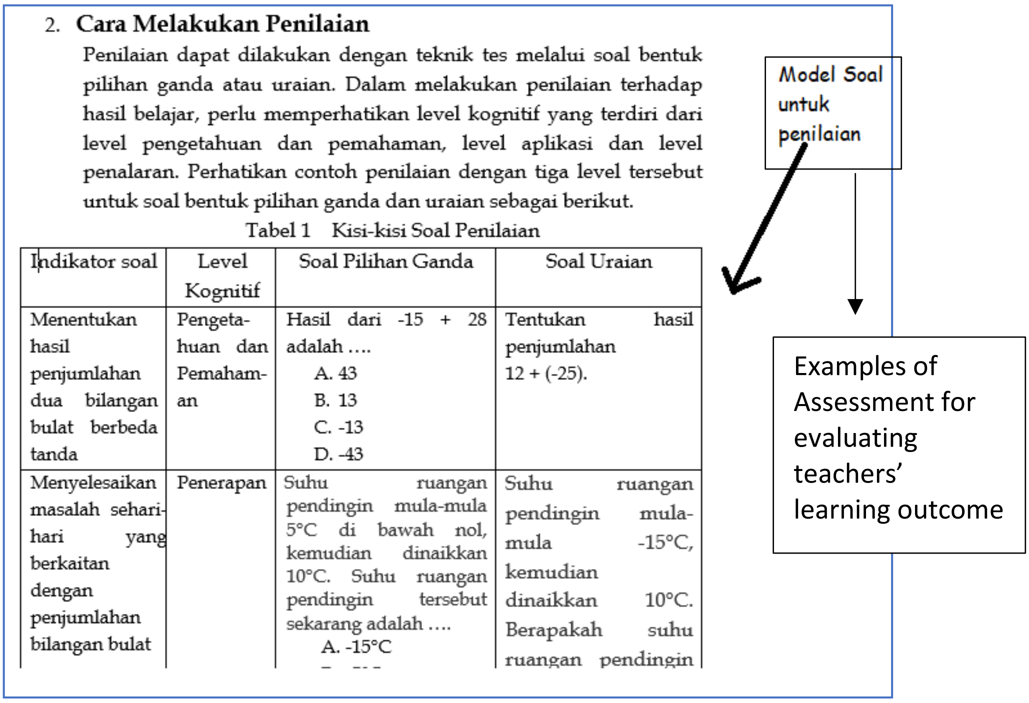
Kota	Suhu
Aceh	25°C
Beijing	-8°C
Jakarta	29°C
London	12°C
Moscow	-5°C
New York	-3°C
Paris	16°C
Tokyo	2°C

Integers

Learning objectives:
After learning this module, teachers are expected to:
1. understand the concept of integer and its operations
2. Design learning for integer and its operation
3. Design the assessment for learning integer and its operation

Essential Material

Figure 3. Presentation of an essential topic



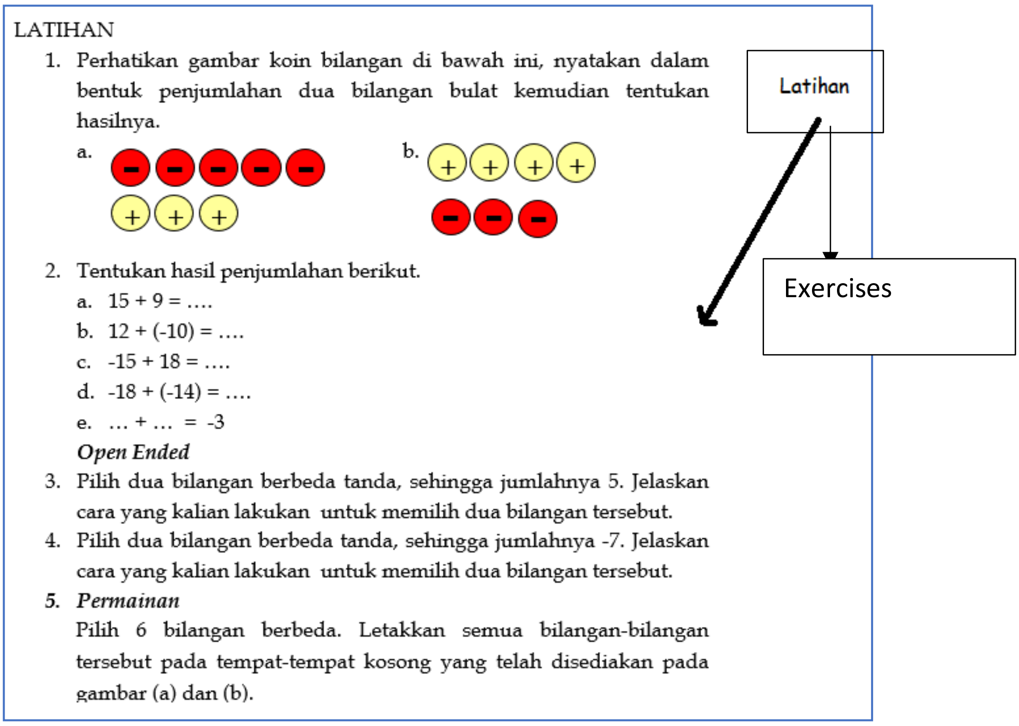
2. **Cara Melakukan Penilaian**

Penilaian dapat dilakukan dengan teknik tes melalui soal bentuk pilihan ganda atau uraian. Dalam melakukan penilaian terhadap hasil belajar, perlu memperhatikan level kognitif yang terdiri dari level pengetahuan dan pemahaman, level aplikasi dan level penalaran. Perhatikan contoh penilaian dengan tiga level tersebut untuk soal bentuk pilihan ganda dan uraian sebagai berikut.

Tabel 1 Kisi-kisi Soal Penilaian

Indikator soal	Level Kognitif	Soal Pilihan Ganda	Soal Uraian
Menentukan hasil penjumlahan dua bilangan bulat berbeda tanda	Pengeta-huan dan Pemaham-an	Hasil dari -15 + 28 adalah …. A. 43 B. 13 C. -13 D. -43	Tentukan hasil penjumlahan 12 + (-25).
Menyelesaikan masalah sehari-hari yang berkaitan dengan penjumlahan bilangan bulat	Penerapan	Suhu ruangan pendingin mula-mula 5°C di bawah nol, kemudian dinaikkan 10°C. Suhu ruangan pendingin tersebut sekarang adalah …. A. -15°C	Suhu ruangan pendingin mula-mula -15°C, kemudian dinaikkan 10°C. Berapakah suhu ruangan pendingin

Figure 4. Assessment model for evaluating teacher's misconception



LATIHAN

1. Perhatikan gambar koin bilangan di bawah ini, nyatakan dalam bentuk penjumlahan dua bilangan bulat kemudian tentukan hasilnya.

2. Tentukan hasil penjumlahan berikut.
   a. 15 + 9 = ….
   b. 12 + (-10) = ….
   c. -15 + 18 = ….
   d. -18 + (-14) = ….
   e. … + … = -3

   Open Ended
3. Pilih dua bilangan berbeda tanda, sehingga jumlahnya 5. Jelaskan cara yang kalian lakukan untuk memilih dua bilangan tersebut.
4. Pilih dua bilangan berbeda tanda, sehingga jumlahnya -7. Jelaskan cara yang kalian lakukan untuk memilih dua bilangan tersebut.
5. ***Permainan***
   Pilih 6 bilangan berbeda. Letakkan semua bilangan-bilangan tersebut pada tempat-tempat kosong yang telah disediakan pada gambar (a) dan (b).

Figure 5. Exercise

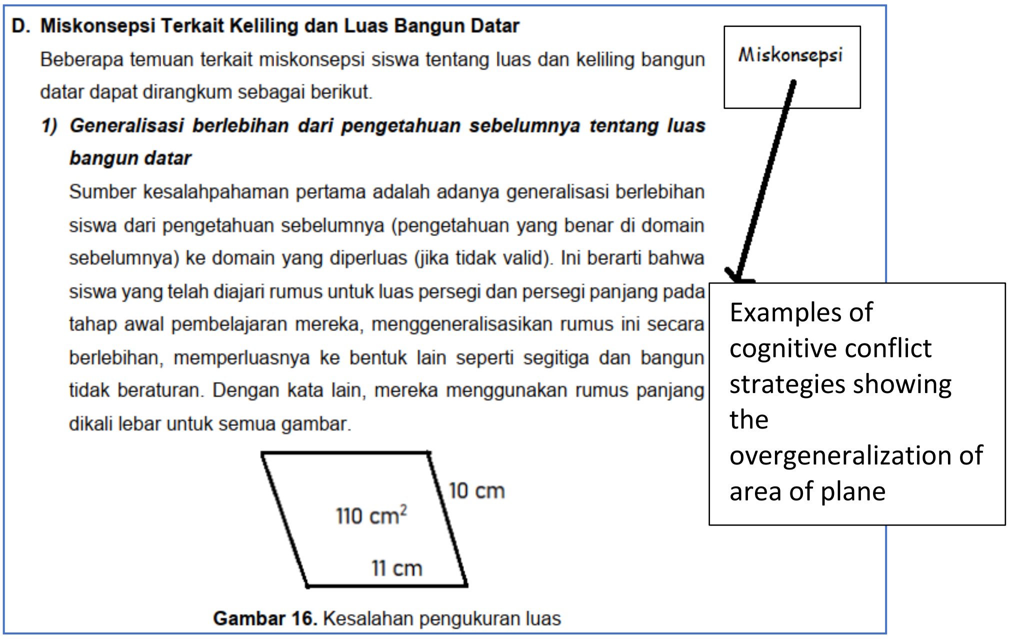


Figure 6. Cognitive Conflict Features

2. Research instruments, namely tools used to obtain research data consisting of teaching material supplement validation sheets; assessment sheets which were also developed as teaching material supplements, and teacher response questionnaires.

Test, evaluation, and revision phase

In the implementation of the test, evaluation, and revision phases, two stages were carried out, namely teaching material supplement validation and teaching material supplement testing. In the phase of validation, the validation was carried out by supplementary teaching material experts produced in the realization phase. The teaching material supplements which are part of prototype 1 were then given to the experts for validation. The results of the validation of teaching material supplements are presented in the following description. The validation results are presented in Table 3.

Table 3. Validation results for the teaching supplements

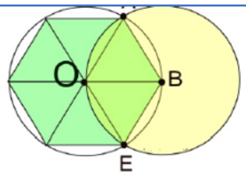
No	Assessment Aspects	I	II	Mean	Interpretation
I	Format				
	1. Clarity of material distribution	4	4	4.00	very valid
	2. Demonstrate a balance between text and illustrations	4	4	3.00	valid
	3. Visually appealing	4	5	4.50	very valid
	4. Use the appropriate font and font size	4	4	4.00	very valid
	5. Numbering system is clear (using a mixture of numbers and letters)	4	5	3.50	valid
	6. Space arrangement/layout is clear	4	4	4.44	very valid
	7. Consistency in the distribution of material	4	4	4.00	very valid

No	Assessment Aspects	I	II	Mean	Interpretation
II	Language				
	1. Conformity of language with correct English grammar	4	4	4.00	very valid
	2. The English used is simple and easy for students to understand	3	4	3.50	very valid
	3. Clarity of instructions/directions related to the completion of a task/problem	4	3	3.50	very valid
	4. Communicative language (raising intimate communication with students)	4	4	4.00	very valid
III	Illustration				
	1. Support to make the material easier to understand	4	4	4.00	very valid
	2. Have a clear view	4	4	4.00	very valid
	3. Easy to understand	4	4	4.00	very valid
	4. Associated with the student environment	4	4	4.00	very valid
	5. Provide visual stimulation	5	4	4.50	very valid
IV	Content				
	1. Truth content	4	4	4.00	very valid
	2. Consistency in the distribution of material	4	4	4.00	very valid
	3. Load related information scattered in the components of the book	4	4	4.00	very valid
	4. Load appropriate exercises to measure students' understanding of the sub-chapters being studied	4	4	4.00	very valid
	5. Contains examples or practice questions that invite students to think critically	4	4	4.00	very valid
	6. Facilitating teachers to overcome misconceptions in students	3	4	3.50	valid
	7. Facilitating teachers to teach material to students	4	4	4.00	very valid
	8. Examples of learning given are in accordance with the development of thinking of elementary students	4	5	4.50	very valid
	9. Facilitating teachers to provide an overview of conducting assessments of students	4	4	4.00	very valid
	AV			**4.02**	**very valid**

Based on Table 3, the average total validity criteria of teaching materials are in the very valid category. This is in accordance with the validity criteria for supplementary teaching materials which are in the category of very valid. Thus, it can be concluded that the student book can be used during trials on the condition that several components of the teaching material supplement have been revised.

In addition to providing an assessment, the validators also provided suggestions for improvements to the student books being developed. These suggestions include improving English grammar in the balance of text and illustrations, choosing common vocabulary, and choosing common mathematical symbols for students. Furthermore, these suggestions were followed up by researchers by revising teaching material supplements. The following descriptions explain some examples of student book revision results.

1. Language improvement



Gambar 11

Perhatikan Gambar 11. Gambarlah lingkaran O. Dari titik B, pada lingkaran, gambar lingkaran lain dengan pusat di B, dan jari-jari OB. Perpotongan dua lingkaran di A dan E membentuk segitiga-segitiga sama sisi ΔAOB dan ΔEOB karena mereka terdiri dari 3 jari-jari kongruen. Rentangkan jari-jari yang membentuk segitiga-segitiga ini melalui lingkaran O untuk membentuk segienam beraturan dengan 6 segitiga sama sisi.

Keliling segienam adalah enam kali jari-jari lingkaran. Tetapi dapat dilihat bahwa keliling lingkaran sebenarnya "lebih besar" dari batas ini. Dengan demikian, keliling lingkaran harus lebih dari 6r. Keterangan: Dalam K = 2π • r, temuan menunjukkan bahwa π harus sedikit lebih besar dari 3, yaitu 2π r> 6r ⇒ 2π r> 2 • 3r ⇒ π> 3.

Windows User
Lingkaran dengan pusat titik O.

Windows User
?? apa maksud rentangkan jari jari??

Windows User
Kata "Tetapi" dihilangkan saja, krn tdk ada pertentangan... Langsung saja: Dapat dilihat bahwa....

Windows User
Bagian ini justru ingin menurunkan rumus keliliang linhkaran....

Figure 7. Review regarding the language issue

Revision

Consider Figure 11. Draw a circle with center O. From point B, on the circle, draw another circle with a center at B, and radius OB. The intersection of two circles at A and E forms equilateral triangles ΔAOB and ΔEOB because they consist of 3 congruent radii. Extend the spokes forming these triangles through circle O to form a regular hexagon with 6 equilateral triangles.

The perimeter of the hexagon is six times the radius of the circle. The circumference of a circle is actually "larger" than this limit. Thus, the circumference of the circle should be more than 6r. Note: In K = 2π • r, the findings show that π must be slightly larger than 3, ie 2π r > 6r ⇒ 2π r > 2 • 3r ⇒ π > 3.

2. The coherence of presenting mathematical contents

Perhatikan bahwa OC adalah panjang dari tinggi segitiga sama sisi, membuat ∠OCA sudut yang siku-siku, dan membentuk ΔAOC kongruen dengan ΔBOC. Untuk kemudahan perhitungan, misalkan panjang jari-jari, AO, dalam diagram ini sama dengan 12 unit. Kita tahu bahwa AB = OA (radius), jadi AC, yang merupakan setengah dari AB, sama dengan 6.

Catatan:
Di bagian mana akhirnya dapat menurunkan rumus keliling lingkaran??

Penurunan Rumus Luas Lingkaran

Untuk mempelajari bagaimana rumus luas lingkaran diperoleh. Perhatikan ilsutrasi pada rangkaian proses yang ditunjukkan oleh rangkaian gambar berikut.

(Review Note, indicated by the yellow-highlighted part: At what part these stages can finally end up with the circle circumference)

Figure 8. Review the presentation of the math content.

Revision

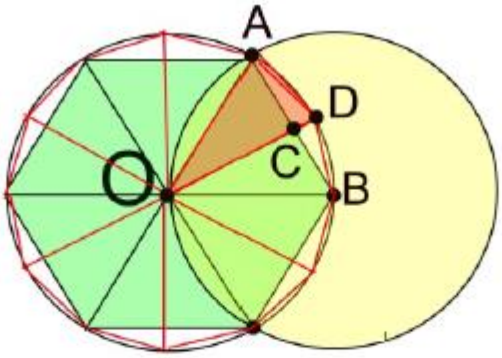
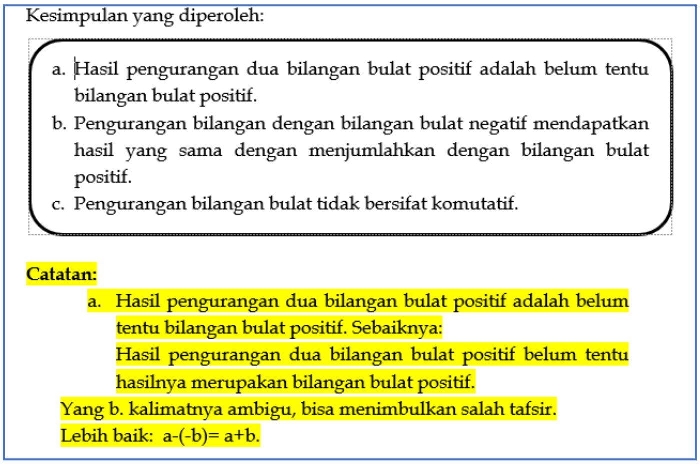


Figure 12

To approximate the true value of π, divide each of the central corners of the hexagon in half to get the sides of the 12-sided figure. See Figure 12. It shows getting closer to the true circumference of the circle.

Note that OC is the length of the height of an equilateral triangle, making ∠OCA a right angle, and forming ΔAOC congruent with ΔBOC. For ease of calculation, suppose that the radius, AO, in this diagram equals 12 units. We know that AB = OA (radius), so AC, which is half of AB, equals 6.

3. Mathematical accuracy



Kesimpulan yang diperoleh:

a. Hasil pengurangan dua bilangan bulat positif adalah belum tentu bilangan bulat positif.
b. Pengurangan bilangan dengan bilangan bulat negatif mendapatkan hasil yang sama dengan menjumlahkan dengan bilangan bulat positif.
c. Pengurangan bilangan bulat tidak bersifat komutatif.

Catatan:

a. Hasil pengurangan dua bilangan bulat positif adalah belum tentu bilangan bulat positif. Sebaiknya:
Hasil pengurangan dua bilangan bulat positif belum tentu hasilnya merupakan bilangan bulat positif.

Yang b. kalimatnya ambigu, bisa menimbulkan salah tafsir.
Lebih baik: a-(-b)= a+b.

(Review note, indicated by the yellow-highlighted part: a) The subtraction of two positive integers is not always positive and vice versa)

Figure 9. Review regarding the accuracy of mathematical content

Revision

a. The result of subtracting two positive integers is not necessarily a positive integer.
b. The result of subtracting a positive integer, say a with a negative integer, say -b is equal to the sum of two positive numbers a+b
c. Subtraction of integers is not commutative.

Practicality and effectiveness test results

Practicality test results

Table 4 presents the results of the practicality test with an assessment of the teacher's response (30 teachers) to the developed teaching materials.

Table 4. Practicality test results

Aspects	Sub Aspects	Rating Scale (Number of responses)				
		1	2	3	4	5
Format	Clarity of material distribution			2	10	18
	Shows a balance between text and illustrations			1	6	23
	Visually appealing			3	13	14
	Using the appropriate font and font size				10	20
	Clarity of numbering system (using a mix of numbers and letters)				4	26
	Clear room arrangement/layout			2	14	14
Language	The suitability of the language used with correct Indonesian grammar			1	12	17
	The Indonesian language used is simple and easy for students to understand				13	17
	Clarity of instructions/directions related to the completion of a task/problem			2	8	20
	Communicative language (generates friendly communication with students)				12	18
Illustration	Support to make material easier to understand			1	8	21
	Have a clear display			4	12	14
	Interesting presentation				11	19
	Connect with the student environment			2	15	13
	Provide visual stimulation			1	5	24
Contents	Accuracy of content				12	18
	Consistency in the distribution of material				15	15
	Load-related information scattered in book components				13	17
	Load appropriate exercises to measure students' understanding of the sub-chapters being studied				21	9
	Contains examples or practice questions that invite students to think critically				8	22
	Facilitating teachers to overcome misconceptions in students			1	11	18
	Facilitating teachers to teach material to students			3	13	14
	Learning examples given are in accordance with the development of thinking of elementary students				14	16
	Facilitating teachers to provide an overview of conducting assessments of students			1	21	8
General Evaluation						
Criteria		**A**	**B**	**C**	**D**	**E**
		3	**27**			

Table 4 indicates that most of the teacher participants (27 out of 30) examine that the teaching materials are practically used to understand and reduce possible misconceptions in mathematics. Note that the general evaluation of the teaching supplements is categorized into five categories: A for practical with no revisions required, B for practical with minor revisions required, while C and D respectively indicate that the supplements are practical with major revisions and almost all aspects are required. E, on the other hand, for no practical.

Comments and improvement suggestions

1. The module is in accordance with integer material. The module presentation is accompanied using learning media and how to teach material that is considered difficult, and mistakes often occur. The practice questions vary with a lot of involving the context around students.
2. Practice questions appear that invite students to think critically and involve games so that they make students feel happy
3. It is necessary to use colorful pictures so that students are happy to see them
4. Explanation of solving the example questions on pages 15 and 17 so that the origins of each step are clarified

Effectiveness

To see the effectiveness of teaching material supplements that have been developed, it can be seen based on the results of teachers' understanding in solving problems related to misconceptions experienced before and after studying teaching material supplements. Teaching material supplements were given to 30 elementary school teachers who had misconceptions (upper and lower grades). Based on the results of the teacher's work, it can be identified that there are still misconceptions that occur after the teacher studies supplementary teaching materials. The misconceptions that occur can be detailed in Table 5.

Table 5. Teacher participants' responses on the assessment sheet for misconception

No	Topics	Total Number of Responses from teachers	Early Grade Teachers	Upper Grade Teachers
1.	Application of the volume of a flat side shape (story problems)	16	10	6
2.	Root number operations	9	6	3
3.	Converts decimal (repeating) fractions to rational numbers	8	5	3
4.	The relationship between surface area and volume of curved side shapes	4	2	2

Based on the data above, it shows that there are still misconceptions about mathematics among elementary school teachers. Most occur in applications related to the volume of flat side shapes presented in the form of word problems, which is as much as 53%. Next, misconceptions occur in the number operations in the form of roots, as much as 30%. While the misconceptions about converting decimal (repeating) fractions into rational numbers and the relationship between surface area and volume of a flat side shape are 26.4% and 13.2%, respectively.

Discussion

The results of this study indicate that the supplementary teaching materials developed are valid, practical, and effective for use as learning resources for elementary school teachers to reduce their misconceptions about basic mathematical material or to serve as inspiration for them to design teaching materials that can prevent students from having mathematical misconceptions. The hallmark of this teacher supplement to teaching is the application of a conceptual change framework, namely in terms of student learning paths from certain issues of their pre-instructional conceptions to mathematical concepts (Gooding & Metz, 2011). This is because there are many cases known from the literature in that students understand the views of mathematics in the right way but do not believe in them (Treagust et al., 2000). Therefore, conceptual changes must be instilled into conditions that support the development of students' ideas. This module provides features that allow students to voice their ideas and to exchange views with other students, where students' ideas are generally considered as a serious attempt to make sense of certain phenomena by the teacher. Compared to previous similar learning resources that aim to reduce the potential of learners' misconceptions (e.g., Kajander and Lovric (2009)), the learning resources developed in this study were deliberately designed using the ideas of implementing conflict cognitive strategies. While the use of these strategies is proven successful to attract the teacher participants' positive responses, the teaching supplement could also assist them improve their knowledge about misconceptions and how to reduce these in their classroom teaching. This is essential due to these two types of knowledge are encouraged to be mastered as parts of their pedagogical content knowledge, especially knowledge of content and students and knowledge of the content of teaching (Ball et al., 2008).

In many of the new teaching and learning strategies available in the literature, instruction begins by generating students' ideas and by building on their experiences with the phenomenon in question. The method used in this supplement developed by utilizing cognitive conflict-based tasks, although generally superior to the traditional oriented approach (Carr, 2010), suffers from a number of difficulties. Most importantly, it is often difficult to get students to see the conflict. It may also be the case that elicitation and lengthy discussion of students' pre-instructional views may reinforce these views alone. Hence, there is a search for strategies that avoid cognitive conflict, that is, those that start from the perspective of students' pre-instructional conceptions that share at least some basic problems with views of mathematics, and from this core of conformity progress to views of mathematics through an essentially continuous path.

Conclusion

The results of research resulted in a set of teaching materials to help reduce misconceptions about mathematics in four chapters, namely integers, fractions, plane shapes, and geometric shapes for elementary school mathematics. The results of trials starting from the initial investigation phase, design, realization, testing, evaluation, and revision showed that the teaching material supplements developed met valid, practical, and effective criteria. These results are based on the results of expert validation, teacher responses, and the results of the teacher's work on the assessment sheet provided. Supplemental teaching materials produced in this study can be used as a teacher to enrich knowledge about school mathematics material, especially at the elementary level. The features developed are useful for reducing teachers' misconceptions about math material, especially the material for integers, fractions, plane shapes, and geometric shapes. However, future research should be able to develop teaching material supplements that are more complete by covering more and more material components for each material developed.

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Conflicts of Interest

The authors declare no conflict of interest regarding the publication of this manuscript. In addition, the ethical issues, including plagiarism, misconduct, data fabrication and/or falsification, double publication and/or submission, and redundancies, have been completed by the authors.

Author Contributions

Endah Budi Rahaju: Conceptualization, research methodology, writing—original draft, writing—review, and editing revised version. **Dwi Iriyani:** Data gathering, formal analysis, review, validation, and supervision. **Ahmad Wachidul Kohar:** Methodology, review and editing, formal analysis.

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