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Students' Mathematical Proofing Ability in Number Theory Material Based On Argumentation Theory

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Abstract

Mathematical argumentation is a part of problem-solving and reasoning that aims to convey ideas and convince others. This involves building conclusions, providing warrants, and supporting conclusions with proof and reasoning. This study aims to determine the involvement of aspects of Toulmin's argument, which consists of data, claims, warrants, backing, rebuttals, and qualifiers in mathematical proving in number theory material. The research subjects were mathematics education students who had taken a number theory course at a university in Aceh Province. The study used a qualitative method with a case study design: students' mathematical proving self-efficacy and proving abilities were grouped according to self-efficacy in the high, medium, and low categories. Collecting data using self-efficacy questionnaires and mathematical proof test instruments, the data triangulation used was an in-depth interview. The results of the study revealed that the self-efficacy of mathematical proving is linear with students' mathematical proving abilities. The involvement of the complete argumentation aspect in high self-efficacy students, while the medium and low self-efficacy student groups have not been able to prove statements, and the involvement of the argumentation aspect is incomplete.

Keywords: mathematical proof; mathematical argumentation; proving self-efficacy; number theory

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Introduction

Higher Education National Standards (SNPT) have set competency standards for university graduates. This standard covers achievement in knowledge, especially through reasoning activities in the learning process, to master concepts and theories in depth and systematically (Kemendikbud, 2020). In addition, SNPT also explains that learning must be interactive, effective, and focused on students through problem-solving activities. Effectiveness in this context means that students must be able to internalize material properly and correctly, according to scientific and rational principles. Furthermore, college graduates are expected to be able to solve problems in their field of expertise by analyzing information and data and ensuring their validity to reach conclusions. They are also expected to be able to formulate ideas, thoughts, and scientific arguments responsibly and in accordance with academic ethics and be able to communicate them using scientific procedures. Reasoning activities, problem-solving, mastery of in-depth concepts and theories, as well as effective communication by students can be achieved through various types of activities, one of which is argumentation.

Mathematical argumentation aims to convey ideas and convince others that the idea is rational. This involves building conclusions, providing warrants, and supporting conclusions with proof and reasoning. This definition aligns with the explanation in the Encyclopedia of Mathematics Education by Sriraman and Umland (2020), which states that argumentation in mathematics is an argumentative process that aims to reach conclusions through a series of reasoning based on mathematical knowledge. Sriraman and Umland also added that this argument is useful for showing and explaining the problem-solving process. Staples and Conner (2022) define mathematical argumentation as a set of processes for making mathematical claims and providing evidence to support them. In this case, one's argument can be supplemented with definitions, theorems, or results whose truth is known to develop and verify the truth of a mathematical problem (Fukawa-Connelly & Silverman, 2015). Because argumentation in mathematics is closely related to learning, argumentation ability is one of the goals in the process of learning mathematics. NCTM (2000) reveals that the objectives of learning mathematics include: (1) problem-solving, (2) argumentation and reasoning, (3) communication, (4) connection, and (5) representation.

In an effort to communicate mathematical ideas and provide evidence that confirms the validity of these ideas, we need a scheme that can be used to identify the structure of an argument. One of the argumentation schemes recommended by Toulmin is often used. Toulmin (1958) recommends an analysis of the argumentation structure, which consists of: claims (C), data (D), warrants (W), backing (B), qualifiers (Q), and rebuttal (R). In mathematical argumentation, warrants are used as a basis for mathematical knowledge, verification, validation, interpretation, patterns and methods, calculations, visualization, informal mathematical knowledge, and other information supporting these claims (Conner in Dede, 2019). This argumentation scheme helps establish a consistent line of thinking and provides a solid basis for conveying mathematical ideas and demonstrating their correctness.

The way a person schemas an argument can reveal the quality of that argument. The proving process in mathematical problems can illustrate this, as Fukawa-Connelly and Silverman (2015) mentioned, that the validity of mathematical proof can be evaluated through argumentation. Arguments and evidence are interrelated and cannot be separated. Aberdein and Dove (2013) support this view by saying that evidence can be divided into two types, formal and informal, and argumentation is included in informal evidence. Arguments are considered evidence when they meet the criteria of valid statements, correct reasoning, and representations in accordance with the related concepts (Gutiérrez et al., 2016). A math teacher needs to have good argument skills in order to be able to explain math material effectively to his students. In addition, the demand for mathematics teachers is to facilitate students to be able to argue mathematically through oral and written communication so that students can understand the extent of understanding of the concepts that have been received. However, in reality, in general, students' abilities in reasoning, especially in the field of argumentation, have not achieved satisfactory results.

Several researchers, such as Hamdani and Subarinah (2020), have found that students still experience difficulties in constructing evidence in almost all courses that require proving and argumentative reasoning. According to Hamdani and Subarinah, this problem is caused by students' lack of understanding regarding the integration of mathematical concepts such as axioms, definitions, lemmas, and theorems that can help build new knowledge. Similar findings were also reported by Kwon et al. (2015), who found that students still had difficulty connecting argumentation structures to produce final claims in math problems related to partial derivatives. In addition, Sadieda (2019) reported that only about 27% of students were able to understand concepts well during four semesters, especially in the algebraic structure course. According to Sadieda, this problem has an impact on students' lack of ability to reason and present arguments effectively.

Research on mathematical proofs that require student arguments has not been widely carried out, especially in the performance of students who have different abilities. The heterogeneity of these abilities makes the quality of the arguments produced have their own characteristics, especially in mathematical proof. Thus, it is necessary to have an in-depth review of mathematical proof based on Toulmin's argumentation theory to see the performance of students with good, sufficient, and poor abilities. A person's mathematical performance can be predicted by self-efficacy or self-confidence. A person's belief in his ability to organize and carry out a series of actions needed to complete a particular task is called self-efficacy (Bandura, 1997). Self-confidence greatly influences the results of students' mathematical proof construction, and it is not uncommon to find that a person is still unsure of his proof construction abilities (Viholainen et al., 2019). Herizal (2021) reveals that self-efficacy is an important key component in efforts to carry out mathematical proof. In addition, confidence in reasoning is also important in supporting the ability to prove and achieve the desired goals. Measuring students' self-efficacy in proving is important because proof construction is the practice of understanding mathematical concepts and how mathematical knowledge is built. This can be used as evaluation material for lecturers to improve future-proof learning, design appropriate designs, and place more emphasis on argumentation-based learning.

Methods

The purpose of this study was to evaluate students' mathematical proving abilities based on argumentation theory which was influenced by their level of mathematical self-efficacy. Therefore, this study uses qualitative methods. Qualitative methods are used to understand and explore the meaning of individuals or groups of people involved in social or humanitarian issues (Creswell, 2017). This study uses a case study design because the study focuses on the case of student self-efficacy and its relation to the ability to prove mathematical problems and argue. This approach is in line with the views of Johnson and Christensen (2014), which state that case study designs are used to examine in detail some instances in qualitative research.

Mathematics education students taking a number theory course at a university in Aceh Province are the subject of this research. Number theory was chosen as the material because the material has a lot to do with proof, such as theorems, lemmas, and practice questions. Data collection was carried out using a mathematical proving test technique and a non-test instrument in the form of a mathematical proving self-efficacy questionnaire to see the level of student self-efficacy. To confirm the data obtained, this study also used data triangulation through in-depth interviews with research subjects. The following guidelines are used to categorize student self-efficacy levels.

Table 1. Guidelines for Levels of Self-Efficacy

Interval	Category
$x < \bar{x} - \frac{1}{2}s$	Low
$\bar{x} - \frac{1}{2}s \leq x \leq \bar{x} + \frac{1}{2}s$	Moderate
$x > \bar{x} + \frac{1}{2}s$	Good

Source: Budiyono (2015)

Meanwhile, argumentation theory to structure student proof construction uses the following Toulmin argumentation scheme:

Table 2. Toulmin's Argumentation Scheme

Argumentation Aspect	Operational
Data	Students are able to organize facts, manipulate, or summarize existing information explicitly or implicitly.
Claim	Students are able to provide final claims or sub-claims regarding the part to be proven in the statement to support arriving at the final claim.
Warrant(s)	Students are able to provide guarantees/support for the answers they write, namely to make a bridge between the facts and the conclusions to be reached.

Backing	The student is able to provide additional collateral either in support of the warrants that have been disclosed.
Rebbutal	Students are able to reject statements and explain the conditions under which these statements do not apply
Qualifier	Students are able to select all the things that have been disclosed/written whether there are errors or not so that the final conclusion can be accepted.

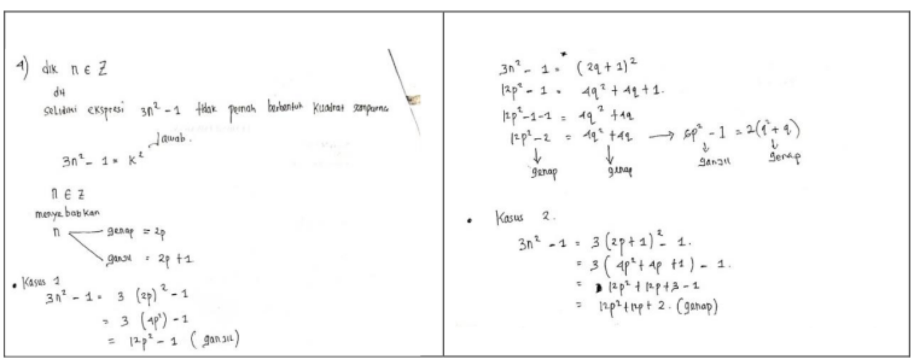
Results

In the first stage, students are given a questionnaire to measure the level of mathematical self-efficacy in the context of proof via the Google Forms website. The results of the questionnaire were then processed using Microsoft Excel with the successful interval (MSI) method and grouped using the guidelines proposed by Budiyono (2015). The following is a summary of the results of students' mathematical self-efficacy.

Table 3. Results of Student Self-Efficacy in Mathematical Proof

Interval	Level	Freq.	%
$X_i \geq 86.5$	Good	14	32.55%
$71 \leq X_i < 86.5$	Moderate	15	34.88%
$X_i < 71$	Low	14	32.55%
Total		**43**	**100%**

Based on Table 3. The number of students who enter each category tends to be similar. The results of the analysis were then reviewed from the results of the answers given by the students and taken by the subject representatives at each level, namely one subject with a good level of self-efficacy, one subject with a moderate level of self-efficacy, and one subject with a low level of self-efficacy. Each of these levels will be described qualitatively using argumentation theory. The following is an analysis of mathematical proving data based on student self-efficacy.



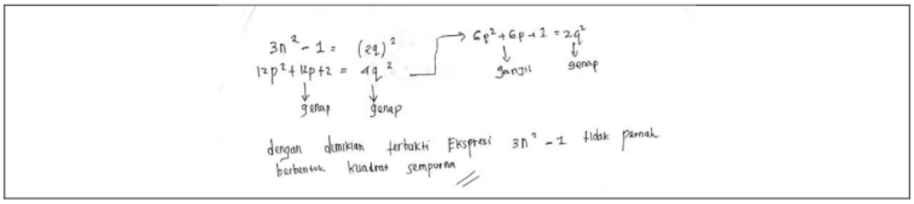


Figure 1. Proof of Mathematical Self-Efficacy in Good Category

In the data aspect, students have been able to convey the information contained in the problem, namely $n \in \mathbb{Z}$, when asked through interviews students can also convey that Z it means the set of integers. Students also found implicit information, namely modeling perfect squares into k^2. On the aspect of claims, through interviews students convey that form $3n^2 - 1$ are never perfect squares, and this can be achieved by circumstantial proof, i.e., proof of contradiction. In proving claim he has made, the student submits several guarantees or warrants, warrants for the contradictory evidence he constructs in the form of: 1) division of cases for n when n is even in the form $2p$, and when n is odd in the form $2p + 1$, in interviews the students explained that the divisions when combined would form a set of integers $\mathbb{Z}$ as referred to in the question. The student's answer is then included in the backing category because it can explain the second layer of the first warrant. In addition, it guarantees $6q^2 - 1$ is odd and $2(q^2 + q)$ is even, 2) for n is eveb, student found $3n^2 - 1$ odd, so k^2 odd implies k odd, student subtitutes $k = 2q + 1$ into the equation. The same is also obtained for cases n odd, so that the warrant given by the student is appropriate for the final claim he wants to prove, it's just that at the initial stage of proof the student does not state in writing that he will prove by contradiction. In the rebbutal aspect, students find a contradiction in the statement $6p^2 - 1 = 2(q^2 + q)$ and $6p^2 + 6p + 1 = 2q^2$ in each case investigated. Students explain that on the left side, the result will always be odd, while on the right side, the result will always be even, so a situation is found where the statement does not apply. This aspect is a rebuttal that is sought by students in proving using the contradiction method. In addition, when viewed from the results of his work and compared with the results of interviews, it is found that students have been able to select the results they have constructed from start to finish and are convinced that all the steps are sufficient to prove the final claim that they want to prove, students realize that they need to declare it first. The method it will use before directly exemplifying $3n^2 - 1 = k^2$. The student's reflection and belief are his ability in the qualifier aspect. Because in the qualifier aspect, the student does not find any more wrong parts and needs to be added/reduced, the student concludes in general that the claim he is aiming for is true and the proof is complete.

From the description regarding the mathematical proving of students with good self-efficacy categories, a conclusion can be drawn that students have brought up all aspects of argumentation, namely data, claims, warrants, backing, rebuttal, and qualifiers. From the construction results, the proof is conceptually correct. Students prove with the indirect proof method, namely proof of contradiction, through a problem-solving approach, namely for cases for even and odd numbers so that two different parities are found, which are seen as a rebuttal; furthermore, students are able to select all of their work and find no errors except for the initial declaration proof. Next, we will describe students' mathematical proving abilities with moderate self-efficacy categories.

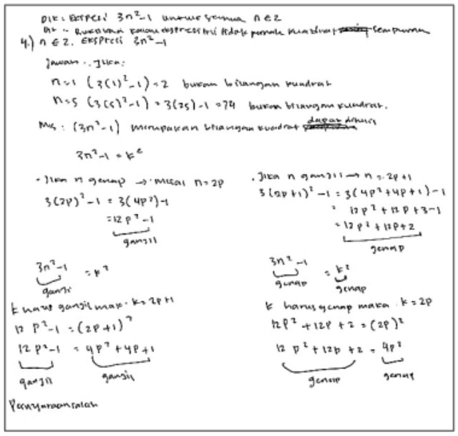


Figure 3. Moderate Category Student Self-Efficacy Mathematical Proof

In the data aspect students write statements that are known namely "expression $3n^2 - 1$ for all $n \in \mathbb{Z}$", confirmation interview students know that n is the set of integers. Furthermore, Students provide claims that they want to prove, namely expressions $3n^2 - 1$ never perfect square. In bridging existing data and claims that the truth/false wants to prove, students give inductive warrants first as shown in Figure 3, students try to provide warrants for $n = 1$ obtained is not a square number, then for $n = 5$ also obtained is not a square number. Furthermore, because the student has not found a denial, he proves it with indirect evidence, which can be seen from the proof he constructs in the sentence "let $3n^2 - 1$ is a square number" which is the negation of the problem. To find the contradiction, students divide the case into two parts, namely when n even, $n = 2p$ and when n is odd, $n = 2p + 1$ which is a deductive warrant. When asked why he divided n into two parts, the student said that working with integers can be made easier with odd and even parities, different from real numbers. This indicates that it has the ability to bring up the backing aspect in support of the deductive warrants it provides. In the qualifier aspect, students have not been able to select all the steps in the process, for example, in the even n case, in the statement $12p^2 - 1 = 4p^2 + 4p + 1$, students stated that both sides were odd, but for $n = 2$, this statement does not apply, and also, students write odd parity on both sides, but this still needs to show a contradiction of something, especially if both sides are odd. It is not sure that the value is the same. Thus, the qualifier aspect has not been able to be maximally raised by students because they have not been able to select all of the results. In addition, students make claims that the statements given are wrong at the end of the settlement. The aspect of the qualifier that the student was unable to bring up made the rebuttal aspect also not appear because he was unable to reject the statement and explain the conditions in which the statement does not apply (for example, two odd numbers, the quantity is not exactly the same). Thus, the results of the evidence provided are not true.

From the description above, students with a moderate level of self-efficacy have come up with several aspects of argumentation in the mathematical proof they make, namely regarding data, claims, warrants, and backings. Three main aspects of the argumentation, namely data-claim-warrant, have been raised. At the beginning, the students tried first for some small values of n to look for denials but were not found, then continued with warrants dividing the even-odd cases. It's just the qualifier aspect, and the rebuttal that it should have raised has not been well seen. Students have found ideas to construct proofs of these mathematical statements through indirect evidence, namely the contradiction method. Unfortunately, students have not found the intended contradiction and at the end claim that the statement is wrong when it should be true. Furthermore, it will be studied regarding the mathematical proving abilities of students who have less self-efficacy levels.

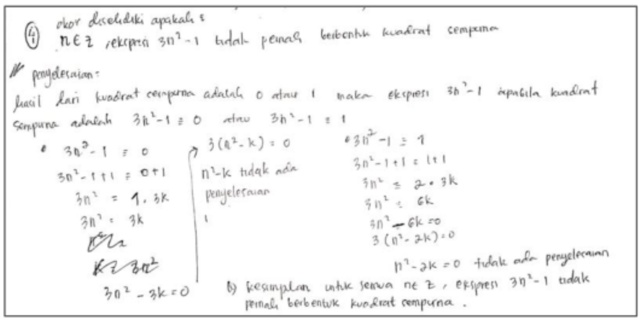


Figure 4. Student Mathematical Proof of Self-Efficacy Low Category

Based on Figure 4, the data aspects were put forward by students at the beginning of the proof, besides that the claims they wanted to prove were also raised, namely "the expression $3n^2 - 1$ never be perfect square". In the aspect of warrants, students try to use direct proof, as can be seen from the sentence stated that "the result of a perfect square is 0 or 1" but this is not very clear what the intended result is; the data on student interviews states that there is a statement that he remembers that perfect squares are always congruent with 0 or 1 (in modulo 3). It's just that students do not write the modulo symbol in the construction of the proof. These are warrants from student's low self-efficacy, it's just that the warrants given are incomplete and ambiguous, and it can be seen from the construction of evidence and interviews that perfect square knowledge is always congruent with 0 or 1 (in modulo 3) only procedural memory, not conceptual because students cannot justify and write it in mathematical sentences according to the rules. Next, students write the congruence symbols "$3n^2 - 1 \equiv 0$", but not written in what modulo. if we pay attention, the construction of student proof has not followed good and correct mathematical rules, conceptually several errors were found which were made as in line" $3n^2 \equiv$

$1.3k$", students cannot clarify their own arguments, so the backing aspect is not raised. In addition, in examining and selecting the overall results, students have not found and cannot explain/justify the results they have constructed and cannot determine things that are still conceptually wrong, so the qualifier aspect is not owned properly. Because the qualifier aspect does not appear, the rebuttal aspect also does not appear at the same time. Overall, the construction of the evidence is neither procedurally nor conceptually valid, so the final conclusion is given that expression $3n^2 - 1$ not being a perfect square is unacceptable. This is in accordance with the purpose of the argument, which is to convince others of the rationality of the idea; with the answers in Figure 4 above, the warrants and backings given have not been able to convince others for the following reasons: 1) warrants are weak, and their meaning is not clear, 2) mathematical procedures cannot be explained, and 3) the ideas conveyed do not lead to the required claims so that conclusions cannot be drawn from constructed reasoning.

From the description above, students with low self-efficacy abilities cannot bring up basic argumentation aspects, namely data, claims, and warrants. The data provided follow the information contained in the problem, as well as the claims requested by the problem. It's just that the given warrants are unclear, and students cannot explain them. The mathematical idea of warrants for square numbers always being congruent with 0 or 1 modulo 3 can prove this statement. It's just that students cannot construct further proofs using this theorem. The backing, qualifier, and rebuttal aspects have yet to be raised in the construction of the evidence, so the conclusions given cannot be accepted. It is also based on procedural, conceptual flow and the use of mathematical rules, which still contain errors and are unacceptable.

Discussion

The ability to prove is a crucial aspect of learning at the higher education level. The subjects presented in the higher education mathematics education study program require more reading skills or constructing evidence. The construction of evidence from the secondary school to tertiary stages is a transition that needs further review because high school learning is still general and not specific to mathematical proof. This was also confirmed by Fadiana et al. (2021) that second-year math teacher candidates still experience difficulties in compiling mathematical proofs at the tertiary level. (Reflina, 2020) found that students needed help responding to the intent and purpose of the questions, determining the initial steps of proof and ideas, needing help with applying definitions, characteristics, theorems and constructing correct proof steps. If it is related to argumentation theory, it is closely related to data, claims, warrants, backing, rebuttal, and qualifiers.

Several studies have shown that mathematical self-efficacy influences a person's abilities, especially in mathematical reasoning (Haerunnisa & Imami, 2022; Santosa & Bahri, 2022; Umaroh et al., 2020). Self-confidence greatly influences the results of students' mathematical proof construction, and it is not uncommon to find that someone is still unsure of their proof construction abilities (Viholainen et al., 2019). Measuring students' self-efficacy in proving is essential because the construction of evidence is the practice of understanding mathematical concepts and how mathematical knowledge is built.

In this study, the ability to construct evidence from argumentation theory is described based on students' self-efficacy level. Students who have a good level of self-efficacy can bring up all aspects of argumentation, namely data, claims, warrants, backing, rebuttal, and qualifiers, and conceptually the construction of the evidence that is produced fulfills mathematical rules and rules, so students with good self-efficacy can prove mathematical statements with good argument structure. Accordance with Maslahah et al. (2019) found that self-efficacy positively affected students' mathematical proving abilities in applying concepts. Qualitative research on student self-efficacy on evidence has yet to be widely studied. However, the association of self-efficacy with mathematical reasoning has shown that self-efficacy is linear with existing mathematical reasoning abilities. Students with moderate self-efficacy raise fundamental argumentation through data, claims, warrants, and backing. However, the qualifier and rebuttal aspects have yet to be able to select all the results, so the results obtained are still conceptually incorrect. Furthermore, students with less self-efficacy only show data and claims, the warrants given tend to be scientifically weak and not following valid mathematical concepts. The weak warrants and backing indicate that students have not mastered the proof method and the relation of mathematical concepts well. This must be followed up immediately. According to (Laamena et al., 2018) the role of backing is very tight in mathematical proof, namely strengthening warrants, finding counter-examples, and providing qualifiers for claims.

Conclusion

This study aims to link mathematical proving abilities based on Toulmin's argumentation theory: data, claims, warrants, backing, qualifiers, and rebuttals are based on different self-efficacy. The difference in self-efficacy turns out to be a predictor of the results of mathematical proof and one's argument; self-efficacy in the good category produces valid mathematical proof construction and raises all aspects of argumentation correctly, while self-efficacy with moderate and poor categories only fulfills the fundamental aspects of argumentation, the construction of the resulting proof has not been able to convince other people that this is true, this is because the warrants given are still weak and not justified. From that, students must improve their mathematical proving abilities in bridging the data and claims they want to prove. For lecturers, this research is expected to be an evaluation material in areas where the ability to construct mathematical proofs is lacking, pay attention to self-efficacy in learning, and emphasize the use of warrants and backing for teaching proof construction.

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Conflicts of Interest

The authors declare no conflict of interest

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Author Contributions

Author One: Conceptualization, writing - original draft, editing, field research and visualization; **Author Two and Three:** review & editing, formal analysis, validation and supervision.

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PAGE 3

Article Error You may need to remove this article.

Missing "," Review the rules for using punctuation marks.

P/V You have used the passive voice in this sentence. You may want to revise it using the active voice.

P/V You have used the passive voice in this sentence. You may want to revise it using the active voice.

S/V This subject and verb may not agree. Proofread the sentence to make sure the subject agrees with the verb.

PAGE 4

P/V You have used the passive voice in this sentence. You may want to revise it using the active voice.

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P/V You have used the passive voice in this sentence. You may want to revise it using the active voice.

Article Error You may need to use an article before this word. Consider using the article **the**.

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Wrong Form You may have used the wrong form of this word.

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Prep. You may be using the wrong preposition.

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Missing "," Review the rules for using punctuation marks.

Article Error You may need to use an article before this word.

Run-on This sentence may be a run-on sentence.

Proofread This part of the sentence contains an error or misspelling that makes your meaning unclear.

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Run-on This sentence may be a run-on sentence.

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