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Students' mathematical argumentation ability when proving mathematical statements based on self-efficacy

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Abstract

Argumentation as an aspect of problem-solving has been studied in mathematics education. However, mathematical proof still needs to be resolved further. This study investigates students' mathematical argumentation skills when proving mathematical statements based on their self-efficacy. The research subjects were 43 mathematics education students at a university in Aceh Province who had taken a number theory course. The study used a qualitative approach with a case study design: students' mathematical proving self-efficacy. Data was obtained using self-efficacy questionnaires and mathematical proof test instruments that experts have validated, while the data triangulation used was an in-depth interview. The results of this study reveal that students' mathematical argumentation skills in proving mathematical statements have differences based on their self-efficacy. The mathematical argumentation ability of students with high self-efficacy involves all aspects of argumentation well so that the construction of the proof is scientifically correct. Meanwhile, the argumentation ability of students with moderate or low self-efficacy still does not involve essential aspects of argumentation. So, the proof results are not scientifically correct because they have not arrived at the proper conclusion.

Keywords: mathematical argumentation; mathematical proof; number theory; proving self-efficacy

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Introduction

Argumentation ability related to reasoning and problem-solving. Mathematical argumentation aims to convey ideas and convince others that the argument is rational; students will unquestionably take deep conceptual understanding through argumentation. This involves building conclusions, providing warrants, and supporting decisions with proof and reasoning. This definition aligns with Sriraman and Umland (2020) that argumentation in mathematics is an argumentative process to reach the findings through reasoning based on mathematical knowledge. They added that argumentation is one of the tools for showing and explaining the problem-solving process.

One's argumentation can be supplemented with definitions, theorems, or results whose truth is known to develop and verify the truth of a mathematical problem (Fukawa-Connelly & Silverman, 2015). Because argumentation in mathematics is closely related to learning, argumentation ability is one of the goals of learning mathematics. NCTM (2000) reveals that the objectives of learning mathematics include problem-solving, reasoning, and argumentation. Toulmin (1958) recommends analyzing the argumentation structure, which consists of claims, data, warrants, backing, qualifiers, and rebuttal. In mathematical argumentation, warrants are used as a basis for mathematical knowledge, verification, validation, interpretation, patterns and methods, calculations, visualization, informal mathematical understanding, and other information supporting these claims (Conner in Dede, 2019). This argumentation scheme helps establish a consistent line of thinking. It provides a solid basis for conveying mathematical ideas and demonstrating their correctness, one of which is through mathematical proof activities.

The proving process in mathematical problems can assess students' argumentation; as Fukawa-Connelly et al. (2015) mentioned, the validity of mathematical proof can be evaluated through argumentation. All students should have opportunities to construct proof for mathematical statements, conjecturing, breaking problems into cases, using counter-examples for refuse statements, justifying answers, and drawing conclusions (Zambak & Magiera, 2020). Argumentation and proof are interrelated and cannot be separated. Aberdein and Dove (2013) support this by arguing that proof can be divided into two types, formal and informal, and argumentation is included in everyday evidence. Argumentations are considered proof when they meet the criteria of valid statements, correct reasoning, and representations by the related concepts (Gutiérrez et al., 2016). Thus, it can be concluded that argumentation needs to be mastered by teachers and prospective mathematics teachers.

A mathematics teacher needs to have good argumentation skills to explain math material effectively to his students. In addition, the demand for mathematics teachers is to facilitate students to argue mathematically through oral and written communication so that students can understand the extent of understanding of the concepts that have been received. To enhance students' comprehension of mathematical argumentation and proof, educators require substantial knowledge of both the content and pedagogy of proof and a positive mindset for teaching proof (Lesseig & Hine, 2022). However, in general reality, pre-service mathematics teacher students' abilities in reasoning, especially in the field of argumentation, have yet to achieve satisfactory results.

Several research results, such as Hamdani and Subarinah (2020), have found that students still experience difficulties in constructing proof in almost all courses that require proving and argumentative reasoning; this problem is caused by students' lack of understanding regarding the integration of mathematical concepts such as axioms, definitions, lemmas, and theorems that can help build new knowledge. Similar findings were reported by Kwon et al. (2015), who found that students still had difficulty connecting argumentation structures to produce final claims in math problems related to partial derivatives. In addition, Sadieda (2019) reported that only about 27% of students understood concepts well during four semesters, especially in the algebraic structure course; this problem impacts students' lack of ability to reason and present arguments effectively.

Based on the findings of the research above, research on mathematical proofs that require student arguments has yet to be widely carried out, especially in the performance of students who have different abilities. The heterogeneity of these abilities makes the quality of the arguments produced have their characteristics, especially in mathematical proof. Thus, it is necessary to review students' mathematical argumentation skills when proving mathematical statements based on their self-efficacy to see the performance of students with different abilities. A person's mathematical performance can be predicted by self-efficacy. Self-efficacy is a person's belief in his ability to organize and carry out a series of actions needed to complete a particular task (Bandura, 1997). Self-efficacy greatly influences the results of students' mathematical proof construction, and it is not uncommon to find that a person is still unsure of his proof construction abilities (Viholainen et al., 2019). In addition, efficacy in reasoning is also essential in supporting the ability to prove and achieve the desired goals. Measuring students' self-efficacy in proving is important because proof construction is the practice of understanding mathematical concepts and how mathematical knowledge is built. Bandura argues that self-efficacy is divided into three dimensions: belief in task difficulty (magnitude), belief that one can complete the task even though it is difficult (strength), and extended belief. Relatively non-routine proof assignments require such efficacy that students do not give up easily in the process.

Several studies still need to integrate mathematical proof with argument structure; the two concepts are still being studied separately. This study bridges the results of previous research and adds affective aspects that affect the ability to prove mathematically based on argumentation theory. This study aims to reveal students' mathematical argumentation abilities when confirming mathematical statements based on self-efficacy. This can be used as evaluation material for lecturers to improve future-proof learning, design appropriate designs, emphasize argumentation-based learning more, and pay attention to the affective aspect of learning.

Methods

This study used a qualitative approach with a case study design because it focused on students' mathematical argumentation abilities when proving mathematical statements based on their self-efficacy. The research subjects are 43 mathematics education students at a university in

Aceh Province, Indonesia (35 females and eight males) who have been taking a number theory course. Data collection was carried out using a mathematical proving test technique and a non-test instrument in the form of a mathematical proving self-efficacy questionnaire to see the level of student self-efficacy. The data was analyzed using stages: 1) data reduction, 2) data presentation/categorization, and 3) conclusion drawing. The mathematical proving test is as follows: "Prove the following statement: elements $n \in \mathbb{Z}$, the expression $3n^2 - 1$ is never a perfect square."

The mathematical proving test has been validated by experts i.e. two number theory lecturers in mathematics education program. To confirm the data obtained, this study also used data triangulation through in-depth interviews with research subjects. The following guidelines are used to categorize student self-efficacy levels after calculation by Microsoft Excel software with Method Successive Interval (MSI) add-ins.

Table 1. Guidelines for levels of self-efficacy (Budiyono, 2015)

Interval	Category
$x < \bar{x} - \frac{1}{2}s$	Low
$\bar{x} - \frac{1}{2}s \leq x < \bar{x} + \frac{1}{2}s$	Moderate
$x \geq \bar{x} + \frac{1}{2}s$	Good

Meanwhile, argumentation theory to structure student proof construction uses the following Toulmin argumentation scheme:

Table 2. Toulmin's argumentation scheme and its relationship with mathematical proving ability

Mathematical Proof Construction Ability	
Indicators	**Related Aspects of Toulmin's Argumentation**
Identify what is the data of the statement	**Data** Students can organize facts, manipulate, or summarize existing information explicitly or implicitly.
Identify the conclusion of the statement (what you want to prove).	**Claim(s)** Students can provide final claims or sub-claims regarding the part to be proven in the statement to support arriving at the final declaration.
States the relationship between data and conclusions by showing a warrant.	**Warrant(s)** Students can provide guarantees/support for the answers they write, namely to bridge the facts and the conclusions to be reached. **Backing** The student can provide additional collateral in support of the warrants that have been disclosed.
Critically evaluate the rules for concluding from given or obtained facts (inference rules)	**Rebuttal** Students can reject statements and explain the conditions under which these statements do not apply. **Qualifier** Students can select all the things that have been disclosed/written, whether there are errors or not so that the conclusion can be accepted.

Source: Modification from Sumarmo in Septiati (2021)

Results

In the first stage, students are given a questionnaire to measure the level of mathematical self-efficacy in the context of proof via the Google Forms website. The questionnaire results were then processed using Microsoft Excel with the successive interval (MSI) method. The following is a summary of the effects of students' mathematical self-efficacy.

Table 3. Results of student self-efficacy in mathematical proof

Interval	Level	Freq.	%
$X_i \geq 86.5$	Good	14	32.56%
$71 \leq X_i < 86.5$	Moderate	15	34.88%
$X_i < 71$	Low	14	32.56%
Total		**43**	**100%**

Based on Table 3. The number of students in each category tends to be similar. The results of the analysis were then reviewed from the results of the answers given by the students and taken by the subject representatives at each level: one subject with a good level of self-efficacy, one issue with a moderate level of self-efficacy, and one subject with a low level of self-efficacy. Each will be described qualitatively using argumentation theory. The following is an analysis of mathematical proving data based on student self-efficacy.

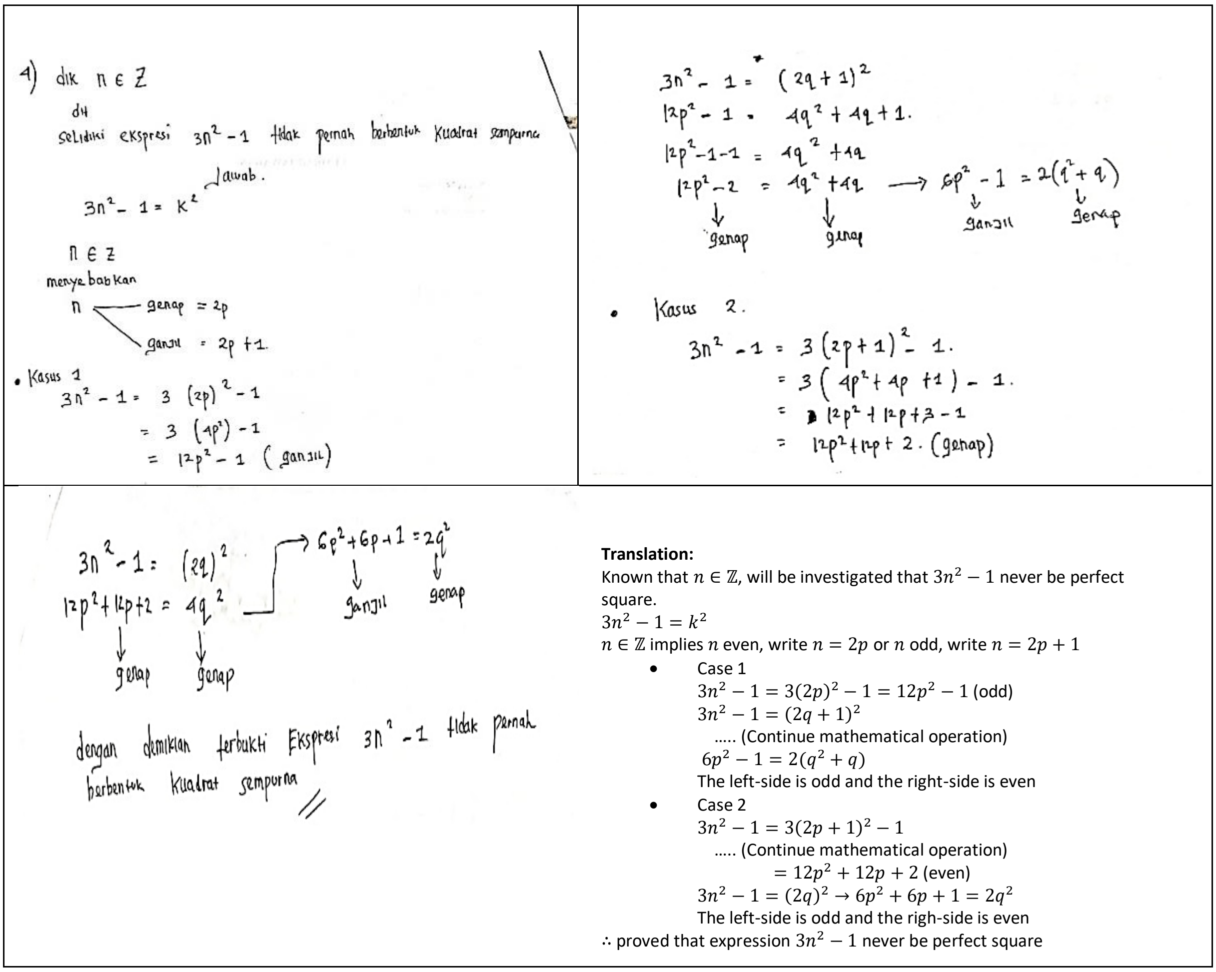


Figure 1. Proof of mathematical self-efficacy in good category

Identify what is the data of the statement

In the data aspect, students have been able to convey the information contained in the problem, namely, $n \in \mathbb{Z}$,; when asked through interviews students can also convey that Z it means the set of integers. Students also found implicit information, namely modeling perfect squares into k^2.

Identify what want to proof

On the aspect of claims, through interviews, students convey that form $3n^2 - 1$ are never perfect squares, and this can be achieved by circumstantial proof, i.e., proof of contradiction. In proving his claim, the student submits several guarantees or warrants.

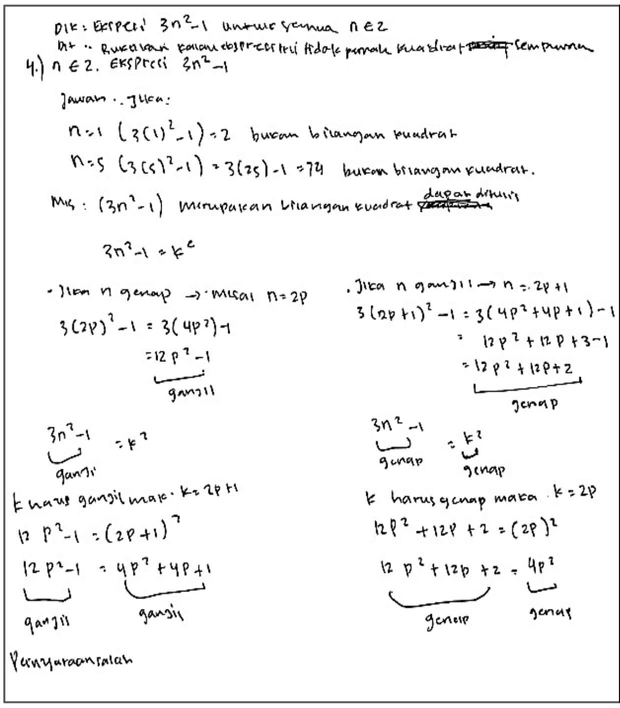
States the relationship between data and conclusions by showing a warrant

Warrants for the contradiction proof he constructs in the form of: 1) division of cases for n when n is even in the form $2p$, and when n is odd in the form $2p + 1$, in interviews the students explained that the divisions when combined would form a set of integers $\mathbb{Z}$ as referred to in the question. The student's answer is then included in the backing category because it can explain the second layer of the first warrant. In addition, it guarantees $6q^2 - 1$ is odd and $2(q^2 + q)$ is even, 2) for n is even, student found $3n^2 - 1$ odd, so k^2 odd implies k odd, student subtitutes $k = 2q + 1$ into the equation. The same is also obtained for cases n odd, so that the warrant given by the student is appropriate for the final claim he wants to prove, it's just that at the initial stage of proof the student does not state in writing that he will prove by contradiction.

Critically evaluate the rules for drawing conclusions from given or obtained facts

In the rebbutal aspect, students find a contradiction in the statement $6p^2 - 1 = 2(q^2 + q)$ and $6p^2 + 6p + 1 = 2q^2$ in each case investigated. Students explain that on the left side, the result will always be odd, while on the right side, the result will always be even, so a situation is found where the statement does not apply. This aspect is a rebuttal that is sought by students in proving using the contradiction method. In addition, when viewed from the results of his work and compared with the results of interviews, it is found that students have been able to select the results they have constructed from start to finish and are convinced that all the steps are sufficient to prove the final claim that they want to prove, students realize that they need to declare it first. The method it will use before directly exemplifying $3n^2 - 1 = k^2$. The student's reflection and belief are his ability in the qualifier aspect. Because in the qualifier aspect, the student does not find any more wrong parts and needs to be added/reduced, the student concludes in general that the claim he is aiming for is true and the proof is complete.

From the analysis regarding the mathematical proving of students with good self-efficacy categories, a conclusion can be drawn that students have brought up all aspects of argumentation, namely data, claims, warrants, backing, rebuttal, and qualifiers. From the construction results, the proof is conceptually correct. Students prove with the indirect proof method, namely proof of contradiction, through a problem-solving approach, namely for cases for even and odd numbers so that two different parities are found, which are seen as a rebuttal; furthermore, students are able to select all of their work and find no errors except for the initial declaration proof. Next, we will describe students' mathematical proving abilities with moderate self-efficacy categories.



Translation:
It is known that: Expression $3n^2 - 1$ for all $n \in \mathbb{Z}$
What to prove: Proof that the expression never be perfect square.
Answer: if $n = 1$ then $3(1)^2 - 1 = 2$ not perfect square
If $n = 5$ then $3(5)^2 - 1 = 74$ not perfect square
Let $3n^2 - 1$ is a perfect square, then write it $3n^2 - 1 = k^2$

- If n is even, let $n = 2p$
  $3(2p)^2 - 1 = 3(4p^2) - 1 = 12p^2 - 1$ which is odd
  $3n^2 - 1 = k^2$, $3n^2 - 1$ itself is odd
  k should be odd, let $k = 2p + 1$
  $12p^2 - 1 = (2p + 1)^2$
  $12p^2 - 1 = 4p^2 + 4p + 1$
  Where the paritiy of each side is odd
- If n is odd, let $n = 2p + 1$
  $3(2p + 1)^2 - 1 = 12p^2 + 12p + 2$ (right-side is even)
  $3n^2 - 1 = k^2$ (Each side is even) then k should be even, let $k = 2p$
  $12p^2 + 12p + 2 = (2p)^2$
  $12p^2 + 12p + 2 = 4p^2$
  Each side is even

∴ The statement is false

Figure 3. Moderate category student self-efficacy mathematical proof

Identify what is the data of the statement & what want to proof

In the data aspect students write statements that are known namely "expression $3n^2 - 1$ for all $n \in \mathbb{Z}$", confirmation interview students know that n is the set of integers. Students provide claims that they want to prove, namely expressions $3n^2 - 1$ never perfect square. In bridging existing data and claims that the truth/false wants to prove, students give inductive warrants first as shown in Figure 3.

States the relationship between data and conclusions by showing a warrant

Students try to provide warrants for $n = 1$ obtained is not a square number, then for $n = 5$ also obtained is not a square number. Furthermore, because the student has not found a denial, he proves it with indirect evidence, which can be seen from the proof he constructs in the sentence "let $3n^2 - 1$ is a square number" which is the negation of the problem. To find the contradiction, students divide the case into two parts, namely when n even, $n = 2p$ and when n is odd, $n = 2p + 1$ which is a deductive warrant. When asked why he divided n into two parts, the student said that working with integers can be made easier with odd and even parities, different from real numbers. This indicates that it has the ability to bring up the backing aspect in support of the deductive warrants it provides.

Critically evaluate the rules for drawing conclusions from given or obtained facts

In the qualifier aspect, students have not been able to select all the steps in the process, for example, in the even n case, in the statement $12p^2 - 1 = 4p^2 + 4p + 1$, students stated that both sides were odd, but for $n = 2$, this statement does not apply, and also, students write odd parity on both sides, but this still needs to show a contradiction of something, especially if both sides are odd. It is not sure that the value is the same. Thus, the qualifier aspect has not been able to be maximally raised by students because they have yet to be able to select all of the results. In addition, students claim that the statements given are wrong at the end of the settlement. The aspect of the qualifier that the student was unable to bring up made the rebuttal aspect also not appear because he was unable to reject the statement and explain the conditions in which the statement does not apply (for example, two odd numbers, the quantity is not precisely the same). Thus, the results of the evidence provided need to be more accurate.

From the analysis, students with a moderate level of self-efficacy have developed some aspects of argumentation in the mathematical proof: data, claims, warrants, and backings. Three main elements of the argumentation, namely data-claim-warrant, have been raised. In the beginning, the students tried first for some small values of n to look for denials but were not found, then continued with warrants dividing the even-odd cases. It's just the qualifier aspect, and the rebuttal that it should have raised has not been well seen. Students have found ideas to construct proofs of these mathematical statements through indirect proof: the contradiction method. Unfortunately, students have not found the intended contradiction and, at the end, claim that the statement is wrong when it should be true. Furthermore, it will be studied regarding the mathematical proving abilities of students with less self-efficacy.

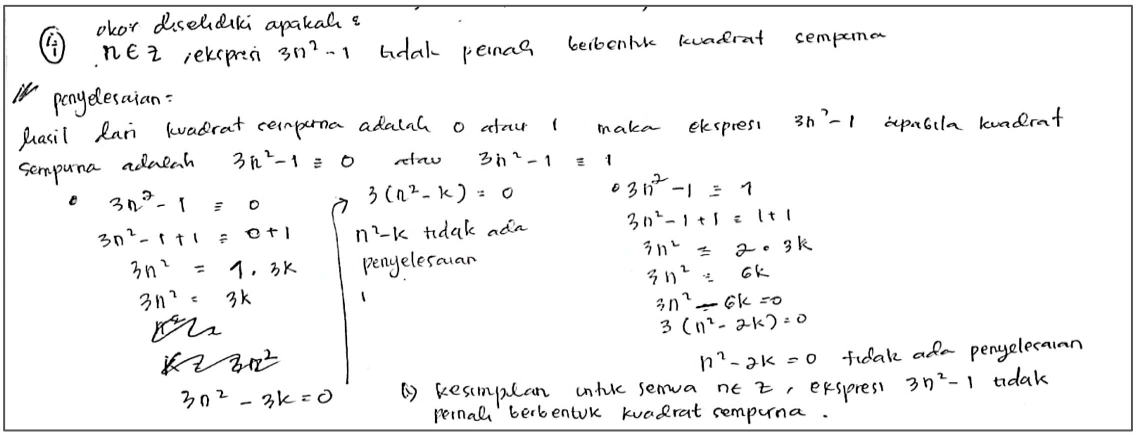


Figure 4. Student mathematical proof of self-efficacy low category

Translation:
Will be investigated whether $n \in \mathbb{Z}$, expresion $3n^2 - 1$ never be perfect square
Solution:
the result of perfect squares is 0 or 1, then if expression $3n^2 - 1$ is perfect square, it should be $3n^2 - 1 \equiv 0$ or $3n^2 - 1 \equiv 1$

- $3n^2 - 1 \equiv 1$
  $3n^2 - 1 + 1 \equiv 1 + 1$
  $3n^2 \equiv 2.3k$
  $3n^2 \equiv 6k$
  $3(n^2 - 2k) = 0$
  $n^2 - 2k$ has no solution
- $3n^2 - 1 \equiv 0$
  $3n^2 - 1 + 1 \equiv 0 + 1$
  $3n^2 \equiv 1.3k$
  $3n^2 \equiv 3k$
  $3n^2 - 3k = 0$
  $3(n^2 - k) = 0$
  $n^2 - k$ has no solution

∴ for all $n \in \mathbb{Z}$, expression $3n^2 - 1$ never be perfect square

Identify what is the data of the statement & what you want to proof

Based on Figure 4, the data aspects were put forward by students at the beginning of the proof, the claims they wanted to prove were also raised, namely “the expression $3n^2 - 1$ never be perfect square”. On the aspect of claim, student write “the expression $3n^2 - 1$ never be perfect square” and trying to solve with some warrants below.

States the relationship between data and conclusions by showing a warrant

In the aspect of warrants, students try to use direct proof, as can be seen from the sentence stated that “the result of a perfect square is 0 or 1” but this is not very clear what the intended result is; the data on student interviews states that there is a statement that he remembers that perfect squares are always congruent with 0 or 1 (in modulo 3). It's just that students do not write the modulo symbol in the construction of the proof. These are warrants from student’s low self-efficacy, it's just that the warrants given are incomplete and ambiguous, and it can be seen from the construction of evidence and interviews that perfect square knowledge is always congruent with 0 or 1 (in modulo 3) only procedural memory, not conceptual because students cannot justify and write it in mathematical sentences according to the rules. Next, students write the congruence symbols “$3n^2 - 1 \equiv 0$”, but not written in what modulo. if we pay attention, the construction of student proof has not followed good and correct mathematical rules, conceptually several errors were found which were made as in line" $3n^2 \equiv 1.3k$", students cannot clarify their own arguments, so the backing aspect is not raised.

Critically evaluate the rules for drawing conclusions from given or obtained facts

In addition, in examining and selecting the overall results, students have not found and cannot explain/justify the results they have constructed and cannot determine things that are still conceptually wrong, so the qualifier aspect is not owned correctly. Because the qualifier aspect does not appear, the rebuttal aspect also does not occur simultaneously. Overall, the construction of the evidence could be more procedurally and conceptually valid, so the final conclusion is given that expression $3n^2 - 1$ not being a perfect square is unacceptable. This is by the purpose of the argument, which is to convince others of the rationality of the idea; with the answers in Figure 4 above, the warrants and backings given have not been able to convince others for the following reasons: 1) warrants are weak, and their meaning is not clear, 2) mathematical procedures cannot be explained, and 3) the ideas conveyed do not lead to the required claims so that conclusions cannot be drawn from constructed reasoning. From the analysis above, students with low self-efficacy cannot bring up essential argumentation aspects, namely data, claims, and warrants. The data provided follows the information contained in the problem and the claims requested by the situation. It's just that the given warrants need to be clarified, and students cannot explain them. The mathematical idea of warrants for square numbers always being congruent with 0 or 1 modulo 3 can prove this statement. It's just that students cannot construct further proofs using this theorem. The backing, qualifier, and rebuttal aspects have yet to be raised in the construction of the evidence, so the conclusions given cannot be accepted. It is also based on procedural, conceptual flow and the use of mathematical rules, which still contain errors and are unacceptable.

Discussion

Students good level of self-efficacy argumentation when proving the mathematical statement

This type of student can bring up all aspects of argumentation, namely data, claims, warrants, backing, rebuttal, and qualifiers, and conceptually the construction of the proof that is produced fulfills mathematical rules, so students with good self-efficacy can prove mathematical statements with good argument structure. These results are by Laamena et al. (2018) that students who can prove a mathematical statement correctly involve all aspects of argumentation, which means that students have good mathematical argumentation skills. Accordance with Maslahah et al. (2019) found that self-efficacy positively affected students' mathematical proving abilities in applying concepts. Qualitative research on student self-efficacy on proof has yet to be widely studied. However, the association of self-efficacy with mathematical reasoning has shown that self-efficacy is linear with existing mathematical reasoning abilities. In this study, students with high self-efficacy had minor errors, such as variable equations that relatively used the same letter and did not declare the proof method they used at the beginning of the proof stage. In line with Indrawatiningsih et al. (2020) that some students still experience slight errors in mathematical argumentation

Students have a moderate level of self-efficacy argumentation when proving the mathematical statement

This type of student raises fundamental argumentation through data, claims, warrants, and backing. However, the qualifier and rebuttal aspects have yet to verify all the results, so the results still need to be conceptually incorrect. In this study, students with moderate self-efficacy used deductive warrants first to find a counter-example, which is good to use. It's just that proof by example sometimes cannot be used because such inductive proofs sometimes cannot improve students' proof construction skills (Alcock & Weber, 2010). In addition, the inductive warrant seeks to show that specific examples can be helpful during exploration, only that they do not have general reasoning value (Fernández-León et al., 2021). In addition, students cannot arrive at the correct conclusion. Fukawa-Connelly (2016) found that students can determine whether they are complete or accurate proofs by increasing their experience of various proofs.

Students low level of self-efficacy argumentation when proving the mathematical statement

This type of student only shows valid data and claims; the warrants given tend to be scientifically weak and need to follow valid mathematical concepts. The worthless contracts and backing indicate that students have not mastered the proof method and the relation of mathematical concepts well. This must be followed up immediately. According to Laamena et al. (2018), the role of backing is very tight in mathematical proof, namely strengthening warrants, finding counter-examples, and providing qualifiers for claims. Therefore, if the guarantee and support are correctly given, the results will be more robust scientifically and mathematically. In the answers given, students know the theorem that can be used to reach the final claim: that a perfect square number always has 0 or 1 left when divided by 3. Still, they need help executing it to other mathematical ideas. Sears (2019) states that although teachers know the theorems or different mathematical rules that can be used, they need to understand how to connect mathematical concepts to reach a valid conclusion. Mathematical knowledge dramatically influences the results of proof construction and the validity of the arguments; researchers agree that low efficacy needs adequate mathematical knowledge and, thus, is associated with low marks in the work (Voica et al., 2020). More extraordinary efforts are required to develop pre-service mathematics teachers' knowledge relative to proof by 1) improving content (mathematical) knowledge and 2) improving their self-efficacy.

Conclusion

The difference in self-efficacy turns out to be a predictor of the results of mathematical proof and one's argument; self-efficacy in the excellent category produces valid mathematical proof construction and raises all aspects of argumentation correctly, while self-efficacy with moderate and poor types only fulfills the fundamental elements of argumentation, the structure of the resulting proof has not been able to convince other people that this is true, this is because the warrants given are still weak and not justified. From that, students must improve their mathematical proving abilities in bridging the data and claims they want to prove.

The results of this study have limitations, namely only taking one subject from each level of self-efficacy so that the sample is only a few. Besides that, the research subjects came from only one university in the mathematics education study program, and the material tested was adjusted to student knowledge, not involving problems that must use complex theorems. Future studies can apply and compare several universities, use collective argumentation, and enlarge the sample to obtain more comprehensive results. The results of this study can be used to improve the teaching of proof methods to students by taking into account the level of self-efficacy. For lecturers, this research is expected to be an evaluation material in areas where the ability to construct mathematical proofs is lacking, pay attention to self-efficacy in learning, and emphasize the use of warrants and backing for teaching proof construction.

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Conflicts of Interest

The authors declare no conflict of interest

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Author Contributions

Surya Kurniawan: Conceptualization, writing - original draft, editing, field research, and visualization; **Rizky Rosjanuardi and Suhendra:** reviewing & editing, formal analysis, validation and supervision.

References

Aberdein, A., & Dove, I. J. (2013). *The Argument of Mathematics* (A. Aberdein & I. J. Dove (eds.)). Springer Netherlands. https://doi.org/10.1007/978-94-007-6534-4

Alcock, L., & Weber, K. (2010). Undergraduates' example use in proof construction: purposes and effectiveness. *Investigations in Mathematics Learning*, *3*(1), 1–22. https://doi.org/10.1080/24727466.2010.11790298

Bandura, A. (1997). *Self-efficacy: The exercise of control.* W.H. Freeman.

Budiyono. (2015). *Pengantar penilaian hasil belajar [Introduction to assessment of learning outcomes]*. UNS Press.

Dede, A. T. (2019). Arguments constructed within the mathematical modelling cycle. *International Journal of Mathematical Education in Science and Technology*, *50*(2), 292–314. https://doi.org/10.1080/0020739X.2018.1501825

Fernández-León, A., Gavilán-Izquierdo, J. M., & Toscano, R. (2021). A case study of the practices of conjecturing and proving of research mathematicians. *International Journal of Mathematical Education in Science and Technology*, *52*(5), 767–781. https://doi.org/10.1080/0020739X.2020.1717658

Fukawa-Connelly, T. (2016). Responsibility for proving and defining in abstract algebra class. *International Journal of Mathematical Education in Science and Technology*, *47*(5), 733–749. https://doi.org/10.1080/0020739X.2015.1114159

Fukawa-Connelly, T., & Silverman, J. (2015). The development of mathematical argumentation in an unmoderated, asynchronous multi-user dynamic geometry environment. *Contemporary Issues in Technology & Teacher Education*, *15*, 445–488.

Gutiérrez, Á., Leder, G. C., & Boero, P. (2016). *The Second Handbook of Research on the Psychology of Mathematics Education*. Sense Publishers. https://doi.org/10.1007/978-94-6300-561-6

Hamdani, D., & Subarinah, S. (2020). Argumen deduktif mahasiswa dalam mengonstruksi bukti [Students' deductive arguments in constructing evidence]. *The 2th National Conference on Education, Social Science, and Humaniora Proceeding*, *2*(1), 21–32.

Indrawatiningsih, N., Purwanto, As'ari, A. R., & Sa'dijah, C. (2020). Mathematical argumentation ability: Error analysis in solving mathematical arguments. *Journal for the Education of Gifted Young Scientists*, *8*(2), 711–721. https://doi.org/10.17478/jegys.654460

Kwon, O. N., Bae, Y., & Oh, K. H. (2015). Design research on inquiry-based multivariable calculus: focusing on students' argumentation and instructional design. *ZDM*, *47*(6), 997–1011. https://doi.org/10.1007/s11858-015-0726-z

Laamena, C. M., Nusantara, T., Irawan, E. B., & Muksar, M. (2018). How do the undergraduate students use an example in mathematical proof construction: A study based on argumentation and proving activity. *International Electronic Journal of Mathematics Education*, *13*(3), 185–198. https://doi.org/10.12973/iejme/3836

Lesseig, K., & Hine, G. (2022). Teaching mathematical proof at secondary school: an exploration of pre-service teachers' situative beliefs. *International Journal of Mathematical Education in Science and Technology*, *53*(9), 2465–2481. https://doi.org/10.1080/0020739X.2021.1895338

Maslahah, Ni'matul, F., Abadi, A., & Maman, A. (2019). *Analisis kemampuan pembuktian, kemampuan berpikir kreatif dan self-efficacy mahasiswa pendidikan matematika pada mata kuliah aljabar abstrak di yogyakarta [Analysis of proving ability, creative thinking ability and self-efficacy of mathematics educations*. Tesis pada Jurusan Pendidikan Matematika, Universitas Negeri Yogyakarta.

NCTM. (2000). *Principles and standards for school mathematics*. NCTM.

Sadieda, L. U. (2019). Kemampuan argumentasi mahasiswa melalui model berpikir induktif dengan metode probing-prompting learning [Students' argumentation skills through inductive thinking models with the probing-prompting learning method]. *Pythagoras: Jurnal Pendidikan Matematika*, *14*(1), 23–32. https://doi.org/10.21831/pg.v14i1.24038

Sears, R. (2019). Proof schemes of pre-service middle and secondary mathematics teachers. *Investigations in Mathematics Learning*, *11*(4), 258–274. https://doi.org/10.1080/19477503.2018.1467106

Septiati, E. (2021). Kemampuan mahasiswa dalam mengkonstruksi bukti matematis pada mata kuliah analisis real [Students' ability to construct mathematical evidence in real analysis courses]. *Indiktika : Jurnal Inovasi Pendidikan Matematika*, *4*(1), 64–72. https://doi.org/10.31851/indiktika.v4i1.6761

Sriraman, B., & Umland, K. (2020). Argumentation in mathematicseducation. In *Encyclopedia of mathematics education* (pp. 63–66). Springer International Publishing. https://doi.org/10.1007/978-3-030-15789-0_11

Toulmin, S. E. (1958). The uses of argument. *Philosophy*, *34*(130).

Viholainen, A., Tossavainen, T., Viitala, H., & Johansson, M. (2019). University mathematics students' self-efficacy beliefs about proof and proving. *Lumat: International Journal of Math, Science and Technology Education*, *7*(1), 148–164. https://doi.org/10.31129/LUMAT.7.1.406

Voica, C., Singer, F. M., & Stan, E. (2020). How are motivation and self-efficacy interacting in problem-solving and problem-posing? *Educational Studies in Mathematics*, *105*(3), 487–517. https://doi.org/10.1007/s10649-020-10005-0

Zambak, V. S., & Magiera, M. T. (2020). Supporting grades 1–8 pre-service teachers' argumentation skills: constructing mathematical arguments in situations that facilitate analyzing cases. *International Journal of Mathematical Education in Science and Technology*, *51*(8), 1196–1223. https://doi.org/10.1080/0020739X.2020.1762938