



## How do students solve reversible thinking problems in mathematics?

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### Abstract

Reversible thinking is a cognitive activity in finding a solution to a problem by arranging the direction of logical thinking from the end to the starting point. Reversible thinking requires a student to think logically in two ways. Therefore, reversible thinking influences students' success in solving problems. This study aims to identify students' thinking processes in solving problems that require reversible thinking ability. This research was conducted on junior high school students in Bandung, West Java, Indonesia, using test instruments, interviews, and documentation studies. The tests given consisted of two types of problems, including tests on forward-thinking problems and tests on reversible thinking problems. The research subjects were students with high average mathematics scores in their class. The study found that students could answer the tests on forward-thinking problem-solving very well but could not work on similar questions with the backward-thinking process. Based on the interview results, one of the causes for the need for more backward-thinking ability is the limited learning resources or context when students first learn the concept.

**Keywords:** backward-thinking; forward-thinking; reversible-thinking

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## Introduction

Problem-solving ability is a key direction in learning mathematics (Flanders, 2014). This is based on the statement stated in the decree of the Head of the Education Standards, Curriculum and Assessment Agency (Ministry of Education, Culture, Research and Technology, 2022) that one of the objectives of mathematics learning is to build students' competencies to be able to solve problems which consist of the ability to understand problems, design mathematical models, solve models or interpret the solutions obtained. In addition, mathematical problem-solving ability is also correlated with other mathematical abilities such as reasoning, decision-making, critical thinking, and creative thinking. According to Krutetskii et al. (1976) and Daulay et al. (2019), one of the abilities related to problem-solving is the ability to work/think backward. This process is called reversible thinking, which was first introduced by Piaget (Oakley, 2004).

Reversible thinking is the mathematical ability to reverse the sequence of events or to set the direction of logical thinking from the end to return to the starting point (Saparwadi et al., 2020; Steffe & Olive, 2009). Thus, reversible thinking is a process of cognitive activity in finding a problem's solution after the final result has been determined and being asked to see the initial condition. Reversible thinking involves mental activities that require individuals to think logically in two reversible ways and make two-way connections between concepts, principles, and procedures to strengthen schema (Flanders, 2014). This means that students must be able to think logically in two interrelated directions. A thorough understanding of concepts and logical thinking creativity will significantly help students work in two ways of solving (forward-thinking and backward-thinking). Students are required to think twice from opposite points of view to minimize the possibility of errors in decision-making (Maf'ulah et al., 2019). Furthermore, Bruner also stated that reversible thinking is a process that influences students' cognition or knowledge (Simon et al., 2016).

Piaget (2005) developed the reversible thinking concept into two indicators: negation and reciprocity. As the negation indicator shows, he said every direct operation could be canceled. In other words, every straightforward procedure has its opposite. For example, the multiplication operation is cancelled by the division operation. Furthermore, the reciprocity indicator displays equal treatment of an equation or inequality. For example, in an equation  $4 + 1 = 2 + 2 + 1$ , this formula can be perceived as the set of numbers on the right side is the compound of the numbers on opposite sides.

Tzur (2004) defines the reversibility in the fraction domain. He developed the concept of  $n/m$  as a part of the unit in relation to a specified whole unit. For illustration, the size of the half of a triangle chopped from a piece of wood is the same as the size of the half of a rectangle chopped from the same piece of wood. Students are typically misled by this approach into thinking that the triangle's half is larger than the rectangle's.

Reversible reasoning is required in understanding that the whole combines each part and, conversely, each part is integrated to form a whole. This part-whole scheme can be examined in the fraction domain as conducted in this study. For example, a rectangular shape thirds of a shape; what is the shape as a whole?

Furthermore, Hackenberg (2005) defines the concept of reversibility as a part having a relationship to the whole whose solution can be described as a schema of quantitative equations. She constructs questions and solutions that are based on quantitative situations. For example, twenty-eight ounces of juice is four times the amount you drank; how much did you drink? The process of finding one of these quantities can be written in the form of a linear equation  $ax = b$  to multiplicatively connect the unknown and known quantities. In this case, students need to reason "inversely" to solve the splitting problem. According to Steffe (2001), the problem can be constructed more quickly in the splitting operation because splitting involves disembedding a posited part from the given whole.

Related to mathematics, topics that require an understanding of reversible thinking include comparison. Regarding comparison, problems can be constructed based on the concept of part-whole. According to Ramful's research, reversible thinking processes are usually related to mathematical operations, fractions, comparisons, algebra, and other mathematical cases (Ramful & Olive, 2008). Mathematical topics regarding contrast are also essential things that need to be analyzed because it is one of the main subjects students study at school. On the other hand, comparison problems also appear in PISA and TIMSS questions. The following is an example of a mathematical problem in the PISA problem. It is known that there are two electric companies known for the average percentage of faulty players in a day, with a different average number of players from each company. Students are instructed to show which company has the least percentage of faulty players.

Research in several countries related to the reversible thinking ability of elementary school students to higher education students showed there was an importance to developing this thinking process to minimize the gap in this thinking process (Maf'ulah & Juniati, 2019); Maf'ulah et al., 2016; Ramful, 2014; Sangwin & Jones, 2017). Research conducted by Maf'ulah Juniati (2019) stated that students could not build reversible relationships between functions and graphs. Likewise, students had difficulty solving problems with reversible thinking concepts in arithmetic. The types of errors students make include errors in operating numbers, calculation errors, and compiling a solution algorithm. On the other hand, students failed to conceptualize the reversible multiplication relationship in the context of fractions. Therefore, to solve the problem, students are required to spend higher costs.

In this study, the researchers intend to identify the characteristics of reversible thinking in junior high school students. This research is expected to provide an overview of the ways and processes of reversible thinking of junior high school students so it can be used as a basis for teacher intervention in mathematics learning practices. The research questions about this objective were as follows: (1) How is the process of reversible thinking in junior high school students? (2) What are the thinking characteristics of junior high school students?

## Methods

### Research design

This research used qualitative methods with a phenomenology research design to explore and interpret students' reversible thinking process in obtaining alternative solutions. We adapted Bungin's stage (2003) in conducting this study. There were: (1) Collecting data from the results of students working on reversible problems and interviews; (2) Data reduction to focus the data that would be used and relevant for research. The data obtained will be analyzed using reversible thinking process indicators. (3) Displaying data, showing the result of calculated data, so it was possible to produce a conclusion and decision-making. (4) Conclusion: determining the meaning of the data that had been collected and analyzed. At this stage, results were obtained that could answer the research question.

The indicators of reversible thinking used in this study were synthesized based on the indicators proposed by Maf'ulah et al. (2017) and Hackenberg (2010). Table 1 below illustrates the indicators of reversible thinking used in this study:

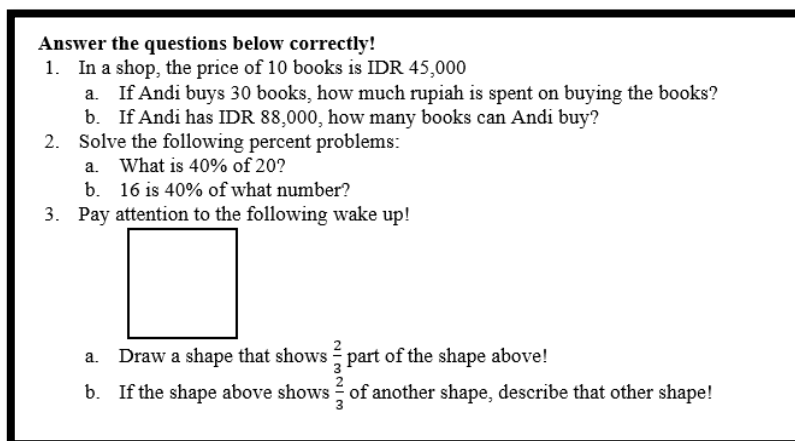
**Table 1.** Indicators of reversible thinking ability

<i>Reversible Thinking Process</i>	<i>Sub Indicator of Reversible</i>	<i>Thinking Indicator</i>
Forwards	Negation	When the subject uses inversion of the related operations in making equations
	Reciprocity	When the subject uses compensation or other relationships that are equivalent to the given equation in making the equation
Backwards	Negation	When the subject uses the inversion of the related operation in creating the equation
	Reciprocity	When the subject uses compensation or other relationships that are equivalent to the given equation in making the equation
	Ability to return to initial data after obtaining results	When the subject can return the equations made to the initial equations using the correct procedures

### Participant and data collection

Data collection was carried out based on the triangulation technique (Sugiyono, 2010) with the following stages: (1) Take a test on a comparison topic, (2) Conduct interviews to deepen students' experiences in reversible thinking processes when working on problems and explore students' experiences in acquiring knowledge related to reversible thinking processes, (3) Documentation during processing, and (4) documentation study of mathematics textbooks. Data was collected at a junior high school in West Java Province, Indonesia, which excels academically. Students who became research subjects had good academic averages and were placed in a particular class. Data collection in the form of tests was conducted on 20 grade 8

junior high school students. The students consisted of 12 female students and eight male students. Participants who took the test were students who had received the comparison topic. After all the students took the test, the students with the best answers were analyzed and interviewed. The test consists of three questions related to reversible thinking processes. Through the previously mentioned indicators, three questions were made to measure students' reversible thinking processability. The details of the questions are depicted in Figure 1 as follows:



**Figure 1.** Questions that require students' reversible thinking processes

Problems presented to students were made based on the following criteria: (a) by the indicators of reversible thinking; (2) each problem contains a solution using the concept of forwards-thinking for type A problems and backward-thinking for type B problems; (3) the level of difficulties, that is from less to more difficult.

In addition to the test results, interviews were conducted with the students with the best scores to get a deeper meaning of the student's answers. It turned out that several student perspectives could be expressed verbally, which supported or even contradicted the answers they wrote. During the tests and interviews, documentation was carried out.

To further explore how students solve reversible thinking problems, researchers also tried to study a series of tasks in the textbooks. The book was commonly used by students in the learning process.

## Data analysis

Data obtained from tests, interviews, and documentation studies were collected to analyze the suitability of the data. The results of students' answers are the primary source in assessing students' reversible thinking process. Furthermore, the results of the answers were confirmed to students through interviews. Then, the data was transcribed to be read repeatedly to match the answers written by students. In addition to analyzing the match between the written answers and the interview results, the researcher also clarified the students' answers based on the task design in the textbook, which was the primary source of students in the learning process in the classroom. The analysis process also considered the results of interviews and studies of students' books. This was done to create good credibility in qualitative studies (Fraenkel et al.,

2012). Data from the test results, interviews, and documentation studies were analyzed based on reversible thinking indicators to measure students' abilities.

## Results

In general, problems containing reversible thinking were made in two workflows. Students worked on the problem with the forward-thinking flow and then worked on it again with the reverse flow. So, in the three tests given, type A questions were completed with a forward-thinking flow, and type B questions were completed with a backward-thinking flow. The concept used in both questions was the same.

### Person ability

Seven of the twenty students who took the test had the highest scores. Other students who had low scores were due to students not being able to work on problems that required forward-thinking, thus leaving blank the problems that required backward-thinking processes. In general, students' forwards-thinking ability based on indicator reversible thinking reviewed from the tests that students have done is shown in Table 2 below:

**Table 2.** Students' forwards-thinking ability

<i>Reversible Thinking Process</i>	<i>Sub Indicator of Reversible</i>	<b>Number of students</b>	<b>Percentages</b>
<b>Forwards-thinking</b>	Negation	16	80%
	Reciprocity	13	65%
<b>Criteria</b>	Students are active in constructing knowledge.		

Furthermore, students' backward-thinking ability based on indicator reversible thinking reviewed from the tests that students have done is shown in Table 3 below:

**Table 3.** students' backwards-thinking ability

<i>Reversible Thinking Process</i>	<i>Sub Indicator of Reversible</i>	<b>Number of students</b>	<b>Percentages</b>
<b>Backward-thinking</b>	Negation	9	45%
	Reciprocity	7	35%
	Ability to return to initial data	7	35%
<b>Criteria</b>	Students' low ability to construct knowledge		

The test results above show that students are more dominant in the forward-thinking process of constructing knowledge. In the sub-indicators of reversible thinking, the negation thinking indicator is more prevalent in both thinking characteristics (forward and backward). Beyond that, some students need help working on the problem properly, choosing not to fill in the answer sheet provided.

The characteristics of reversible thinking of junior high school students can be seen from the data on the results of student work in answering questions that require this thinking process. From the test data, seven students could do well in the backward-thinking process on the indicators of reciprocity and return to the initial data. Information on students with the best

ability is then identified through in-depth interviews to explore students' mindsets in backward-thinking and reversible thinking abilities, including negation and reciprocal thinking abilities.

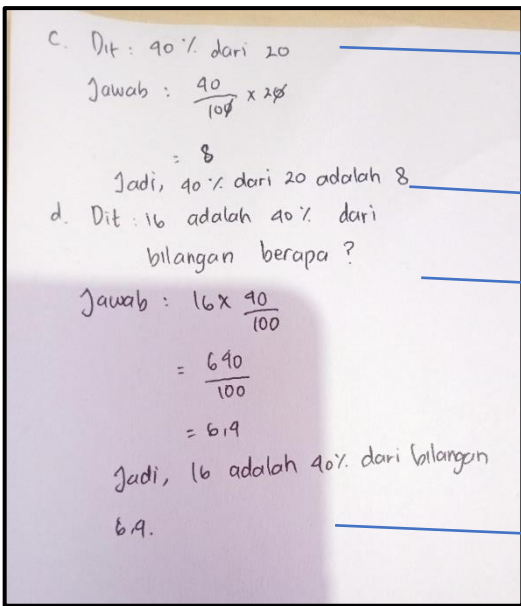
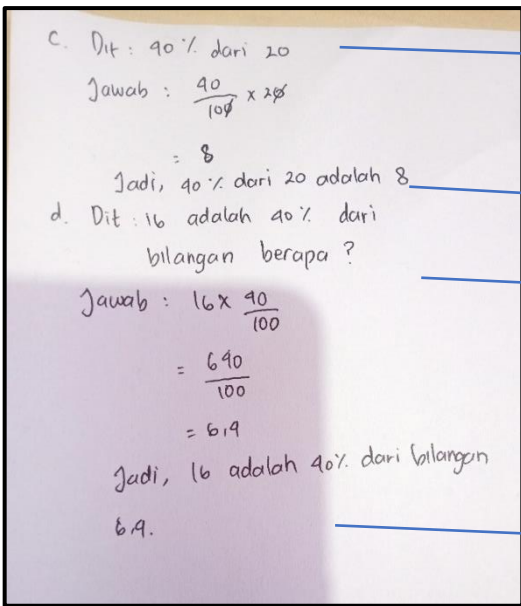
The following will present the results of the answers of students who have good abilities (reached the reciprocity indicator and returned to the initial data) in solving reversible thinking problems in problem number 2, presented in Figure 2.

Solve the following percent problems:  
 a. What is 40% of 20?  
 b. 16 is 40% of what number?

**Figure 2.** Questions related to the reversible thinking process

The following were the answers from several participants to question number 2:

**Student 1's answer**

Student Answer	English Translation
	<p>40% from 20 is...</p> <p>So, 40% from 20 is 8</p>
	<p>16 is 40% from?</p> <p>So, 16 is 40% from 6,4</p>

**Figure 3.** Student work results 1

Based on student 1's answer, the student was correct in solving type A problems (able to do the forward-thinking process). Student 1 was able to define the problem well and used the correct operation to solve the problem. Meanwhile, to answer type B problems, students needed help determining the solution strategy. In this case, student 1 had yet to complete the invertible or backward-thinking process successfully. The student planned to do the invertible thinking process but failed because the student needed to understand the whole quantitative equation (Hackenberg, 2005). This was characterized by student 1 needing help to use the negation process in multiplication operations. The following interview results supported these results:

Researcher: How do you do to answer no? 2a?

Student 1: I need to multiply the 40% by 20. The 20 is divided by 10; the 100 is divided by 10. It remains  $40 \times 2 : 10 = 8$

Researcher: Then, for part b, why do it that way?

Student 1: I'm confused because the questions were reversed; I just reversed the multiplication. For the answer no. 2a is  $40\% \times 20$ , so for the answer no. 2b is  $16 \times 40\%$

### Student 2's answer

Student Answer	English Translation
<p>C. Ditanyakan: berapakah 40% dari 20?                  Dijawab: <math>\frac{40}{100} \times 20 = \frac{40}{5} = 8</math> jadi 40% dari 20 adalah 8</p> <p>D. Ditanyakan: 16 adalah 40% dari bilangan berapa?                  Dijawab: <math>16 : \frac{40}{100} = 16 \times \frac{100}{40} = 40</math></p>	40% from 20 is ...
	So, 40% from 20 is 8
	16 is 40% from?

Figure 4. Student work results 2

Based on the results of student 2's answers, that student was correct in solving questions of types A and B. Student 2 had been able to do an inversion of the operation by changing  $16 : \frac{40}{100}$  to  $16 \times \frac{100}{40}$ . In addition, student 2 was able to carry out a reciprocity process of number operations. He was able to make equivalent relationships in the equations. It could be seen in the work that  $\frac{40}{100} \times 20$  was operated by simplifying fractions by dividing by the same number. He divided it by 20 to become  $\frac{40}{5} \times 1$ .

However, from the results of in-depth interviews, student 2 needed to understand the concept of comparison fully. The student 2 did not realize that 16 was a relative measure of 40%. Thus, in answering type B questions, the opposite of type A, the process was only done by trial and error because students still needed clarification. The interview process indicated that student 2 was required to fully understand the thinking aspect to return to the initial data after getting the results. The data was supported by interviews as follows:

- Researcher : How to do number 2a?  
 Student 2 : I multiplied it and simplified how to find the answer. 20 and 100 are both divided by 20 and remains  $\frac{40}{5} \times 1 = \frac{40}{5} = 8$
- Researcher : Then how to answer 2b? Why is the operation divided?  
 Student 2 : That's just random, I try it. Because I don't know how to do it either.
- Researcher : Because the previous question is multiplied, and the next question is reversed, the operations are also reversed.  
 Student 2 : Yes, I think so.



**Student 3's answer**

Student Answer	English Translation
<p>c. Dik: 40% dari 20</p> <p>Jawab: <math>\frac{40}{100} \times 20 = 8</math></p> <p>Jadi, 40% dari 20 adalah 8.</p>	<p>So, 16 is 40% from 40</p> <p>So, 40% from 20 is 8</p>
<p>Dik: 16 adalah 40% dari</p> <p><math>= \frac{40}{100} \times x = 16</math></p> <p><math>40 \times x = 1600</math></p> <p><math>x = \frac{1600}{40}</math></p> <p><math>= 40</math></p> <p>Jadi 16 adalah 40% dari 40.</p>	<p>16 is 40% from?</p> <p>40% from 20 is</p>

**Figure 5.** Student work results 3

Based on the results of student 3's answers, that student was correct in solving questions of types A and B. In carrying out operations, student 3 made equivalent operations by multiplying the equation  $\frac{40}{100} \times x = 16$  by the number 100. It becomes  $40 \times x = 1600$ . For the other part, student 3 created a complete equation with the appropriate placement, becoming  $40 \times x = 1600$  into  $x = \frac{1600}{40}$ . When student 3 was correct in defining the quantitative equation, they had a good understanding of the part-whole relationship. These results were supported by interviews as follows:

- Researcher : How to do number 2a?  
 Student 3 : I multiply directly 40% x 20  
 Researcher : How about 2b?  
 Student 3 : the same method as 2a but asked a different question. So, I write 40% multiplied by  $x = 16$ . Then, all I have to do is find the value for  $x$ .  
 Researcher : How do you get the value of  $x$ ?  
 Student 3 : First, I multiply the two sides by 100. Then divide it by 40.

**Documentation study**

Additional information obtained from the sourcebook used by students also illustrates that the problems given in the book are only limited to the context of forwards-thinking, as in the following Figure 6 From the book used by students, students are only led to be able to work on problems that require a forwards-thinking process, from a total of 16 sample problems in one chapter, there are no problems that guide students to think reversibly.

Determine whether the set of number pairs is under proportion or not!

first number	2	4	6	8	10
second number	4	6	8	10	12
first number	3	6	9	12	15
second number	4	8	12	16	20

Figure 6. Students' ability to think forward and reversible

## Discussion

### Forward-thinking more dominant rather than backward-thinking

In general, the test results and interviews show that the processes of students' thinking are more dominant in the forward-thinking process rather than the reversible-thinking process. The seven students with the highest scores successfully answered type A problems and had difficulty answering type B problems. The other thirteen students had low scores because they needed help working on variety A problems well, so they left blank type B problems blank. At the same time, this reversible thinking is one of the mathematical competencies students need to improve their problem-solving ability (Simon et al., 2016; Saparwadi et al., 2020; Prabawanto, 2023). Through reversible thinking, students must understand the concept as a whole to work backward and think inversely or the opposite of a procedure. In addition, students are also required to be able to build relationships between concepts so they can think in two directions to find alternative solutions. This ability will be very influential in solving non-routine problems that require high problem-solving ability. Students who have reversible thinking ability will look at problems not only from one point of view to solve them.

The limitation in reversible thinking is due to students' inability to make meaningful connections between mathematical concepts and not being able to build two-way interrelated relationships. Maf'ulah et al. (2019) explained that high school students cannot establish meaningful two-way correlations between functions and their graphs. Ramful (2014) also suggested that students fail to conceptualize the multiplicative relations in reverse, making students choose a more primitive strategy. At the same time, there are problems that are simpler if done by division (reverse of multiplication). This finding is reinforced by other studies which state that students at various levels of study, ranging from elementary and secondary school students to prospective mathematics teachers, have difficulty in working on problems that require reversible thinking ability (Maf'ulah & Juniati, 2020; Maf'ulah et al., 2016; Sutiarmo, 2020; Sangwin & Jones, 2017).

In other cases, in this study, the errors that occur in students are generally caused by students being accustomed to working on problems by memorizing procedures (forward

procedures) and memorizing formulas. This results in students needing help to relate the concepts they have to solve problems under the same concepts in other types that require a reverse thinking process. The two types of problems presented show that students are successful in working on type A problems that require forward-thinking processes but fail in solving type B problems that require backward-thinking processes.

One of the student's mistakes in the backward-thinking process is when students give the answer  $\frac{40}{100} \times 16$  for type A problems and give the answer  $16 \times \frac{40}{100}$  for type B problems. In this case, students have not been able to think back to the initial data after knowing the final value of the equation. Students who have a complete understanding of the concept of comparison should be able to construct the equation using the concept of invertible thinking become  $\frac{40}{100} \times x = 16$  so as to get the comparison equation  $\frac{40}{100} = \frac{16}{x}$ . From the equation, students can determine the value of  $x$  using the negation and reciprocity process of the multiplication operation. Conditions like this show that students need more context when working on different types of problems (Suryadi, 2019) or experience faulty intuition in defining prior knowledge (Kandaga et al., 2022). The existence of context limitations will generate perceptions in students that the problem being worked on is a new problem that has never been studied before. This can be indicated that students need help understanding the concept well and comprehensively (Wahyuningrum et al., 2019).

Furthermore, regarding the additional information obtained from the textbook used by students, this also resulted in the limited context of the concept of comparison that students learned (Abung & Herman, 2023; Nova et al., 2023). The series of tasks given in the book only requires students to work by thinking forward. This causes students' prior knowledge of the concept to be limited. This statement is to the results of research by Fitriati et al. (2020) that a series of tasks yang implemented will influence how students think and their level of understanding. Books are one of the primary sources that students usually use in directing student learning activities (Kajander & Lovric, 2009). Based on this, students who state that they memorize the procedure for working on a context will have great difficulty working on problems that have never been discussed in the book.

### **Negation sub-indicator is more dominant used by students**

Based on the two thinking abilities of students (forward and backward thinking), in the negation sub-indicator, this ability is more owned by students than the reciprocity ability and the ability to return to the initial data. Negation ability is already owned by elementary school students, such as doing subtraction as the negation of addition or division as the negation of multiplication (Hackenberg, 2010). Furthermore, reciprocity ability is an ability that has also been introduced since elementary school through the concept of similarity. This makes it easy for students to understand. However, after students recognize the concept of variables, performing a balanced process in an equation requires good knowledge of defining the variables. Defining variables is also challenging for students (Syarah et al., 2023).

## Conclusion

Based on the analysis of seven students with good initial mathematical abilities, students tend to succeed in forward-thinking. On the other hand, students experienced difficulties constructing answers that required reversible thinking processes. The main thing that causes errors in the solution process is when students make mistakes in making quantitative equations. Students have obstacles in the reversible thinking process due, among others, to the limited context when students first learn the concept. Therefore, it can be a concern for teachers and researchers to design a learning process to build a comprehensive concept.

## Conflicts of Interest

The authors declare no conflict of interest regarding the publication of this manuscript. In addition, the ethical issues, including plagiarism, misconduct, data fabrication and falsification, double publication and submission, and redundancies, have been completed by the authors.

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## Author Contributions

**Aneu Pebrianti:** Writing - review & editing, formal analysis, and methodology; **Sufyani Prabwanto:** Conceptualization, writing - original draft, editing, and visualization; **Elah Nurlaelah:** Validation and supervision.

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