



Epistemological learning obstacles on fractions in elementary school

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Abstract

Fractions are one of the most essential concepts in mathematics learning. In elementary school, a student's understanding of fractions does not fully involve the meaning of various representations, so it could cause learning obstacles. The purpose of this study was to identify and analyze students' learning obstacles on the topic of fractions in grade 5 of elementary school. This study uses a qualitative method. The participants of this study were 21 elementary school students with different characteristics. Data was collected using test techniques, interviews, observation, and document study. Data were analyzed qualitatively to identify learning obstacles experienced by students. The research results are an overview of learning obstacles with the type of epistemological obstacle in fractions, including errors in operating fractions, needing to understand the purpose of the questions, and not understanding the problem with a different context. The research results obtained can be used to develop a hypothetical learning trajectory.

Keywords: epistemological obstacles; fractions; learning trajectory

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Introduction

Some students consider mathematics lessons difficult because mathematics involves formulas and calculations. The difficulties faced by students result in poor reasoning and test results. It is reflected in the mathematics tests conducted by the Trends in International Mathematics and Science Study (TIMSS) and the Program for International Student Assessment (PISA), Indonesia, which are represented by students within a specific age range and have obtained poor results. Indonesia's ranking in TIMSS in 2023 was 44th out of 49 participants (Mimbarwati et al., 2023), and Indonesia's ranking in PISA in 2018 was 70th out of 78 participants (Maulida et al., 2018).

Unfavorable results like the above might lead to an erroneous process of learning mathematics in the classroom. Learning mathematics is providing learning experiences to students through a series of planned activities so that students gain competence in the mathematics material being studied (Fuadiah et al., 2019). The process of learning mathematics is the most essential part of learning mathematics. The learning process is called teaching and learning, in which there is intentional interaction between students and teachers (Fuadiah et al., 2019). The teaching and learning process is the core of the entire educational process, with the teacher as the primary role holder. Brunner (1966) states that learning mathematics is learning about mathematical concepts and structures contained in the material being studied and looking for relationships between concepts and mathematical structures.

Fractions are one of the materials that are considered essential for learning mathematics. Fractions and decimals are critical to later mathematical achievement and career success (Lortie-Forgues et al., 2015). Fractions are numbers that can be denoted $\frac{a}{b}$ is called the quantifier, and b is called the denominator where a and b are integers and $b \neq 0$. The form $\frac{a}{b}$ can also be interpreted as $a : b$ (a divided by b). In other words, an object is divided into several equal parts, and comparing each part with the whole object creates the primary symbol of a fraction (Karso, 2013; Heruman, 2017). Based on the Decree of the Minister of Education and Culture of the Republic of Indonesia No. 37 of 2018, learning fractions in elementary schools starts from grade 2 to grade 6 (Kemdikbud, 2018). It means that fractional material is essential for elementary students to understand, even though, in reality, problems often occur due to difficulties in learning it. Hunting and Gary Davis (1991) argue that children seem to have problems learning the correct meaning of fractions because teaching fractions if delayed too long, allows the language of fractions and their symbolism to be learned through knowledge of whole numbers.

Learning fractions that are difficult for students creates learning obstacles. Suryadi (2013) learning obstacles are learning obstacles faced by students during the learning process. Brousseau (1997) can be caused by several factors, namely the obstacle of ontogenic origin (mental learning readiness), the obstacle of didactical origin (effect of the education system), and the obstacle of epistemological origin (knowledge of students who have limited application contexts). Ontogeny obstacles (obstacles of ontogenic origin) occur because the learning process is not on the child's readiness. Therefore, ontogeny obstacles are closely related to students' mental development, which is related to age and level of development. If the obstacle

arises only because of slow mental development and not because of a congenital disease, then the obstacle will disappear as the student grows. Didactical obstacles (obstacles of didactical origin) occur due to errors in the learning process originating from the learning system in the school itself. Epistemological obstacles (obstacles of epistemological origin) are essentially someone's knowledge that is only limited to specific contexts. If the person is exposed to a different context, his knowledge becomes unusable or needs help. In this case, students' views of one concept with another could be more cohesive and comprehensive.

In research conducted by Romdhani & Suryadi (2017), learning obstacles that were found when learning fractions in class III were: 1) students' ability to represent fractions from the presented geometric figure (fractional concept image), 2) the concept of drawing fractions in geometric shapes presented, 3) understanding of bigger and smaller signs in comparison of fractions, 4) concept of the meaning of fractions and procedures for comparing two fractions, ability to translate and solve fraction story problems. Research conducted by Diputra et al. (2023) showed that students experienced three types of learning obstacles: epistemological, didactical, and ontogenically. The students faced Epistemological obstacles in constructing fractions as parts of a whole in the visualization of the model area. Didactical obstacles occurred in constructing fractions as parts of a whole in visualizing discrete models and improper fractions. Ontogenic obstacles occurred in constructing fractions as measures, especially placing fractions on the number line.

Based on the research above, learning mathematics about fractions still faces learning obstacles in grades 3 and 4. To get a comprehensive picture of the problem, the researchers conducted a preliminary study in grade 5 at one of the Islamic elementary schools in Serang City. Based on the observations and interviews with class V teachers, many students still need help with fraction material. Of the 22 students, only eight could understand fraction material, especially operations on fractions. Apart from that, students need help understanding story questions. Students confirmed this by interviewing the questions given. Some students are still confused about answering the story questions given. Below are the questions and student answers.

What is the result of adding $\frac{6}{8} + \frac{5}{7} =$

Berapakah hasil dari penjumlahan dari $\frac{6}{8} + \frac{5}{7} =$ ~~$\frac{6}{8} \times \frac{5}{7} = \frac{6}{2} \times \frac{5}{7} = \frac{40}{42} : 2 = \frac{20}{21}$~~

Figure 1. Student's difficulties in answering questions

In Figure 1 above, students can add fractions by cross-multiplying or by the numerator multiplied by the denominator and the denominator multiplied by the numerator, then divided by two. This error occurs because students need help understanding the meaning of the addition operation on fractions. Students assume that all operations on fractions are by cross multiplication. After being confirmed by interviews, students understand how to do it like that.

It is based on the understanding that was taught to him. However, when asked by the teacher, the teacher said that he had taught fraction operations using cross multiplication, but that was the operation of dividing fractions.

Based on the problems that occur, this research aims to describe the epistemological learning obstacles experienced by grade 5 elementary school students. The differences between this research and previous research are: 1) this research is limited to epistemological obstacles, and 2) this research was conducted in grade 5 of elementary school.

Methods

The research method used in this research is qualitative research with a hermeneutic-phenomenological approach. A *hermeneutic phenomenology* is an approach based on the philosophy of phenomenology and hermeneutic philosophy. Langdrige (Suryadi, 2019a) states that *phenomenology* is defined as a philosophical study focused on revealing a person's understanding of the world in which he lives and what it means for those involved. Apart from that, phenomenology aims to describe human life experiences regarding certain phenomena as explained by participants (Creswell, 2012). On the other hand, phenomenology can only fully and comprehensively understand various phenomena by understanding the experiences of its participants. Because of this, hermeneutics is needed (Suryadi, 2019a). *Hermeneutics* is defined as the philosophy of interpretation of meaning. Phenomenology and hermeneutics complement each other.

This research reveals and describes fractional learning obstacles and the factors that cause learning obstacles. The participants in this research were 21 fifth-grade elementary school students. Data was collected using test techniques, interviews, observation, and document study. This research procedure begins with observing 21 students, then giving a mathematical ability test on fractions to 21 elementary school students, interviewing nine students to reveal learning obstacles and their factors, analyzing learning obstacles, then compiling a hypothetical learning trajectory (HLT) of fractions that can be applied to learning. Mathematics fraction material in class V elementary school. The questions given cover students' abilities in interpreting fractions, which include 1) Fractions as part of a whole, 2) Fractions as operators, 3) Fractions as a measure, 4) Fractions as a quotient, 5) Fractions as ratios, (Lamon, 2012; Purnomo, 2015).

Results

The research results include findings of learning obstacles experienced by students when working on questions in the learning process. Based on the analysis of student answers from 21 students, data was obtained on students who answered incorrectly when working on the questions given. Some students did not answer, and some students answered with wrong answers. Based on this, students do not understand the fraction questions given, especially since the questions given are questions with a different context from what the class teacher usually

gives. The following is a description of data regarding learning obstacles experienced by students when working on adding and subtracting fractions on each question given.

Question number one

Express the shaded area in the image below into a fraction, then operate on the fraction!

Question number one is a matter of fractions of parts of the whole. Many students can understand this question and have no difficulty. However, of the 21 students, eight experienced difficulties. The mistakes are: 1) Students have written down the value of the fraction correctly but still operate the denominator with the denominator and the numerator with the numerator. 2) Students write down the shaded part's numerator and the not-shaded part's denominator. 3) Students only write down the shaded amount in integer form and operate it. One of the answers given by students is as follows:

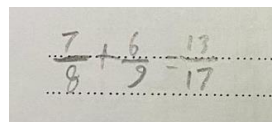


Figure 2. Student answer on no 1

Based on the results of the student's answers with the three types of mistakes made, the researcher uncovered the students' difficulties through interviews. After being confirmed, the student said that he did not focus on the question instructions; he only focused on the picture, so he did not work according to the instructions on the problem because questions like these are new questions for students.

Question number two

Two buckets are filled with water at the same time, every 2 minutes the first bucket is filled with $2\frac{1}{4}$ liters of water, and the second bucket is filled with $1\frac{1}{2}$ liters of water, after 8 minutes how many liters of water have been filled in total from the two buckets.

Question number two is a matter of fractions as a measure, one of the answers given by students is as follows:

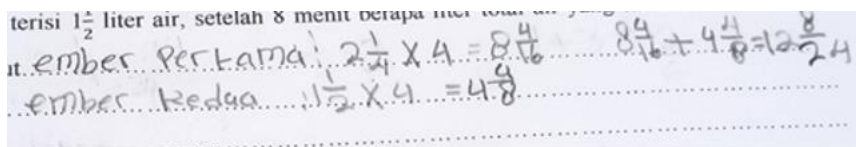


Figure 3. Student answers to question no 2

In Figure 3 above, it can be seen that students are working on fractions as a measure. The above problem involves multiplication and addition of fractions. When carrying out multiplication operations, students directly multiply mixed fractions with integers, but there is an error in the steps taken, so the results given need to be corrected. Students multiply fractional numbers with

integers directly without changing one of them or multiplying by the numerator only. This error occurs because the student needs help understanding the question's meaning. Students work according to their abilities based on experience working on fractions directly. After being confirmed by interviews, the students understood how to do it like that, based on the understanding that they had been taught through fraction problems, which were straight to the numbers, not story problems. However, when asked the teacher, the teacher said the questions that were often given to students were in the form of direct questions. Fractions word problems are rarely given.

Question number three

A gallon filled with water weighs 20 kg, if half the water is poured out, then the gallon containing water becomes $10\frac{1}{4}$ kg, how much does the gallon weigh if it doesn't contain water?

Problem number three is a matter of fractions as a quotient, one of the answers given by students is as follows:

The image shows a student's handwritten work on a piece of paper. The work is as follows:

$$20 - 10 = 10$$

$$\frac{10}{4} = 2.5$$

$$20 - 2.5 = 17.5 = 17\frac{1}{2}$$
There are some faint scribbles and corrections above the main steps.

Figure 4. Student answers to question no. 3

In Figure 4 above, we can see students working on fractions as quotients. The problem involves the operation of subtraction and addition of fractions. However, students need help understanding the meaning of the problem of subtraction by subtracting the numbers in the question. Then, students also made mistakes when changing the form of mixed fractions. This confusion occurs because students need help understanding the meaning of the questions, what operations should be used, and how to do them. After being confirmed by interviews, the students needed help understanding the purpose of the questions, so they needed to learn how to do them.

Question number four

Two candles burn simultaneously, every hour the candle will decrease. The first candle will decrease by $\frac{1}{6}$ and the second candle will decrease by $\frac{1}{3}$. After 2 hours of burning, the first candle will be twice the length of the second candle. Show the situation with a diagram and how much is left over from each candle?

Problem number four is a matter of fractions as ratios, one of the answers given by students is as follows:

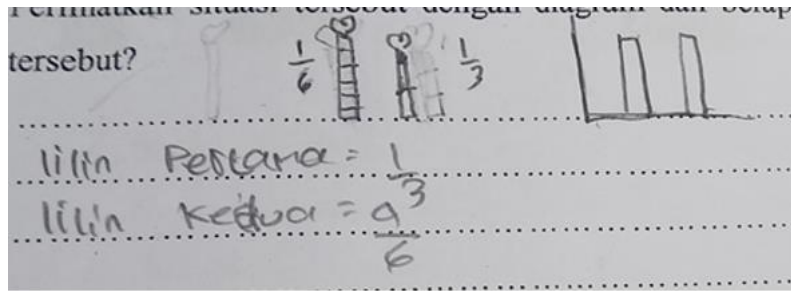


Figure 5. Student answers to question no. 4

In Figure 5 above, it can be seen that students are working on fractions as ratios. The problem involves subtraction operations and student ratios. Students drew candles and started to divide them into pieces, but students needed help understanding how to do it because this was the first time they had done a problem like this. When writing down their answers, they go along with their classmates without understanding the meaning of the questions. This mistake occurred because the students needed help understanding the purpose of the questions. After being confirmed by interviews, the students did not understand the meaning of the questions, so they did not know how to answer them because it was their first time working on questions like this.

Question number five

On Wednesday dad came home with pizza. The pizza is divided into 10 equal portions. Mom gets 2 portions of pizza, sister gets 2 portions of pizza, 20% goes to aunt. How many slices of pizza are left? Make a picture pattern from the problem!

Problem number five is a matter of fractions as an operator, one of the answers given by the students is as follows:

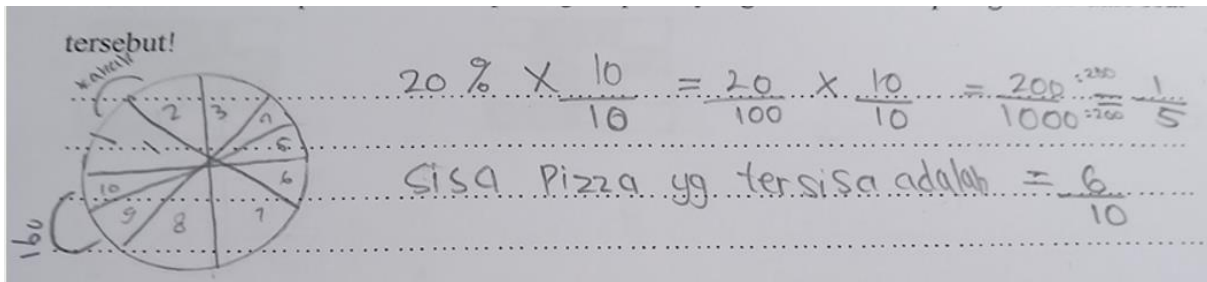


Figure 6. Student answers to question no. 5

Figure 6 above shows students working on fraction problems as operators. The above problem involves division and multiplication operations on fractions; when dividing, the students immediately divided by describing the pizza each made. When it is divided into $\frac{1}{10}$ parts, the students do not understand that one part is the pizza; the students also had difficulties when the percentage number reduced the portion of the pizza. Students focus on finding the 20% value, so they must be corrected to determine the final result of the reductions made. Student errors occur because students need to understand the meaning of the questions fully. Students work according to their abilities based on experience working on previous fraction problems. After

being confirmed by interviews, the students needed help understanding the question's meaning. The questions given involve several arithmetic operations and different forms of numbers.

Discussion

The discussion includes learning obstacles students experience when working on problems in the learning process, then discussed and reinforced with related theories. The following is a discussion based on the questions posed.

Question number one is a matter of fractions of parts of the whole. The mistakes are: 1) Students have written down the value of the fraction correctly but still operate the denominator with the denominator and the numerator with the numerator. 2) Students write down the shaded part's numerator and the not-shaded part's denominator. 3) Students only write down the shaded amount in integer form and operate it. Based on confirmation of interviews with students, these students said they did not focus on the question instructions; they only focused on the pictures, so they did not work according to the instructions on the questions. Because questions like these are new questions for students, with these conditions, students are declared to experience epistemological obstacles. Epistemological obstacles (obstacles of epistemological origin) are essentially someone's knowledge that is only limited to specific contexts. If the person is exposed to a different context, his knowledge becomes unusable or needs help (Brousseau, 1997; Suryadi, 2013; Schneider, M. 2014).

Question number two is a matter of fractions as a measure. The problem involves multiplication and addition of fractions. When carrying out multiplication operations, students directly multiply mixed fractions with integers, but there is an error in the steps taken, so the results given are wrong. Students multiply fractional numbers with integers directly without changing one of them or multiplying by the numerator only. This error occurs because the student needs help understanding the meaning of the question. Students work according to their abilities based on experience working on fractions directly. After being confirmed by interviews, the students understood how to do it like that, based on the understanding taught them through fractional problems, which were straight to the numbers, not story questions. When asked the teacher, the teacher said the questions that were often given to students were in the form of direct questions. Fractions word problems are rarely given. Thus, students' abilities cannot be used, and students are forced to experience epistemological obstacles. Epistemological obstacles (obstacles of epistemological origin) are essentially someone's knowledge that is only limited to specific contexts. If the person is exposed to a different context, his knowledge becomes unusable or has difficulty using it (Suryadi, 2013; Prihandhika et al., 2020; Sulastri et al., 2022).

Question number three is a matter of fractions as quotients. The problem involves the operation of subtraction and addition of fractions. However, students perform subtraction by subtracting the numbers in the question without understanding the meaning of the problem. Then, students also made mistakes when changing the form of mixed fractions. This confusion occurs because students do not understand the meaning of the questions, what operations should be used, and how to do them. After being confirmed by interviews, the students needed help

understanding the purpose of the questions, so they needed to learn how to do them. Obstacles experienced by students include epistemological obstacles. Epistemological obstacles (obstacles of epistemological origin) are essentially someone's knowledge that is only limited to specific contexts. If the person is exposed to a different context, his knowledge becomes unusable, or he needs help with using it (Brousseau, 1997; Suryadi, 2013; Syahril et al., 2021).

Question number four is a matter of fractions as ratios. The problem involves subtraction operations and student ratios. Students draw candles and start dividing them into $\frac{1}{6}$ and $\frac{1}{3}$, but students need help understanding how to do this because this is the first time they have done a problem like this. When they wrote down their answers, they just went along with their classmates without understanding the meaning of the questions. This mistake occurred because the students did not understand the purpose of the questions. After being confirmed by interviews, the students did not understand the meaning of the questions, so they did not know how to answer them because it was their first time working on questions like this. Obstacles experienced by students include epistemological obstacles. Epistemological obstacles (obstacles of epistemological origin) are essentially someone's knowledge that is only limited to specific contexts. If the person is exposed to a different context, his knowledge becomes unusable or needs help (Brousseau, 1997; Suryadi, 2013; Hariyani et al., 2022).

Question number five is a matter of fractions as operators. The problem involves division and multiplication operations on fractions; when dividing, the students immediately divided by describing the pizza each made. When it is divided into ten parts, the students must understand that one part of the pizza is $\frac{1}{10}$. Then, the students also had difficulties when the percentage number reduced the portion of the pizza. Students focus on finding the 20% value, so they are mistaken in determining the final result of the reductions made. Student errors occur because students do not fully understand the meaning of the questions. Students work according to their abilities based on experience working on previous fraction problems. After being confirmed by interviews, the students needed help understanding the question's meaning. Because the questions given involve several arithmetic operations and different forms of numbers, obstacles experienced by students include epistemological obstacles. Epistemological obstacles (obstacles of epistemological origin) are essentially someone's knowledge that is only limited to specific contexts. If the person is exposed to a different context, his knowledge becomes unusable or has difficulty using it (Brousseau, 1997; Suryadi, 2013; Unaenah et al., 2023).

The novelty of this research is the analysis of learning epistemology obtained using fraction interpretation questions. So the student results in this study describe students' abilities in the five meanings of fractions, namely: 1) Fractions as part of a whole; 2) Fractions as operators; 3) Fractions as a measure; 4) Fractions as a quotient; 5) Fractions as ratios, (Susan et al., 2012; Purnomo, 2015).

Conclusion

This study reveals that epistemological obstacles affect students' understanding and interpretation of fractions. It was also found that students' understanding of fractions was related to procedural skills in how to solve problems. Based on the study's results, the errors in students'

answers come from epistemological constraints such as understanding fraction operations, changing the form of fractions, and changing fractions to decimals. A lack of understanding of the concept of fractions or the inability to understand is an epistemological obstacle in understanding operations on fractions, equating denominators to fractions, and changing the form of fractions.

Obstacles can also lead to more fundamental educational problems. Many didactic practices justified by the simple additive classical model must be reviewed and rejected. However, this model influences internal (within and between classes) and external (between teachers and society) negotiations with the education system regarding the teaching curriculum. However, the didactic problem is not only that the diagnosis of errors, their explanations, and the following descriptions have changed but also that the roles and obligations of teachers and students have been shifted.

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Conflicts of Interest

The authors declare no conflict of interest regarding the publication of this manuscript.

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Author Contributions

Een Unaenah: Conceptualization, writing - original draft, editing, visualization, review, editing, formal analysis, and methodology; **Didi Suryadi:** Validation and monitoring; **Turmudi:** validation and monitoring.

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