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Scoring rubric design to measure the ability to prove plane geometry problems not accompanied by image visualization

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Abstract

Proof of the type of problem not accompanied by image visualization will require a longer flow and process than proof of the kind of problem accompanied by image visualization. The ability of students to prove problems, especially problems not accompanied by image visualization, must be adequately expressed and objectively. For that, we need an instrument that can reveal the ability to prove the case of these problems. This research has successfully designed a scoring rubric that can be explicitly used to measure students' proving abilities on problems not accompanied by image visualization. Aspects developed in the scoring rubric include making image visualizations according to the information in the questions. These include sub-aspects of image accuracy and completeness of labels, initial steps of proving, preparation of conjectures, flow of proving, and support for valid arguments for statements made. Based on the validation from the experts, the scoring rubric developed was declared valid and ready to be used to measure the student's proving ability on plane geometry, proving problems not accompanied by image visualization.

Keywords: image visualization; plane geometry; proof; scoring rubric

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Introduction

Plane geometry, in particular, and geometry, in general, is essentially an essential subject in mathematics (Naidoo & Kapofu, 2020) in which many geometric ideas are essential components of mathematical reasoning (Patac et al., 2022) characterized by there is an interaction between the visual appearance of geometric elements and a conceptual understanding of the meaning contained in them (Ramírez-Uclés & Ruiz-Hidalgo, 2022). It is very reasonable because geometry is a subject of mathematical study that can connect mathematical ideas with the physical reality of life (Indrayany & Lestari, 2019). In addition, by studying geometry, the skills needed to examine, ask, guess, and experiment (Naidoo & Kapofu, 2020) can be obtained and can be used as a vehicle to develop different ways of thinking in mathematics (Gridos et al., 2021) and as a vehicle for learning to make valid arguments (Scristia et al., 2022).

Plane geometry, especially Euclidean geometry, is considered the main subject of mathematics, which is the primary source for teaching mathematical argumentation, reasoning, and proving (Mwadzaangati, 2019). Meanwhile, proving and reasoning are essential mathematical skills in geometry subjects (Peligro et al., 2018). As one of the subjects of study in mathematics, plane geometry has special characteristics that make it unique compared to other science subjects, especially in proving (Arifin, 2021).

The problem of proof given in plane geometry can be grouped into two. First, the problem of proof that is accompanied by image visualization, and second, the problem of proof that is not accompanied by image visualization. Of course, each of these types of problems will require a different flow and process of proof. The presence or absence of image visualization on the given proof problem greatly affects the student's ability to prove (Haj-Yahya, 2019). The ability of students to prove problems in both categories of problems must be disclosed properly and objectively. To reveal this ability, an instrument must reveal the ability to prove each type of problem given.

Many studies have been carried out on the use of scoring rubrics to measure proving ability. However, the rubric used is only based on problem-solving steps according to Polya's criteria (Anhar et al., 2019; Kurniawan et al., 2019; Mahfuddin & Caswita, 2021). Meanwhile, other research uses a rubric based only on error analysis based on Newman's criteria (Arifin, 2021; Fallo et al., 2021; Nurikawai et al., 2021). Cirillo and Hummer (2021) use three cognitive domains, knowing, applying, and reasoning, as indicators of these competencies. It shows that until now, there has yet to be found a scoring rubric specifically used to reveal the ability to prove proof problems that are not accompanied by image visualization. Therefore, it is deemed necessary and urgent to develop a scoring rubric that can objectively reveal students' proving abilities related to proof problems that are not accompanied by image visualization. This research aims to develop a valid and reliable scoring rubric that can be used to measure students' ability to prove plane geometry problems, especially problems that are not accompanied by image visualization. It is the novelty of this research.

The proving problems given in plane geometry can be grouped into two. The first is the problem of proof that is accompanied by image visualization. The second is the problem of proof not accompanied by image visualization. In the following, an example of each of these problems is given.

Example 1. The problem of proof accompanied by image visualization

> Prove that the measure of the angle formed by a tangent and a chord that passes through the tangent is half the measure of the intercepted arc. (See image visualization below)

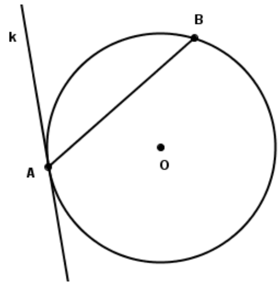

Figure 1. Problem visualization

Example 2. The problem of proof that is not accompanied by image visualization.

> Prove that the measure of the angle formed by a tangent and a chord that passes through the tangent is half the measure of the intercepted arc.

To solve the proof problem in Example 1, students can use the image visualization given to determine the next step of proving. The conditions will be very different when students have to prove the problem, as in Example 2. Students can not only solve the problem in Example 2 by visualizing the image first because the known conditions of the problem have not been clearly described. There needs to be epistemic rationality to make the visualization appropriate (Urhan & Bülbül, 2022). Solving problems not accompanied by visualizations is undoubtedly more complicated than problems accompanied by relevant images (Gagatsis et al., 2022). Furthermore, Gagatsis et al. (2022) stated that making relevant images based on things that were known in the problem was an extra difficult job. It is why related images usually accompany the problems given in geometry textbooks.

Making visualizations when proving is essential because it will produce the desired evidence (VanSpronsen, 2008). Even Zarzycki (2004) explicitly states that visualization is a bridge for proving. By paying attention to the two conditions above, in measuring the ability to prove geometric problems, the type of proving problem given is essential in compiling the scoring rubric.

Arcavi (2003) defines *visualization* as a process and result of the creation of interpretations, images, or diagrams that exist in the mind contained on paper or technological tools intended to describe or communicate information, thoughts about ideas developed that are not yet known beforehand, to have a better understanding. Vavra et al. (2011) define visualization as the process of forming visual graphics and mental images, the process of visually interpreting or representing in a form that can be seen.

Zimmermann and Cunningham (1991) state that mathematical visualization is a process of mentally forming images with paper and pencil or with the help of technology and using these images effectively for mathematical discovery and understanding. Similarly, Pachemska et al. (2016) define *visualization* in mathematics as forming images, such as sketches or drawings used for discoveries or solving mathematical problems. Mudaly and Reddy (2016) state that visualization is sketching diagrams or using symbols when solving Euclidean geometry problems.

Lipovec & Podgoršek (cited in Žakelj & Klančar, 2022, p. 1394) define visualization as a spontaneous identification of mathematical relationships in graphic presentations. From the above definition, there is a match between the definition of visualization stated by Zimmermann & Cunningham (1991) and Mudaly & Reddy (2016). In the context of this study, the definition of visualization referred to refers to the synthesis of definitions stated by Zimmermann and Cunningham (1991), Mudaly and Reddy (2016), and Pachemska et al. (2016) namely, visualization is a process of forming a sketch of an image and using the image effectively to solve the proving problem of proving euclidean geometry.

In the visualization process, students are expected to create, identify, and form visual representations and use them meaningfully to solve problems (Žakelj & Klančar, 2022). Visualization can assist in analysis and problem-solving (Vavra et al., 2011). Diagram visualization plays an essential heuristic role in guiding students' intuition when constructing a reasonable argument or proof (Krajcevski & Sears, 2019). VanSpronsen (2008) states that making a visualization at the time of proving is essential. Creating visualizations will enable the delivery of the desired evidence. The reasons above further emphasize that visualization is critical for solving mathematical problems, including proving. Suppose students can make visualizations related to a given geometry problem. In that case, students' understanding of the problem will increase because they can pay attention to certain concepts not previously observed (Mudaly & Reddy, 2016).

According to Hanna & Sidoli (2007), diagrams or other visual representations have been agreed upon as heuristic accompaniments for proof because visualization not only facilitates understanding of the problem and the evidence but often can also inspire to prove a problem and can be an approach to constructing the evidence alone. Furthermore, Hanna & Sidoli (2007) stated that visual representation, in addition to evidence, is an integral part of proving. Visualization not only organizes data in the form of a meaningful framework but is also an essential factor that can guide the analytical development of the proposed solution (Yilmaz & Argun, 2018).

As stated by Chaachoua (as cited in Mithalal & Balacheff, 2019, p.3),

visualization has two main functions, namely, the illustration function and the experimental function. As an illustration function, visualization can illustrate problem statements, generate hypotheses, or show relevant elements for solutions. Meanwhile, as a practical function, visualization provides a vehicle in which most of the resolution is carried out experimentally. Working on pictures involves identifying new sub-figures or relationships, making or testing conjectures, etc. Sometimes, in proving, students need to make a visualization of the given problem or make a visualization to solve all possible cases. It is necessary to visualize images that have a high level of accuracy (Hadi et al., 2021). The accuracy of the images made will be able to make conjectures and provide correct evidence (Accascina et al., 2005).

Proving is one of the most important mathematical problem-solving techniques (Patac et al., 2022). Therefore, the stages or steps for problem-solving can be applied in proving plane geometry problems (Jupri, 2022). According to Polya's version, the steps in problem-solving consist of 4 stages: understanding the problem, devising a plan, carrying out the plan, and looking back. The four steps in Polya's version of problem-solving can be considered in developing indicators related to the scoring rubric that will be developed.

Several studies have used scoring rubrics to measure problem-solving and proving abilities based on Polya's version of problem-solving steps (Anhar et al., 2019; Kurniawan et al., 2019; Mahfuddin & Caswita, 2021; Riyadi et al., 2021). Another version of using a scoring rubric to measure problem-solving or proving skills can also be adopted from Newman's criteria, even though Newman's criteria are used to see the types of errors that may occur in solving mathematical problems (Arifin, 2021; Cahyani et al., 2020; Clements & Ellerton, 1996; Fallo et al., 2021; Firdaus, 2021; Nurikawai et al., 2021; Singh et al., 2010; Siskawati, 2020; Sutama & Indriyani, 2021; Wardhani & Argaswari, 2022; White, 2009). Related to a person's process in solving problems, Newman (as cited in White, 2009, p. 251) explains that there are five sequential steps that a person must go through in solving problems. The problem-solving process goes through the stages of reading, comprehension, transformation, process skills, and encoding. Although Newman's criteria are used to see the types of errors that may occur in solving mathematical problems, Newman's version of criteria can also be considered in developing indicators related to the rubric that will be developed.

Assessment of the ability to prove problems in geometry is very important. Therefore, the assessment of geometry learning must continue to be developed so that every student can analyze geometric objects into geometric concepts and construct geometric knowledge through formal proving (Noto et al., 2019). To measure the ability to prove geometric problems, Maarif et al. (2020) consider aspects of using sketch diagrams and geometric labels, initial steps, use of conjectures, arguments, flow of thought, and understanding of theorems or related concepts.

In designing and implementing rubrics, using information from the assessment needs to be considered, whether for summative or formative purposes (Panadero & Jonsson, 2020). In developing a scoring rubric to measure students' proving abilities, the type of rubric can be prepared by choosing one of the analytic rubrics or holistic rubrics (Nitko & Brookhart, 2014). The selection must be made carefully by considering various aspects so that the resulting rubric is genuinely adequate, can be used as a valid measuring tool, and can guide the assessment carried out to ensure that the assessment is carried out consistently and transparently (Jonsson, 2014; Nitko, 2001; Nitko & Brookhart, 2014).

Methods

This research used the four-D Models development model developed by Thiagarajan et al. (1974). This model is used because the rubric developed is a tool in the learning system (Azizah et al., 2021; Jaelani & Hasbi, 2022). The 4-D model consists of 4 development stages: Define, Design, Develop, and Disseminate. In this study, the development was only carried out in three stages, namely the Define, Design, and Develop stages, by modifying the aspect analysis and task analysis activities. The stages in the 4-D model were simplified from four to three stages because this research is a multi-year study whose main target is only to reach the stage of compiling a scoring rubric. The concept analysis and task analysis activities in the define stage, which were originally carried out in parallel, were changed hierarchically, starting from concept analysis to task analysis. It is done because the activities must be carried out sequentially to prepare the scoring rubric.

The define stage is intended to analyze the problems and needs an effort to define the aspects that must be considered in preparing the scoring rubric. The design stage is intended for the preparation of aspects or dimensions, the selection of formats, and the initial design of the rubric. The development stage is intended to see the validity of the rubric developed based on the assessment results from the experts.

To determine the validity of the instrument, the method developed by Polit et al. (2007) was adopted using the Modified Kappa k* value, which is formulated with

$$k^* = \frac{ICVI - P_c}{1 - P_c} \qquad (1)$$

k* = kappa approval value

ICVI = Item-level content validity index

Pc = Occurrence probability (Polit et al., 2007; Suhaini et al., 2021)

The ICVI and Pc values are calculated using the following formula, respectively.

$$ICVI = \left(\frac{ne}{N}\right) \qquad (2)$$

$$P_c = \left(\frac{N!}{A!\,(N-A)!}\right).5^N \qquad (3)$$

ne = Number of approvals on the relevant object (3 or 4)

N = Number of expert panels

A = Number of experts agreeing on reasonable items

There are two criteria used to see the validity of the rubric. First, the criteria for each aspect was assessed based on the ICVI value. The existing criteria in each aspect can be used or maintained if the ICVI value is ≥ 0.80 (Polit et al., 2007; Suhaini et al., 2021). Second, the rubric developed is declared valid if the Modified Kappa k* value is in the Excellent category, namely k* 0.75 (Polit et al., 2007; Suhaini et al., 2021).

Results

At the define stage, four activities are carried out: preliminary-final analysis, student analysis, aspect analysis, task analysis, and specification of proving steps.

Start-End Analysis aims to raise the fundamental problems in preparing the scoring rubric. Some of the things that are considered in the start-end analysis are the essence of proof. As explained earlier, proving in geometrical problems is an activity to solve problems based on accurate and logical arguments based on known facts and through a series of logical deductions that finally arrive at what you want to prove. In proving, every statement must always have a valid argument underlying it.

Aspect Analysis Based on The Type of Problem is carried out to systematically compile the aspects that must be considered in compiling the scoring rubric. The aspect that must be considered is the type of problem proposed, whether the problem is accompanied by visualization or not. The stages students must go through when proving problems accompanied by visualization and those without visualization are very different.

The Task Analysis activity was carried out to identify various main aspects that students must master, especially in proving plane geometry problems, especially those related to proving problems not accompanied by image visualization. An analysis of the tasks that students need to do can be presented as shown in the following Figure 2.

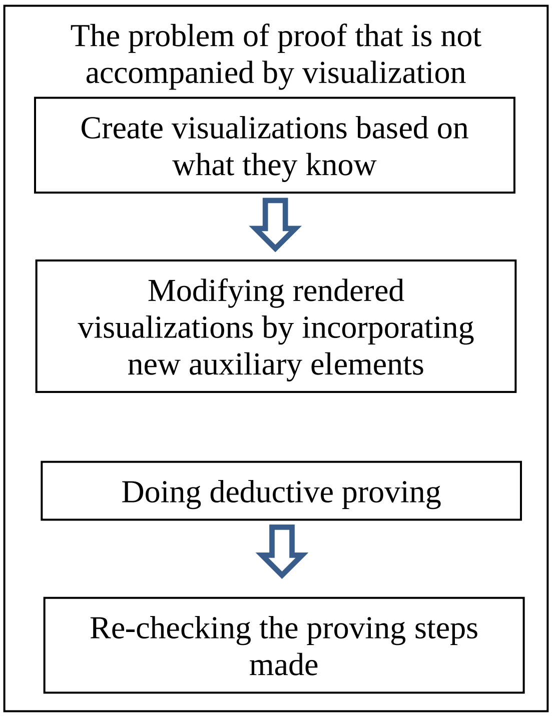

Figure 2. Analysis of the tasks to be performed in proving geometry problems not accompanied by image visualization

The activity in the Specification of Proving step was to write down the main indicators and aspects that students must master in proving plane geometry problems, especially those related to proving problems not accompanied by image visualization. This activity was adjusted to the results of the previous aspect and task analyses. The specification of the purpose of proving was a reference in designing the rubric to be developed.

The final product expected from this research is a scoring rubric that can be used to measure students' ability to solve plane geometry problems, especially related to proving problems not accompanied by image visualization. Activities in the design stage include preparation of aspects or dimensions, selection of formats, and initial design.

The aspect or Dimensional Arrangement stage aims to select the appropriate rubric format to provide scores related to the student's proving ability. Therefore, before determining the rubric format, it is necessary to know in advance the definition of the rubric, the types of rubric formats available, and the weaknesses and advantages of each format.

Based on the previous development stages, namely the Define Stage, Design Stage, and Develop Stage, the scoring rubric of the ability to prove problems can be produced that is not accompanied by image visualization that was successfully formulated in full is presented in the following Table 1.

Table 1. Scoring rubric for measuring ability to prove on problems not accompanied by pictures

No	Aspect	Criteria	Code	Score
1.	Make an image visualization according to the information known in the problem	Make an image visualization appropriately according to the information known in the question	1.a.1	3
	a. Image accuracy	Make a visualization of the image but there is a small part that is not correct or not depicted (less than or equal to 50%) according to the information known in the problem	1.a.2	2
		Making image visualizations but most of them are inaccurate or not depicted (more than 50%) according to the information known in the questions	1.a.3	1
		Make image visualization but everything does not match the information known in the question or does not make image visualization at all	1.a.4	0
	b. Label accessories	Correctly label the images made according to the information known in the questions	1.b.1	3
		There are a small number of labels that are not made or there are a small number of errors in labeling (less than or equal to 50%) according to the information known in the questions.	1.b.2	2
		There are most of the labels are not made or there are most errors in labeling (more than 50%) according to the known information on the question	1.b.3	1
		All labels that are made incorrectly or do not give labels at all to	1.b.4	0

No	Aspect	Criteria	Code	Score
		images that are made according to the information known in the problem		
2.	First step of proof	Start the proof in your own way by making a complete auxiliary element in the image that can support all subsequent proof steps	2.1	3
		Start the proof by making some of the auxiliary elements that are incomplete in the picture that can support all the next steps of the proof	2.2	2
		Starting the proof by creating auxiliary elements in the image but all of them do not support the next proving step	2.3	1
		Absolutely do not create auxiliary elements in the image even though there are auxiliary elements that need to be made to support the next proving step	2.4	0
3.	Conjecture arrangement	All conjectures are arranged according to the purpose of the proof.	3.1	3
		There is a small proportion of conjectures (less than or equal to 50%) that are not arranged according to the purpose of the evidence.	3.2	2
		Most of the conjectures (more than 50%) that are compiled are not in accordance with the purpose of the evidence.	3.3	1
		All conjectures that are arranged do not match the purpose of the evidence or do not make up the conjecture at all	3.4	0
4.	Proof flow	The flow of proof is arranged in an orderly and logical manner	4.1	3
		There is a small part of the proof flow (less than or equal to 50%) which is not arranged in an orderly and logical manner	4.2	2
		Most of the proof flow (more than 50%) is not arranged in an orderly and logical manner	4.3	1

No	Aspect	Criteria	Code	Score
		the flow of proof is not arranged in an orderly and logical manner or there is no flow of proof at all	4.4	0
5.	Support valid argument on statement made	All statements in the proof are supported by valid arguments	5.1	3
		There are a small number of statements (less than or equal to 50%) in the proof that are not supported by valid arguments or even if they are supported by arguments but are not valid	5.2	2
		Most statements (more than 50%) in the proof are not supported by valid arguments or even if they are supported by arguments but are not valid	5.3	1
		All statements in the proof are not supported by valid arguments or do not make a proof statement at all	5.4	0

The development stage aims to produce a draft rubric scoring the ability to prove geometric problems not accompanied by excellent and reliable image visualization according to the criteria. To achieve this goal, in this development stage, an assessment of the product quality is carried out with reference to the product quality criteria developed by Nieveen (1999). According to Nieveen (1999), a product is said to be of high quality if one of the products meets the validity criteria.

From the validation test conducted by three experts, all items on the to-be-retained criteria and based on the value of k* are in the Excellent category. The results of the complete analysis are presented in the following Table 2.

Table 2. Statistical analysis of icvi value and k* value

No	Criteria Code	Number of expert	Number Giving Rating of 3 or 4	ICVI	Rating Level of ICVI	Pc	k*	Rating Level of k*
1	1.a.1	3	3	1	To be retained	0.13	1.000	Excellent
2	1.a.2	3	3	1	To be retained	0.13	1.000	Excellent
3	1.a.3	3	3	1	To be retained	0.13	1.000	Excellent
4	1.a.4	3	3	1	To be retained	0.13	1.000	Excellent
5	1.b.1	3	3	1	To be retained	0.13	1.000	Excellent
6	1.b.2	3	3	1	To be retained	0.13	1.000	Excellent
7	1.b.3	3	3	1	To be retained	0.13	1.000	Excellent

No	Criteria Code	Number of expert	Number Giving Rating of 3 or 4	ICVI	Rating Level of ICVI	Pc	k*	Rating Level of k*
8	1.b.4	3	3	1	To be retained	0.13	1.000	Excellent
9	2.1	3	3	1	To be retained	0.13	1.000	Excellent
10	2.2	3	3	1	To be retained	0.13	1.000	Excellent
11	2.3	3	3	1	To be retained	0.13	1.000	Excellent
12	2.4	3	3	1	To be retained	0.13	1.000	Excellent
13	3.1	3	3	1	To be retained	0.13	1.000	Excellent
14	3.2	3	3	1	To be retained	0.13	1.000	Excellent
15	3.3	3	3	1	To be retained	0.13	1.000	Excellent
16	3.4	3	3	1	To be retained	0.13	1.000	Excellent
17	4.1	3	3	1	To be retained	0.13	1.000	Excellent
18	4.2	3	3	1	To be retained	0.13	1.000	Excellent
19	4.3	3	3	1	To be retained	0.13	1.000	Excellent
20	4.4	3	3	1	To be retained	0.13	1.000	Excellent
21	5.1	3	3	1	To be retained	0.13	1.000	Excellent
22	5.2	3	3	1	To be retained	0.13	1.000	Excellent
23	5.3	3	3	1	To be retained	0.13	1.000	Excellent
24	5.4	3	3	1	To be retained	0.13	1.000	Excellent

Discussion

In the aspect analysis based on the type of problem. When the problem is not accompanied by visualization, the first step that students must take is to make a visualization based on the information about the problem accurately. Making visualizations when proving is crucial because it will produce the desired evidence (VanSpronsen, 2008). Visualization enables students to create meaningful mental images to solve mathematical problems (Moleko, 2021). Making image visualization is an instrumental deconstruction that lays the foundation for problem-solving. Visualizations that serve as illustrations will be able to illustrate problem statements. Generate hypotheses or show relevant elements for resolution (Mithalal & Balacheff, 2019).

In determining the aspects or dimensions of the developed censorship rubric, Polya's version of troubleshooting steps and Newman's version of error type analysis are worth considering. In addition, Maarif et al. (2020) have developed a scoring rubric to measure students' geometric proof construction skills. It is just that this rubric has yet to consider the type of problem given explicitly. Whether the problem is accompanied by visualization or not, by considering the aspects or dimensions developed by Maarif et al. (2020), the aspects or dimensions he has developed need to be adopted in developing this scoring rubric. Thus. in this study, the aspects or dimensions developed in the rubric were carried out by synthesizing the aspects or dimensions of problem-solving in Polya's version—Newman's version of the error type and the dimensions developed by Maarif et al. (2020).

A scoring rubric can be designed in a holistic or analytic form (Nitko & Brookhart, 2014). Holistic rubrics are usually used for summative assessments, while analytic rubrics are more suitable for formative assessments (Brown, 2018). Considering the current learning paradigm, assessment is an integral part of learning that needs to be focused on summative assessment. The scoring rubric format chosen is the analytical rubric format.

The aspects developed in making a scoring rubric to measure the ability to prove geometric problems not accompanied by pictures consist of making image visualizations according to the information in the problem, the initial steps of proof, the preparation of conjectures, the flow of proving, and support for valid arguments on the statements made. Image visualization is the main and most important component that must be considered in compiling the scoring rubric. Problems not accompanied by image visualization cause students to visualize through an identification process to clarify existing mathematical relationships (Lipovec & Podgoršek as cited in Žakelj & Klančar, 2022. p. 1394). they interpret problems through drawings or sketch diagrams (Arcavi, 2003; Mudaly & Reddy, 2016; Vavra et al., 2011) to solve the problem (Pachemska et al., 2016).

In making image visualization according to the information known in the problem, two sub-aspects must be considered: the image's accuracy and the label's completeness. The accuracy of the image in question is how the image is made exactly according to what is known in the problem. For example, whether a known line touches, intersects, or does not intersect the circle, whether a point is inside, outside, or on the circle, whether a point on the line is between two other known points, and so on. At the same time, the completeness of the label in question is the accuracy in naming objects or giving special symbols to geometric objects that are known in the problem. For example, they are labeling a point, a line, or a line segment, naming an angle, giving a special symbol for parallelism, perpendicularity or congruence of two angles or line segments, and others. These two aspects are very important sub-aspects to consider because they will clarify the relationship between existing objects and make it easier to make an analysis to determine the next step of proof (Vavra et al., 2011). They guided students' intuition to build evidence (Krajcevski & Sears, 2019) and facilitate the construction of evidence (Maarif et al., 2019). The accuracy of the images and the completeness of the labels made are the initial sources of information that really must be accurately and correctly described.

The initial step of proving is essential in providing evidence (Maarif et al., 2020; Ozturk, 2021; Sommerhoff & Ufer, 2019). According to Maarif et al. (2020), in constructing geometric proofs, the initial steps of proving will have an essential role in compiling further evidence. To prove problems not accompanied by image visualization in the first step aspect of the criterion, the proof is seen in how students start the proving by utilizing visualizations that had been made correctly by making auxiliary elements. This step can serve as a trick and generate ideas for the next proving step. In simple problems, sometimes this step is unnecessary, so this aspect can be omitted when using the rubric. However, in more complex problems, making auxiliary elements is necessary (Gridos et al., 2021).

Regarding constructing conjectures, the criteria used are that all conjectures compiled in the proof must be by the things to be proven (Maarif et al., 2020). Determining the conjecture is an activity that cannot be separated from constructing evidence (Tripathi, 2020). In proving, accuracy is needed to determine the conjecture of the theorem, which will later be closely related to argumentation and evidence (Mariotti & Pedemonte, 2019).

In the aspect of the flow of evidence, the criteria set are that the flow of evidence must be arranged in an orderly and logical manner. It is because the flow of thinking in proving is a series of ideas contained through steps or ways of proving the problem to be proven (Budiarti, 2014). The logic of the flow of thought will be seen from whether there is a link between the statements in the compiled evidence step (Maarif et al., 2020). In order for the evidence compiled to be easy to understand. a coherent line of thinking is needed without any logical leap (Tripathi, 2020).

In supporting valid arguments for the statements made, the criteria used are that valid arguments must support all statements in the proof. The validity of a mathematical proof is shown by the existence of a series of valid arguments (Tripathi, 2020). In proof, there is no justification for a statement that is not based on a valid argument. Justifying a statement supported by valid arguments will have implications for a conjecture that leads to the desired evidence (Mariotti & Pedemonte, 2019).

The Looking Back aspect of Polya's version of the problem-solving criteria is an important step that must be done when proving. In developing this scoring rubric, it was not considered an aspect. The underlying reason is the activity of checking each step. Re-check whether everything asked has been answered or proven; it will be more accurate to be revealed if the assessor conducts an observation or interview process. Meanwhile, this rubric may be used after the results of the proving work have been collected without making observations when students are working on the proof. In addition, the activity of looking back is an activity that involves mentality, which is likely to be challenging to observe.

Based on the analysis results from the expert test, all items, which are descriptions of the criteria for each aspect of the rubric developed, meet the two criteria for the validity of the rubric. namely the ICVI value ≥ 0.80 and the Modified Kappa k* value ≥ 0.75. meaning that it is in the Excellent category. It confirms that the rubric developed is valid and can be used to measure the ability to prove geometric problems not accompanied by image visualization.

Conclusion

Aspects that need to be developed in making the scoring rubric include making image visualizations according to the information known in the questions, including the sub-aspects of image accuracy and completeness of labels, initial steps of proof, preparation of conjectures, flow of proof, and support for valid arguments for statements made. Based on the validation results, the rubric that has been developed can be applied to measure the intended ability.

The scoring rubric that has been developed has the potential to be applied to measure students' proving abilities, especially in cases of problems that are not accompanied by image visualization. Several aspects developed in the rubric allow this rubric to be used to see potential errors or inability of students, especially related to problems not accompanied by image visualization. For other researchers interested in proof problems that are not accompanied by image visualization, they can also develop a similar assessment rubric so that students' actual abilities in proving such cases can be expressed more thoroughly and accurately.

The rubric developed has just reached the validation test stage. Therefore, it is necessary to continue with the trial phase to see the rubric's practicality. Using the rubric developed is also limited to measuring students' ability to prove problems, especially problems not accompanied by image visualization. This rubric also does not accommodate the looking back aspect (the problem-solving stage in Polya's version) as the final stage in conducting evidence.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this manuscript. Additionally. the ethical issues. including plagiarism. misconduct. data fabrication and/or falsification. double publication and/or submission. and redundancies. have been completed by the authors.

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Author Contributions

I Putu Wisna Ariawan: create the idea, conceptualization, writing - original draft. Visualization, getting the data, formal analysis, and methodology; **I Made Ardana:** conceptualization, writing, review, validation and supervision; **Dewa Gede Hendra Divayana:** writing, review, validation and supervision; **I Made Sugiarta:** conceptualization, writing, review, validation and supervision.

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