Jurnal Elemen, 10(1), 105-120, January 2024
https:/doi.org/10.29408/jel.v10i1.23177
e- ISSN: 2442 - 4226

Check for updates

Exploring decision-making prospective mathematics teacher in solving geometric proof problems

Titin Masfingatin, Davi Apriandi *, Wasilatul Murtafiah, Edy Suprapto, Restu Lusiana

Department of Mathematics Education, Universitas PGRI Madiun, East Java, Indonesia

* Correspondence: davi.mathedu@unipma.ac.id

Abstract

Facing the current digital era, correct and accurate decisions are needed to deal with the problems. This research aimed to explore the decision-making abilities of prospective mathematics teacher students in solving geometric proof problems. This research was a qualitative descriptive study with research subjects 3 out of 32 students of the 2nd semester Mathematics Education study program. The research instrument is the researcher himself, who is assisted with geometric proof problem tests and interview guidelines. Data analysis was including reduction, presentation, and conclusion. The validity of the research data was tested using methods triangulation. The research results show that subjects with a proof ability of K2 can build and clarify geometric concepts/principles and can assess the reasonableness of ideas even though the subject feels unsure about their choice. Subjects with a proof ability of K3 can build and clarify geometric concept/principle ideas but cannot assess the reasonableness of proof ideas. Subjects with a proof ability of K4 can build ideas of geometric concepts/principles, can clarify even though they are incomplete, and cannot assess the reasonableness of proof ideas. Students need to be equipped with decision-making skills in solving proof problems as preparation to become mathematics teachers in the future.

Keywords: decision-making; geometric problem; proof

How to cite: Masfingatin, T., Apriandi, D., Murtafiah, W., Suprapto, E., & Lusiana, R. (2024). Exploring decision-making prospective mathematics teacher in solving geometric proof problems. *Jurnal Elemen*, *10*(1), 105-120. https://doi.org/10.29408/jel.v10i1.23177

Received: 18 October 2023 | Revised: 29 January 2024
Accepted: 5 February 2024 | Published: 8 February 2024

Introduction

Correct and accurate decisions are needed to deal with the problems in the current digital era. A person can find solutions to problems and achieve the desired goals through the right choices. Decision-making is the process of choosing a preferred option or action among a set of alternatives based on given basic criteria or strategies (Vanlommel et al., 2017; Wang & Ruhe, 2007; Yee, 1966; Wang et al., 2006). Decision-making involves the thinking process carried out by a person when solving problems and making decisions (Swartz et al., 1998). Problem-solving activities in mathematics learning allow students and educators to make decisions as answers to learning problems that solve them (Winarso, 2014). When making a decision, a person carries out a thinking process that starts from building ideas, clarifying ideas, and assessing the reasonableness of ideas (Swartz & Parks, 1994). Decision-making in problem-solving is the beginning of all conscious and directed activities (Winarso, 2014).

It is known that a mathematical problem is a problem in mathematics that needs to be determined how to solve it. Mathematical problems students face consist of two types: problems to find and problems to prove. The problem is that students face a problem, so they try to construct all types of objects or information that can be used to obtain a solution. The problem to prove is that students imagine a problem in the form of a statement and then try to prove the truth value of the statement, whether it is true or false. The problem of proving prioritizes a hypothesis or conclusion of a theorem whose truth must be proven. Prospective mathematics teachers often face proof problems, especially in geometry courses. The ability to construct geometric proofs is an absolute requirement for students in the geometry learning process in higher education because one of the determining factors for mathematical maturity is the ability to prove (Maarif et al., 2020; Mariotti & Pedemonte, 2019). The theorem-proving activity requires high reasoning power to combine and analyze information to conclude.

Facts in the field show that many students still need help to prove geometric theorems (Masfingatin et al., 2018, 2020; Murtafiah & Masfingatin, 2015). Students' problems in solving evidentiary problems were also encountered in research (Kurniawan et al., 2023; Mujib, 2019). According to Mujib (2019), one of the obstacles to student learning is formal mathematical writing evidence. According to Herizal (2020), the main factor that influences students' ability to prove is experience. Experience, in this case, is the student's experience in learning to prove and gaining knowledge about how to prove it.

The problems that occur need to be immediately resolved because they are needed to face the challenges of the times. At the higher education level for student teachers, one of the things that can be used as a solution to overcome this problem is to implement freedom to learn-based learning. Freedom to learn (independent learning) is characterized by insatiable curiosity that encourages students to absorb everything they can see, hear, and read (Rogers, 1969). Ki Hajar Dewantara also emphasized independence in learning, that students are not always pioneered or told to acknowledge other people's ideas but are accustomed to seeking their knowledge using their thoughts.

Freedom to learn is based on the constructivist theory that educators act as facilitators who help students form their knowledge and foster independence in students by providing opportunities to act and make decisions (Hendri, 2020). Students make decisions in the process of constructing their knowledge. *Freedom* to learn is defined as freedom to think, freedom to innovate, freedom to learn independently and creatively, and freedom for happiness (Fahmi, 2023; Suhartono, 2021). The freedom to learn policy makes the education system creative, innovative, and characterized (Mazid et al., 2021). The concept of independent learning aims to ensure that students have the freedom to think critically and intelligently.

Several researchers have carried out research related to decision-making in mathematics learning. Previous researchers have examined students' decision-making in solving mathematical literacy problems (Murtafiah et al., 2021). National-level winning student decision-making in designing ICT-based learning media (Murtafiah et al., 2019). Research on decision-making also examines the decision-making of students who win microteaching competitions in designing and implementing mathematics learning (Murtafiah et al., 2022). Other research on decision-making has been carried out on prospective teacher students in designing HOTS questions (Suwarno et al., 2022). However, research on students' decision-making in solving evidentiary problems has yet to be carried out. Thus, it is necessary to know the decision-making abilities of prospective teacher students in solving evidentiary problems.

Methods

This type of research is descriptive with a qualitative approach. *Qualitative descriptive research* is a research method based on the philosophy of postpositivism, which is used to study the condition of objects naturally with the researcher as the main instrument(Creswell, 2015; Creswell, 2012). Qualitative descriptive research aims to describe, explain, and answer in more detail the problems to be studied by studying as many individuals, groups, or events as possible (Johnson & Christensen, 2014). This research describes students' decision-making abilities in solving evidentiary problems.

The subjects of this research were 32 students in the second semester of the PGRI Madiun University Mathematics Education study program for 2022/2023. The selection of research subjects is based on the ability to solve evidentiary problems and considers students' communication skills. Research data was collected using test and interview techniques. The test is a written test about geometric proof problems, adapted from the Elementary Geometry for College Students book (Alexander & Koeberlein, 2011). Semi-structured interviews were conducted with selected subjects to obtain more in-depth and complete data.

The research instrument is the researcher himself, who is assisted with geometric proof problem tests and interview guidelines. The test instrument used in this research is a geometric proof problem relating to the relationship between angles in geometry. The test instruments were essay as follows.

Prove the theorem: "The bisectors of two supplementary angles form a right angle."

Test result data is disclosed and used to investigate student decision-making. The basis for grouping student answers was adapted from (Arnawa et al., 2007; Maarif et al., 2020; Masters, 2010; Middleton, 2009) and is presented in Table 1.

Table 1. Criteria for grouping student answers

Group	Criteria of Students' Answer
K1	The evidence is clear and overall correct. The use of sketch diagrams and geometric labels corresponds to what is used in the proof. The initial proof step has coherence with the subsequent proof steps. Conjectures are prepared according to the purpose of the evidence. Appropriate arguments support all statements. The flow of evidence is arranged logistically. Using appropriate concepts and theorems.
K2	The proof is about 80% correct. Use diagrammatic sketches and geometric labels that correspond to those used in the proof. The initial proof step has coherence with the subsequent proof steps. Conjectures are prepared according to the purpose of the evidence. There is a slight error in determining the proof step argument. Use geometric concepts and theorems correctly.
K3	At least half of the evidence is correct. The use of sketch diagrams and geometric labels corresponds to what is used in the proof. The initial steps are correct, but several steps are not appropriate. Several conjectures are formulated that are not by the purpose of the evidence. Some of the arguments used are inappropriate. Using appropriate concepts and theorems.
K4	Less than half of the evidence is correct. Use sketch diagrams and symbols as appropriate. Errors in determining the initial steps of proof. There is a proof step that is missing. Some arguments are not quite right. There are several concepts and theorems used in false proofs.
K5	Not answering or the evidence prepared is not related to the expected evidence.

Semi-structured interviews were conducted to obtain more complete and in-depth data based on written test results. Interviews were conducted based on the decision-making indicators adapted from (Murtafiah et al., 2021) and are presented in Table 2.

Table 2. Decision-making indicators in resolving evidence problems

Stages of the Decision-Making Process	Description
Building ideas	Students collect all ideas and think about each idea related to the conclusion to be proven.
Clarifying ideas	Analyze the ideas that have been collected and start building proof of ideas. Students classify ideas and can provide reasons and state assumptions of ideas in proof.
Assess the reasonableness of ideas	Assess all existing ideas (which have been built and clarified). Assessments can be made based on existing facts, things/hypotheses, postulates, theorems that have been proven, and definitions or principles that are logical and correct to determine the correct idea.

Research data was analyzed using the stages of data reduction, data presentation, and conclusion (Miles & Huberman, 2007). Data reduction in this research is carried out to select essential data, then present the data according to the indicators and conclude the research results. The validity of the research data was tested using the triangulation method by checking, comparing, and verifying data from written test results and interview results.

Results

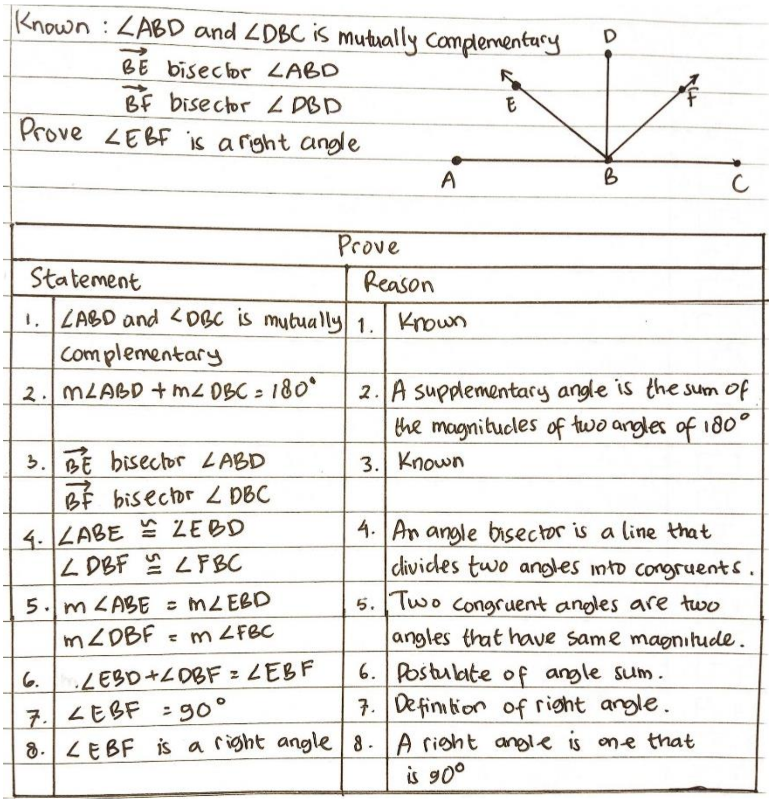
This research began by delivering theorem-proving questions to 32 students in the 2nd semester of Mathematics Education at PGRI Madiun University for the 2022-2023 academic year. The students' geometric proof results were analyzed and reflected based on Table 1. The results of the geometric proof problems test showed that there were no students who K1 (0%), in the K2 group there was one student (3.125%), K3 was three students (9.375%), K4 as many as 11 students (34.375%) and K5 as many as 17 students (54.125%).

Students from the K2, K3, and K4 groups analyze the results of each group of students' written work to obtain information about their decision-making abilities. Students from K5 were not taken as research subjects because the results of the evidence did not match the expected evidence or even did not produce evidence. In this case, it is said that students do not make decisions. Interviews were conducted with subjects who had good communication skills to obtain more in-depth data. Student decision-making in solving proof problems is explained as follows.

Decision-making student K2 group

Building idea stage

The decision-making process by K2 subjects starts from the stage of solving the form of a theorem proof. The ideas developed by S1 in solving theorem-proving problems can be seen in **Figure 1.**


Known : ∠ABD and ∠DBC is mutually complementary
$\overrightarrow{BE}$ bisector ∠ABD
$\overrightarrow{BF}$ bisector ∠DBD
Prove ∠EBF is a right angle

Prove

Statement		Reason	
1.	∠ABD and ∠DBC is mutually complementary	1.	Known
2.	m∠ABD + m∠DBC = 180°	2.	A supplementary angle is the sum of the magnitudes of two angles of 180°
3.	$\overrightarrow{BE}$ bisector ∠ABD $\overrightarrow{BF}$ bisector ∠DBC	3.	Known
4.	∠ABE ≅ ∠EBD ∠DBF ≅ ∠FBC	4.	An angle bisector is a line that divides two angles into congruents.
5.	m∠ABE = m∠EBD m∠DBF = m∠FBC	5.	Two congruent angles are two angles that have same magnitude.
6.	∠EBD+∠DBF = ∠EBF	6.	Postulate of angle sum.
7.	∠EBF = 90°	7.	Definition of right angle.
8.	∠EBF is a right angle	8.	A right angle is one that is 90°

Figure 1. Example of student proof results from K2 group

Figure 1 shows that the subject from K2 group built a proof of idea based on the image he made. The subject builds ideas for the solution by writing down what is known based on the picture and what will be proven. The written test results are explained based on the following interview excerpt.

P : Do you understand the meaning of the question?

K2(1) : this problem is a theorem that is known to be that there are two supplementary angles, each of which has an angle bisector. Then you are asked to prove that the bisector of each angle forms a right angle.

P : How can you prove it?

K2(2) : I made a picture based on the statement in the problem as a first step before proving it. I drew two angles that were side by side, I named them angle ABD and angle DBC. Then I created ray BE, which is the bisector of angle ABD, and ray BF, which is the bisector of angle DBC.

P : What ideas can you find to answer this question?

K2(3) : there are two supplementary angles, namely $\angle ABD$ and $\angle DBC$, each angle has its angle bisector $\overrightarrow{BE}$ is the angle bisector of $\angle ABD$, and $\overrightarrow{BF}$ is the angle bisector of $\angle DBC$ then asked to prove that the angle formed by the angle bisectors ($\angle EBF$) is 90°.

P : Apart from that, are there any other ideas?

K2(4) : definition of supplementary angles, the definition of an angle bisector, definition of complementary angles, postulate of addition of angles, congruent angles, and definition of right angles.

Clarifying idea stage

Based on Figure 1 and the results of the interview, Subject from K2 started solving the problem by creating a picture based on known information. The image that has been constructed by K2 is followed by the process of determining a complete geometric label by the rules existing in the geometric material.

For example, to name points, use capital letters, and for bisectors, it is a ray, namely a line that starts from a point and has an arrow at the other end. Even though image labeling is a simple thing, it is very important. Apart from that, the resulting image will also be the basis for generating ideas for solving evidentiary problems.

S1 writes down known things based on the pictures created. What the subject writes down is that $\angle ABD$ and $\angle DBC$ are mutually complementary; $\overrightarrow{BE}$ is the bisector of $\angle ABD$; $\overrightarrow{BF}$ is the bisector of $\angle DBC$. The subject also wrote that what will be proven is that $\angle EBF$ is a right angle.

Next, K2 arranges the evidence in the form of two columns, namely the statement column and the reasons column. The subject brings up things that are known with geometric concepts, such as definitions and postulates. Each statement is written in the statement column and the supporting reasons are written in the reasons column.

The results of the interview show that student K2 expressed ideas to solve the problem. Starting from making known pictures, writing down the things that will be proven, and arranging the evidence using two columns. The subject mentioned several ideas to support the preparation of proof in the form of the definition of supplementary angles, the definition of bisector angles, the definition of complementary angles, the postulate of addition of angles, congruent angles, and the definition of right angles. Clarifying ideas stage interviews with K2 subjects showed that the subjects were able to clarify the ideas that had been conveyed.

P : Can you explain the ideas you conveyed that were used in the proof?
K2(5) : I make pictures based on things from the theorem, namely the hypothesis or if clause in the theorem. The picture I made will make the proof process easier. If there is no picture, the question sentence is still general, so it is difficult to prove it.
P : What about the next idea?
K2(6) : The next idea is to write down the things that are known and write down the things that I will prove based on the picture that I made. This aims to enable me to easily determine the next proof step. After that, I arranged the proof steps by creating two columns: the statement column and the reasons column.
P : What proof method did you choose to prove the theorem?
K2(7) : I chose the direct proof method because the information I obtained made it possible to carry out direct proof.
P : What is the proof process that you carry out?
K2(8) : from what is known two angles are adjacent to each other and supplement each other according to the picture I made, namely $\angle ABD$ *and* $\angle DBC$, $\overrightarrow{BE}$ *is the bisector of angle* $\angle ABD$, *and* $\overrightarrow{BF}$ *is the line bisector of the angle* $\angle DBC$ *and then asked to prove that the angle formed by the bisector of the angle* ($\angle EBF$) *is* $90°$. *Next, prepare direct evidence from that. Starting from the known* $\angle ABD$ *and* $\angle DBC$ *supplements. Based on the definition of supplementary angles,* $m\angle ABD + m\angle DBC = 180°$. *Definition of the angle bisector so that* $\angle ABE \cong \angle EBD$ *and* $\angle DBF \cong \angle FBC$ *are obtained. Likewise, the definition of congruent angles results in* $m\angle ABE = m\angle EBD$ *and* $m\angle DBF = m\angle FBC$. *Based on the angle addition postulate, we obtain* $\angle EBD + \angle DBF = \angle EBF$, *so it can be concluded that* $\angle EBF = 90°$ *or a right angle.*

The results of the interview showed that subject K2 was able to clarify the ideas he had discovered. The subject can explain each idea presented along with the reasons.

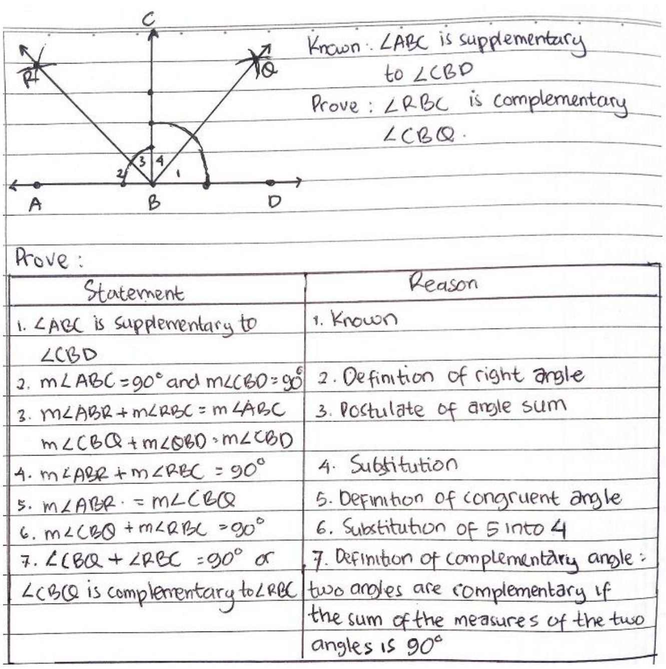
Assess the reasonableness of ideas stage

P : Are you sure that you have proof of the logistics?
K2 (9) : yes, but there is a little less confidence when proving that the EBF angle is a right angle.
P : Why do you feel unsure?
K2 (10) : because there is no logical relationship from the statement $\angle EBD + \angle DBF = \angle EBF$ *it is concluded that* $m\angle EBF = 90°$.

The results of the interview show that subject K2 can assess the reasonableness of his ideas. The material explains the proof process along with the reasons for each step. The subject also explained that he felt doubtful about the final proof step. This means that K2 realized that something was not by the principles or definitions used in compiling the evidence.

Decision-making students of K3 group

The stages of decision-making for Masters subjects are explained as follows.



Statement	Reason
1. ∠ABC is supplementary to ∠CBD	1. Known
2. m∠ABC = 90° and m∠CBD = 90°	2. Definition of right angle
3. m∠ABR + m∠RBC = m∠ABC m∠CBQ + m∠QBD = m∠CBD	3. Postulate of angle sum
4. m∠ABR + m∠RBC = 90°	4. Substitution
5. m∠ABR = m∠CBQ	5. Definition of congruent angle
6. m∠CBQ + m∠RBC = 90°	6. Substitution of 5 into 4
7. ∠CBQ + ∠RBC = 90° or ∠CBQ is complementary to ∠RBC	7. Definition of complementary angle : two angles are complementary if the sum of the measures of the two angles is 90°

Figure 2. Example of student proof results from K3 group

Building idea stage

Subject K3 made a picture based on the statement in the question. Subjects create complete images with labels that comply with the rules of geometry. Interviews with K3 subjects related to the idea-building stage are as follows.

P : Do you know the meaning of the question?
K3(1) : this question is proof, that is, if it is known that two angles are complementary, then you are asked to prove that each angle is complementary.
P : What ideas can be used to solve this problem?
K3(2) : to solve this problem, a picture is needed, things that are known and what will be proven based on the picture, as well as a proof process. Apart from that, the definition of right angles, the definition of complementary angles, congruent angles, the postulate of addition of angles, and the nature of substitution are required.
P : What method of proof will you apply? Why?
K3(3) : I will use the direct proof method because I think it is easier than the indirect method.
P : What are your steps for compiling this evidence?
K3(4) : I made two columns, for statements and reasons. Then I will start compiling proof based on things that are known and with specifications of these things with definitions (definition of congruent angles, right angles, complementary angles), the postulate of addition of angles, and the nature of substitution.

Based on the written results in Figure 2 and the results of the interview show that K3 can express his ideas to provide proof. The interview results showed that K3 expressed ideas to solve problems. K3 mentioned ideas in the form of problem-solving steps, namely making pictures based on what is known, determining what is known and what will be proven based on the picture, and compiling proof. K3 also expresses ideas for proof, namely related to definitions (congruent angles, right angles, complementary angles), the postulate of addition of angles, and the nature of substitution. The subject has the idea of using a direct proof method but for less scientific reasons.

Clarifying ideas stage

Figure 2 is known to show that K3 wrote things based on the pictures he had made. The subject wrote that what is known is $\angle ABC$ supplemented with $\angle CBD$. The subject also wrote down what would be proven, namely that $\angle RBC$ is complementary to $\angle CBQ$. K3 has also prepared evidence in a two-column format, namely the statement column and the reasons column. K3 arranges the evidence in these columns, starting from what is known to what will be proven at the end of the proof. Explanations related to clarifying ideas are as follows.

P : Can you explain the ideas you conveyed earlier for the proof process?

K3(4) : to solve the proof problem, my first step was to make a picture based on the information in the problem. From the question it is stated that 'the bisectors from two angles are mutually complementary'. So I drew angle ABC supplemented with angle CBD. I also drew ray BC as the bisector of angle ABC and ray BQ as the bisector of angle CBD. Then I wrote down the things that are known, namely $\angle ABC$ supplemented with $\angle CBD$, and the things that will be proven, namely $\angle RBC$ complemented with $\angle CBQ$.

P : explain the evidence you have compiled!

K3(5) : The first step I wrote in the statement column was $\angle ABC$ supplemented with $\angle CBD$ and in the reason column I wrote 'known'. Next, based on the definition of a right angle, I wrote $m\angle ABC = 90°$ and $m\angle CBD = 90°$. The third step, $m\angle ABR + m\angle RBC = m\angle ABC$ and $m\angle CBQ + m\angle QBD = m\angle CBD$ is based on the angle addition postulate. $m\angle ABR = m\angle CBQ$ based on the definition of congruent angles. In the 6th step, by substitution we get $m\angle CBQ + m\angle RBC = 90°$ and the conclusion is that $m\angle CBQ$ is complementary to $m\angle RBC$ based on the definition of complementary angles, namely two angles are complementary if the sum of the measures of the two angles is 90°.

Based on the interview results, K3 clarified the ideas found and the reasons, making known images complete with labels based on that information. Next, the subject writes down what he knows but needs to be completed. The subject writes down what will be proven correctly. K3 organizes the proof in a two-column format and connects known things, postulates, definitions and properties, and equations, namely the nature of substitution. In the end, something is obtained that will be proven.

Assessing the reasonableness of the idea stage

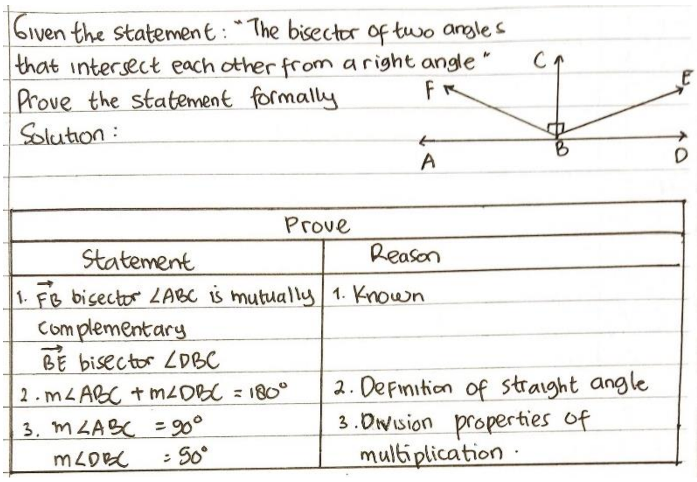
K3 could not assess the reasonableness of the idea, and the subject felt confident in what was done even though what was written was incomplete. Subjects also believe that the evidence compiled is correct, but some of the conclusions obtained are based on something other than logical reasons.

Q : Are you sure that the proof you did is correct?
K3(6) : I am sure my picture is correct because it matches the hypothesis in the question. The proof steps are also correct in each step.
Q : What do you think you know? Have you written it all down?
K3(7) : yes, I have written everything
Q : What about the evidence that you compiled, is every step correct?
K3(8) : yes

K3 prepared the proof with the correct initial steps, but several steps were not appropriate, such as the statement in step number 2, namely $m\angle ABC = 90°$ and $m\angle CBD = 90°$. This is not known in the hypothesis of the problem.

Decision-making students of K4 group

The results of the K4 written work are presented in Figure 3 as follows.


Given the statement: "The bisector of two angles that intersect each other from a right angle"
Prove the statement formally
Solution:

Prove	
Statement	Reason
1. $\overrightarrow{FB}$ bisector ∠ABC is mutually complementary $\overrightarrow{BE}$ bisector ∠DBC	1. Known
2. m∠ABC + m∠DBC = 180°	2. Definition of straight angle
3. m∠ABC = 90° m∠DBC = 90°	3. Division properties of multiplication .

Figure 3. Example of student proof results from K4 group

Building idea stage

K4 re-wrote the question, remaining like the statement in the question. The subject makes a picture based on the question, and then the subject arranges the evidence in a two-column format, namely statement and reason. The written test results are explained based on the following interview excerpt.

P : Do you understand the meaning of the question?
K4(1) : Know the statement: 'the bisector of two supplementary angles forms a right angle'. We are asked to formally prove this statement.
P : How can you prove it?
K4(2) : I made a picture and then compiled the evidence. I draw a right angle ABC at B and a right angle CBD at B adjacent to each other. Then I made the ray BF the bisector of angle ABC and the ray BC the bisector of angle CBD.
P : Are there any other ideas?

K4(3) : Next, I arranged the evidence by making two columns: the statement column and the reasons.

Based on the written results and interviews, K4 developed an idea to solve the proof problem by making an image based on the statement in the theorem. The subject then made a proof in a two-column format.

Clarifying idea stage

Figure 3 shows that K4 wrote back the questions complete with instructions. K4 created an image based on the question and then compiled the evidence by creating two columns: the statement and reasons. The interview results with S3 at the idea clarification stage are as follows.

P : Can you explain the ideas you have conveyed earlier?
K4(4) : first I wrote down the question because it was something that was known. Then I write down what I want to prove, that is, prove the statement formally. Next, I made a picture based on the problem, namely that there were two adjacent angles, each with its bisector.
P : What do you do next regarding what was said earlier?
K4(5) : I prove it by filling in the statement column: 'ray FB bisector angle ABC is supplemented by ray BE bisector angle DBC' and writing what is known in the reason column. In the second step, I wrote the statement: $'m\angle ABC + m\angle DBC = 180°'$ in the reason column I wrote the definition of a straight angle. In the third step, I wrote: $'m\angle ABC = 90°$ and $m\angle DBC = 90°'$ based on the properties of Division in multiplication.
P : Can you explain how you concluded that $m\angle ABC = 90°$ and $m\angle DBC = 90°$?
K4(6) : because the sum of two angles is $180°$, each angle is $90°$

Based on the written results in Figure 3 and the interview results, K4 explained that the first step was to create an image based on the questions to solve the proof questions. The subject explained that he compiled the proof after making the picture. The subject can clarify the ideas presented previously, but some ideas need to be written down, namely things that are known and that will be proven. Subjects only write down things that are known according to the question or re-read the question. Some of the statements made need to be more accurate, for example: 'ray FB, the bisector of angle ABC, is supplemented by ray BE, the bisector of angle DBC'. This statement has no meaning because it does not comply with the rules of geometry. subject K4 was unable to understand the hypothesis so the statement made was incorrect. This also results in the initial steps in compiling evidence being incorrect.

Assessing the reasonableness of an idea stage

K4 believes that the proof process carried out was correct and arrived at a conclusion.

P : Are you sure about that answer?
K4(4) : yes, I'm sure.
P : Has this question been proven?

K4(8) : I think it has been proven.

Subjects cannot judge that the evidence that has been compiled is illogical. Some of the reasons given are not by the previous statement so the statements made are less/illogical.

Discussion

Decision-making ability has indicators that include: students' ability to build ideas, clarify ideas, and assess the reasonableness of ideas (Murtafiah et al., 2019, 2022, 2023). When building ideas, students collect all ideas and think about each idea related to the conclusion that will be proven (Suwarno et al., 2022).

Building idea stage

At the building idea stage, Subject K2 begins problem-solving by creating images based on known information. Likewise, K3 and K4 also build ideas using the strategy of creating images based on known information. It shows that all subjects have ideas in the form of strategies that will be used in proof (Kosko & Herbst, 2012; Nunokawa, 2010).

In contrast to K3 and K4, the image constructed by K2 is followed by a complete geometric label determination process using the rules existing in the geometric material. For example, to name points, use capital letters; for bisectors, it is a ray, a line that starts from a point and has an arrow at the other end. The resulting image will also be the basis for generating ideas for solving evidentiary problems. K2 writes down known things based on the pictures created. This step is in line with (Maarif et al., 2020) that the first step in the proof is to create an image based on the information in the question.

The difference between K2, K3, and K4 in building ideas starting with drawing depends on each student's perception. Students' perceptions will influence the symbols given on the sketch diagram that has been constructed (Maarif et al., 2020).

Clarifying ideas stage

When clarifying ideas, students analyze their collected ideas and begin to build proof of ideas. Idea analysis is carried out to explain the ideas used at the idea-building stage (Colakkadioglu & Celik, 2016). Students classify ideas and can provide reasons and state assumptions of ideas in proof.

Both K2, K3, and K4 can clarify the ideas they have discovered. Subjects K2 and K3 could explain each idea presented along with the reasons. Although K4 can also explain his ideas, some ideas need to be written down, namely things that are known and that will be proven. K4 only writes down known things according to the question or re-reads the question. Students' ability to explain each idea is greatly influenced by their conceptual knowledge regarding geometric facts, concepts, definitions, principles/postulates/theorems (Masfingatin et al., 2020). Geometry is a field of mathematics that is related not only to numbers but also to shapes, data, and space, so reasoning/logic is needed to solve problems (Chua & Wong, 2012).

Additionally, visual-spatial abilities are also needed to understand the concepts and principles (Wahab et al., 2018; Zakaria, 2013).

Assessing the reasonableness of an idea stage

When assessing the reasonableness of ideas, students assess all existing ideas (which have been built and clarified). Assessments can be made based on existing facts, things/hypotheses, postulates, theorems that have been proven, logical, and correct definitions or principles to determine the correct idea (Maarif et al., 2020; Marchisotti et al., 2018).

At this stage, K2 can assess the reasonableness of his idea. The subject explains the proof process along with the reasons for each step. The subject also explained that he felt unsure about the last step of the proof. In contrast to K2, subjects K3 and K4 were unable to assess the idea of reasonableness.

This stage of assessing the reasonableness of an idea is a determinant of the sequence in which a person is said to have good decision-making abilities or not (Murtafiah et al., 2023). This is because, at this stage, the selection or determination of the evidentiary steps to be carried out is carried out. This stage also determines whether a person's decision-making is correct or not. It is proven that K3 and K4, who could not assess the reasonableness of the idea, could not prove the given geometric problem.

Conclusion

With a proof ability of 80% (K2), the subject's decision-making ability can build and clarify ideas on geometric concepts/principles and assess the reasonableness of ideas even though the subject feels unsure about his choice. Subjects with a proof ability of 50% (K3) can build and clarify geometric concept/principal ideas but cannot assess the reasonableness of proof ideas. Subjects with a proof ability of less than 50% (K4) can build ideas on geometric concepts/principles, can clarify even though they are incomplete, and cannot assess the reasonableness of proof ideas. Students need to be equipped with decision-making skills in solving evidentiary problems as preparation to become mathematics teachers in the future. Decision-making abilities for future mathematics teachers are very important because students must also have these abilities to face future challenges that are closely related to rapid changes. Rapid changes require good decision-making skills to determine appropriate, accurate, and efficient choices. Future researchers can develop other mathematical proof problems that are more complex and varied in order to train the decision-making skills of prospective teacher students.

Conflicts of Interest

The authors declare no conflict of interest regarding the publication of this manuscript. In addition, the ethical issues, including plagiarism, misconduct, data fabrication and/or falsification, double publication and/or submission, and redundancies, have been completed by the authors.

Funding Statement

This work received no specific grant from any public, commercial, or not-for-profit funding agency.

Author Contributions

Titin Masfingatin: Conceptualization, writing - original draft, editing, and visualization; **Davi Apriandi:** Writing - review & editing; **Wasilatul Murtafiah:** formal analysis, and methodology; **Edy Suprapto:** Validation; **Restu lusiana:** Supervision.

References

Alexander, D. C., & Koeberlein, G. M. (2011). *Elementary geometry for college students* (5th Ed). Brooks/Cole.

Arnawa, I. M., Sumarno, U., Kartasasmita, B., & Baskoro, E. T. (2007). Applying the APOS theory to improve students ability to prove in elementary abstract algebra. *Journal of the Indonesian Mathematical Society*, 133–148. https://doi.org/10.22342/jims.13.1.80.133-148

Chua, P. H., & Wong, K. Y. (2012). Characteristics of problem posing of grade 9 students on geometric tasks. *Mathematics Education: Expanding Horizons*. the Annual Meeting of the Mathematics Education Research Group of Australasia (MERGA), Singapore.

Colakkadioglu, O., & Celik, D. B. (2016). The effect of decision-making skill training programs on self-esteem and decision-making styles. *eurasian journal of educational research*, *16*(65), 1–35. https://doi.org/10.14689/ejer.2016.65.15

Creswell, J. (2015). *riset pendidikan: perencanaan, pelaksanaan, dan evaluasi riset kualitatif dan kuantitatif* (5th ed.) *[educational research: planning, conducting, and evaluating qualitative and quantitative research]*. Pustaka Pelajar.

Creswell, J. W. (2012). *educational research: Planning, conducting, and evaluating quantitative and qualitative research*.

Fahmi, S. F. S. F. S. (2023). Merdeka belajar-kampus merdeka [Independent learning-independent campus]. *At-Tadris: Journal of Islamic Education*, *2*(1), 92-97. https://doi.org/10.56672/attadris.v2i1.70

Hendri, N. (2020). Merdeka belajar: Antara retorika dan aplikasi [Freedom to learn: Between rhetoric and application]. *E-Tech: Jurnal Ilmiah Teknologi Pendidikan*, *8*(1), 1-29. https://ejournal.unp.ac.id/index.php/e-tech/article/view/107288

Herizal, H. (2020). Faktor yang memengaruhi kemampuan pembuktian matematis siswa [Factors that influence students' mathematical proof ability]. *Vygotsky: Jurnal Pendidikan Matematika dan Matematika*, *2*(1), 33-42. https://doi.org/10.30736/vj.v2i1.187

Johnson, R. B., & Christensen, L. B. (2014). *Educational research: Quantitative, qualitative, and mixed approaches* (5. ed). SAGE.

Kosko, K. W., & Herbst, P. (2012). Evaluating teachers' decisions in posing a proof problem. *Navigating Transitions Along Continuums*. The 34th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, Kalamazoo, Michigan USA.

Kurniawan, S., Rosjanuardi, R., & Suhendra, S. (2023). Students' mathematical argumentation ability when proving mathematical statements based on self-efficacy. *Jurnal Elemen*, *9*(2), 578–590. https://doi.org/10.29408/jel.v9i2.15151

Maarif, S., Wahyudin, W., Alyani, F., & Pradipta, T. R. (2020). Kemampuan mengkonstruksi bukti geometri mahasiswa calon guru matematika pada perkuliahan geometri [Ability to construct geometry proof of prospective mathematics teacher students in geometry lectures]. *Jurnal Elemen*, *6*(2), 211–227. https://doi.org/10.29408/jel.v6i2.2012

Marchisotti, G. G., Domingos, M. D. L. C., & Almeida, R. L. D. (2018). Decision-making at the first management level: The interference of the organizational culture. *RAM. Revista de Administração Mackenzie*, *19*(3). https://doi.org/10.1590/1678-6971/eramr180106

Mariotti, M. A., & Pedemonte, B. (2019). Intuition and proof in the solution of conjecturing problems'. *ZDM*, *51*(5), 759–777. https://doi.org/10.1007/s11858-019-01059-3

Masfingatin, T., Murtafiah, W., & Krisdiana, I. (2018). Kemampuan mahasiswa calon guru matematika dalam pemecahan masalah pembuktian teorema geometri [The ability of prospective mathematics teacher students in solving problems of proving geometric theorems]. *Jurnal Mercumatika : Jurnal Penelitian Matematika dan Pendidikan Matematika*, *2*(2), 41-50. https://doi.org/10.26486/jm.v2i2.272

Masfingatin, T., Murtafiah, W., & Maharani, S. (2020). Exploration of creative mathematical reasoning in solving geometric problems. *Jurnal Pendidikan Matematika*, *14*(2), 155–168. https://doi.org/10.22342/jpm.14.2.7654.155-168

Masters, J. (2010). Automated scoring of an interactive geometry item: A proof-of-concept. *The Journal of Technology, Learning and Assessment*, *8*(7), 1-39. https://ejournals.bc.edu/index.php/jtla/article/view/1626

Mazid, S., Futaqi, S., & Farikah, F. (2021). The concept of "freedom of learning" in a multicultural education perspective. *Ta'dib*, *24*(1), 70-80. https://doi.org/10.31958/jt.v24i1.2759

Middleton, T. J. (2009). *Development of scoring rubrics and pre-service teachers ability to validate mathematical proofs*. The University of New Mexico.

Mujib, A. (2019). Kesulitan mahasiswa dalam pembuktian matematis: Problem matematika diskrit [Student difficulties in mathematical proof: Discrete mathematics problems]. *Jurnal MathEducation Nusantara*, *2*(1), 51-57. https://jurnal.pascaumnaw.ac.id/index.php/JMN/article/view/68

Murtafiah, W., Lestari, N. D. S., Yahya, F. H., Apriandi, D., & Suprapto, E. (2023). How do students' decision-making ability in solving open-ended problems? *Infinity Journal*, *12*(1),133-150. https://doi.org/10.22460/infinity.v12i1.p133-150

Murtafiah, W., Lukitasari, M., & Lestari, N. D. S. (2021). Exploring the decision-making process of pre-service teachers in solving mathematics literacy problems. *Jurnal Pendidikan Matematika*, *15*(2),145-160. https://doi.org/10.22342/jpm.15.2.13908.145-160

Murtafiah, W., Lukitasati, M., Lestari, N. D. S., Zayyadi, M., & Widodo, S. A. (2022). Decision making students winning microteaching contest in design lesson plan and its implementation in mathematics learning. *Cypriot Journal of Educational Sciences*, *17*(6), 2045–2060. https://doi.org/10.18844/cjes.v17i6.7497

Murtafiah, W., & Masfingatin, T. (2015). Thinking process in solve geometry problem of student with low spatial intelligent. *Applied Research of Mathematics and Natural*

Sciences to Improve Its Usefulness for Knowledge and Society. International Conference on Mathematics, Science, and Education 2015 (ICMSE 2015), Semarang.

Murtafiah, W., Sa'dijah, C., Chandra, T. D., & Susiswo, S. (2019). Decision making of the Winner of the National Student Creativity Program in Designing ICT-based Learning Media. *TEM Journal*, *8*(3), 1039–1045. https://doi.org/10.18421/TEM83-49

Nunokawa, K. (2010). Proof, Mathematical Problem-solving, and explanation in mathematics teaching. In G. Hanna, H. N. Jahnke, & H. Pulte (Eds.), *Explanation and Proof in Mathematics* (pp. 223–236). Springer US. https://doi.org/10.1007/978-1-4419-0576-5_15

Rogers, C. R. (1969). *Freedom to Learn*. C. E. Merrill.

Suhartono, O. (2021). Kebijakan merdeka belajar dalam pelaksanaan pendidikan di masa pandemi COVID-19 [Independent learning policy in implementing education during the Covid-19 Pandemic]. *Ar-Rosikhun: Jurnal Manajemen Pendidikan Islam*, *1*(1), 8-19. https://doi.org/10.18860/rosikhun.v1i1.13897

Suwarno, S., Nusantara, T., Susiswo, S., & Irawati, S. (2022). The decision making strategy of prospective mathematics teachers in improving LOTS to be HOTS problem. *International Journal of Nonlinear Analysis and Applications*, *13*(1), 1613–1627. https://doi.org/10.22075/ijnaa.2022.5776

Swartz, R. J., Fischer, S. D., & Parks, S. (1998). *Infusing the teaching of critical and creative thinking into secondary science: A lesson design handbook*. Critical Thinking Books & Software.

Swartz, R. J., & Parks, S. (1994). *Infusing critical and creative thinking into content instruction: A lesson design handbook for the elementary grades*. Critical Thinking Press & Software.

Vanlommel, K., Van Gasse, R., Vanhoof, J., & Van Petegem, P. (2017). Teachers' decision-making: Data based or intuition driven? *International Journal of Educational Research*, *83*, 75–83. https://doi.org/10.1016/j.ijer.2017.02.013

Wahab, R. A., Abdullah, A. H., Abu, M. S., Atan, N. A., Mokhtar, M., & Hamzah, M. H. (2018). A Learning 3d geometry through sketchup make (SPPD-SUM) to enhance visual spatial skills and the level of geometric thinking. *Journal of Fundamental and Applied Sciences*, *10*(6S).

Wang, Y., & Ruhe, G. (2007). The Cognitive Process of Decision Making: *International Journal of Cognitive Informatics and Natural Intelligence*, *1*(2), 73–85. https://doi.org/10.4018/jcini.2007040105

Winarso, W. (2014). Problem Solving, Creativity Dan Decision Making Dalam Pembelajaran Matematika [Problem Solving, Creativity and Decision Making in Mathematics Learning]. *Eduma : Mathematics Education Learning and Teaching*, *3*(1). https://doi.org/10.24235/eduma.v3i1.3

Yee, A. H. (1966). Mathematics probability and decision-making. *The Arithmetic Teacher*, *13*(5), 385–387.

Yingxu Wang, Ying Wang, Patel, S., & Patel, D. (2006). A layered reference model of the brain (LRMB). *IEEE Transactions on Systems, Man and Cybernetics, Part C (Applications and Reviews)*, *36*(2), 124–133. https://doi.org/10.1109/TSMCC.2006.871126

Zakaria, A. H. A. and E. (2013). Enhancing students' level of geometric thinking through Van Hiele's phase-based learning. *Indian Journal of Science and Technology*, *6*(5), 1–15. https://doi.org/10.17485/ijst/2013/v6i5.13