A praxeological review of concept-sequence and series: Comparing Malaysia and Indonesia textbooks

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Abstract

Malaysia, through its curriculum, has implemented mathematics textbooks as the main source. However, not all concepts can be offered to a country. This research aims to analyse the comparison between Indonesian and Malaysian textbooks from a praxeological perspective. The data used is only on the topic of sequences and series. This qualitative research adopts a hermeneutic phenomenological approach, serving as an alternative method. In the context of the Indonesian version of didactic design, this study represents research conducted during the prospective stage of the entire research series. The results of the research show that in the praxis block, there are similarities in the presentation of Indonesian and Malaysian textbooks in introducing the topic of sequences and series. The difference is that in Indonesian mathematics textbooks there are 2 themes, while in Malaysia there is only 1 theme. The design in Malaysian textbooks is more in-depth and comprehensive, and the learning trajectory is arranged more systematically so that Malaysian textbooks allow for no gaps between the types of T used. The use of Malaysian textbooks has the potential to build complete knowledge, so the potential for students to have difficulties will be smaller.

Keywords: praxeology review; sequence and series; textbooks


Received: 15 November 2023 | Revised: 23 April 2024
Accepted: 2 May 2024 | Published: 31 May 2024
Introduction

The education system implemented in a country relies heavily on textbooks. Textbooks used by students can provide good learning opportunities (Baker, 2019; Kharisma, 2016; Pepin et al., 2013; Valverde et al., 2002) so the use of textbooks as teaching materials is very often found in the world of education (Can, 2021). Textbooks are the main curriculum source consisting of an organized set of units (Huang et al., 2022). The content presented in curricula or textbooks is the result of a didactic transposition (Chevallard, 1992; Chevallard & Bosch, 2020) which is influenced by the different cultures and histories in each country (Kang & Kilpatrick, 1992). In mathematics learning, several research results show that mathematics textbooks have a lot of influence on mathematics teaching and learning (Fan et al., 2013; Reys et al., 2004; Tarr et al., 2008) because the quality of mathematics textbooks can influence the way students learn mathematics (Fan, 2013; OECD, 2013; Stein et al., 2007). School textbooks also offer important resources that a mathematics teacher relies on and relies on in choosing the learning approach to be used (Beaton et al., 1996; Fan & Zhu, 2000; Sun et al., 2009). It is therefore important to analyze and compare textbooks between countries to understand the various education systems around the world (Huang et al., 2022). Several studies also show that textbook analysis can provide an important means of explaining differences in student achievement (Reys et al., 2004; Zhu & Fan, 2006) as well as can explain differences in student performance in international comparative studies (Fuson et al., 1988; Li, 2000).

Several researchers have analyzed and compared mathematics textbooks between countries, especially mathematics textbooks. Hendriyanto et al., (2023) have analyzed and compared textbooks used in Indonesia and Singapore on set materials. Another topic studied by Charalambous et al., (2010) was the material on adding and subtracting fractions in three countries, namely Cyprus, Ireland, and Taiwan. Yang et al., (2017) conducted another study, in which they examined geometry problems in middle-grade mathematics textbooks from Taiwan, Singapore, Finland, and the United States. Meanwhile, Mellor et al., (2018) compared how linear function material was presented in South African textbooks and German textbooks. Based on the example above, analyzing and comparing books has become a special concern among mathematics researchers. The topic in this research is so interesting that it makes researchers interested in studying it.

In this research, book analysis was carried out in two Southeast Asian countries, namely Indonesia and Malaysia. Indonesia and Malaysia have ethnic diversity so the implementation of the educational curriculum is adjusted to each ethnic group. Malaysia was chosen because previous research revealed that Indonesia and Malaysia have significant differences in student mathematics performance, for instance, assessments like the Trends in International Mathematics and Science Study (TIMSS) and the Program for International Student Assessment (PISA), (eg Mullis et al., 2000, 2008, 2012; OECD, 2010, 2014, 2016, 2019). Specifically, in the 2022 PISA results, Malaysia was ranked 53th, while Indonesia was only ranked 70th (OECD, 2023). Then, both countries are both located in Southeast Asia, so analyzing and comparing this textbook is apples to apples.
The mathematical topics that will be studied in this research are sequences and series. Initially, this topic was introduced at the elementary school level as number patterns, then continued to material on sequences and series at the middle school and high school levels (MoEC, 2018). This topic is important because it is often encountered in everyday life (Putri et al., 2023; Rachma & Rosjanuardi, 2021), such as sales, purchasing activities, and calculating the proceeds from the sale of goods (Putri et al., 2023). However, previous research reveals that students' abilities in this topic are still low (Maarif et al., 2021; Rachma & Rosjanuardi, 2021). For this reason, the results of this textbook analysis have the potential to provide a broader and deeper picture (Gracin, 2018). This analysis can be a starting point for designing textbooks in the future (Charalambous et al., 2010; Son & Senk, 2010; Yang & Lin, 2015), especially on sequences and series material.

This research needs to be equipped with theoretical and methodological reasons for analyzing mathematics textbooks. In this study, the Anthropological Theory of Didactic (ATD) was selected as the guiding theoretical and methodological framework. (Chevallard, 2006, 2019; Chevallard & Sensevy, 2014). Artigue and Bosh, (2014) define the Anthropological Theory of Didactic (ATD) as a theory for observing mathematical activities through epistemological models of mathematical knowledge. The ATD concept is to manipulate any material with didactic purposes (Juuti & Uitto, 2015). This research adopts praxeology as a contribution to the theoretical aspect (Chevallard et al., 2015) which is an important part of ATD. Using praxeological analysis, this research examines the presentation of the concept of sequences and series in mathematics textbooks in Indonesian and Malaysian textbooks.

In a previous textbook study, research by Siagian et al., (2023) only revealed the presentation of sequence and series material in Indonesian mathematics textbooks. In this research, we consider it necessary to analyze and compare Indonesian mathematics textbooks with other countries, in this case compared with Malaysia. This research offers a comparative analysis from a praxeological perspective and analyzes the differences between Indonesia and Malaysia to produce a general picture of the designs created. The research inquiry revolves around identifying the commonalities and distinctions between Indonesian and Malaysian textbooks concerning sequences and series. Essentially, this research involves a content analysis approach, utilizing a theoretical framework to comprehend the diverse content and educational levels represented in different systems. The ultimate aim is that the analyses and comparisons made within this study will contribute to the improvement of textbooks within the educational system.

Theoretical framework

ATD postulates that every activity related to the production, diffusion, or acquisition of knowledge must be interpreted as an ordinary human activity, and then proposes a general model of human activity built through the idea of praxeology (Bosch & Gascón, 2014). The similarities and differences between an educational system (called an institution) in ATD can be understood and characterized based on the conditions and limitations of the knowledge they use (Chevallard, 2006, 2019; Chevallard & Sensevy, 2014). In ATD, the didactic transposition
process is divided into 4 stages, namely: scholarly knowledge, knowledge to be taught, taught knowledge, and learned knowledge (Bosch & Gascón, 2006, 2014). Each didactic transposition process will involve a particular institution, such as a community of mathematicians, an education system, a mathematics class, or a study community.

The concept of praxeology is a tool offered by ATD (Bosch & Gascón, 2006, 2014; Chevallard et al., 2015). A praxeology comprises two integral components, specifically the praxis block and the logos block, with each of these blocks containing two individual elements, as illustrated in Table 1.

Table 1. Praxeological model of adaptation (Putra & Witri, 2017)

| Praxis Practical Block | Type of Task($T$) | Problems of given
|------------------------|-------------------|-----------------|
|                        | Technique($\tau$) | A way of performing the type of task
| Logos Knowledge Block  | Technology($\theta$) | A way of explaining and justifying (or designing) the technique
|                        | Theory($\Theta$)  | To explain, justify, or generate, whatever part of the technology that my sound is unclear or missing

The praxeology model depicted in Table 1 shows that the knowledge and activities of a praxeology model are denoted as ($T, \tau, \theta, \Theta$) (Chevallard et al., 2015), where ($T$) is the type of assignment given to students, ($\tau$) is the technique used by students in solving problems, ($\theta$) is the technology used, and ($\Theta$) is the theory used to justify a technology. These four elements are interconnected ($T, \tau, \theta, \Theta$).

In a class, assignments ($T$) can come from certain materials in books (Hendriyanto et al., 2023). So, students need techniques ($\tau$) to complete assignments($\tau$) (Nazli Akar & Övez, 2018). In essence, a specific task ($T$) can be accomplished through the application of different techniques ($\tau$), and a technology ($\theta$) can employ several types of techniques ($\tau$) as described by Putra & Witri, (2017). The relationship between a particular task ($T$) and techniques ($\tau$) used to carry it out is termed punctual organization. On the other hand, general technology supports various methods for a range of task types, and this is commonly referred to as local praxeology. However, because a theory ($\Theta$) is always used for several technologies, it is called a regional organization.

Praxeology serves not only as a tool for dissecting mathematical knowledge but also extends its utility to the examination of didactic knowledge. The nature of the didactic task ($T$) is connected to how the teacher instructs mathematics, such as how the teacher organizes situations in the classroom with the aim of so that students can understand several techniques ($\tau$) for completing task ($T$), for example, arithmetic and geometric series. Each teacher applies a variety of didactic techniques, for example, a didactic process that will be teacher- or student-centered. In reality, the technological theory block of a didactic praxeology is used to justify a technique used. These techniques vary greatly depending on knowledge and experience. This didactic praxeology organization is known as a didactic organization.
Methods

Research design
In this research, a qualitative methodology employing a hermeneutical phenomenological approach was utilized. This approach allows for the interpretation of meanings associated with the phenomenon under investigation. Hermeneutic phenomenology draws from both phenomenological and hermeneutic philosophies. Phenomenology's objective is to reveal the interplay between objectivity and subjectivity inherent in contemporary human experiences (Guillen, 2019). Hermeneutics, on the other hand, is employed to fathom the meaning of a particular situation or object from the perspectives of mathematicians, mathematics educators, and students (Keshavarz, 2020; Laverty, 2003). Hermeneutics also aids in clarifying behaviors, encompassing both spoken and unspoken expressions, cultural elements, and organizational systems, and reveals the meaning they contain, all while maintaining their interconnected relationships (Guillen, 2019). The integration of both phenomenology and hermeneutics complements each other effectively. Specifically, this research investigates the phenomenon of examining how the topic of sequences and series is presented in mathematics textbooks in two countries, Indonesia and Malaysia. Employing a hermeneutic phenomenological approach, this research holds significant relevance for scholars and professionals in the field of education (Friesen et al., 2012).

This praxeological research is situated within the Didactical Design Research (DDR) framework, which was introduced by Didi Suryadi in 2010 (Suryadi, 2019a). DDR serves as a research approach that can be applied to uncover specific phenomena. DDR functions are based on two fundamental paradigms: the interpretive and critical paradigms. The interpretive paradigm focuses on the analysis of real-world phenomena arising from the influence of didactic design on cognitive processes. In contrast, the critical paradigm is oriented toward effecting change by suggesting alternative solutions through hypothetical learning designs. The DDR framework, in its execution, consists of three distinct analytical stages: prospective, metapedagogical, and retrospective phases. These stages collectively facilitate a comprehensive investigation of didactical phenomena and the development of innovative educational solutions.

Praxeological research within DDR is situated within the prospective stage. This research is the initial research in a series of interconnected DDR research. In this study, investigators aim to scrutinize the emerging phenomena that underlie the development of hypothetical learning trajectories (HLT), derived from the didactic transposition analysis of sequence and series materials. The primary focus of this research is to explore the implications of employing textbooks in the didactic transposition process, particularly in Indonesian and Malaysian mathematics textbooks.

Selection textbooks for comparison
The textbooks chosen as the primary data for this study are those serving as the primary reference in schools in both Indonesia and Malaysia. Table 2 provides a presentation of mathematics textbooks from both countries.
Table 2. Selection of books from Indonesia and Malaysia

<table>
<thead>
<tr>
<th>Country</th>
<th>Books used</th>
</tr>
</thead>
</table>

Mathematics textbooks from Indonesia refer to the revised edition for eleventh grade of SMA/MA students published by the Ministry of Education and Culture in 2017. This book is a representation of the implementation of revised K13 (Curriculum 2013), which develops 21st century abilities, including critical thinking and problem-solving abilities; communication and collaboration skills; as well as creativity and innovation (Trilling & Fadel, 2009). This book is a book that is used as the main reference in schools in Indonesia to increase the efficiency and effectiveness of learning in Indonesia (Kemdikbud, 2016).

According to a survey conducted by the Indonesian Ministry of Education and Culture, it is reported that textbooks issued by the Indonesian government are extensively utilized by mathematics teachers in Indonesia (Yang & Sianturi, 2017). Meanwhile, the Malaysian textbook used as a data source in this research is a level 2 mathematics book in the Standard Curriculum Secondary Curriculum issued by the Malaysian Ministry of Education in 2017. Both Indonesian and Malaysian textbooks are available online and can be accessed online free. In Indonesian mathematics textbooks, the topic studied in this research is named "Barisan". Meanwhile, in textbooks from Malaysia, it is given the name "Pola dan Jujukan". Both countries place material at the secondary education level.

Data collection procedures

In line with the research objectives, the data utilized in this study pertains to document analysis. Document analysis is a method employed to explore various research aspects by examining written materials, notes, and documents. This approach is commonly employed in educational research, particularly when textbooks or curricula serve as the primary source of data. In this specific case, the documents being analyzed are secondary-level mathematics textbooks from Indonesia and Malaysia. The goal is to scrutinize the content and design within these documents, guided by a suitable conceptual framework.

Data analysis methods

The process of data analysis involves systematically exploring the meaning or significance of the collected information (Hatch, 2002). Cohen et al., (2007) revealed that qualitative data analysis includes the act of organizing, calculating, and explaining the data that has been collected. The analysis of data in qualitative research is an interconnected process, intricately linked with other stages in the progression of qualitative research, including data collection, exploration, and the writing phase (Creswell, 2015). The data analysis procedure in this research consists of 3 main phases, namely: the first phase, the design of arbitrary unit tasks in Indonesian and Malaysian textbooks is selected. Meanwhile, we include topics related to task design in the praxeology table. In the second phase, we created codes related to test design
independently. Coding is done sequentially. In the third phase, intercoder reliability is determined. In this phase, we will reconverge and analyze location inconsistencies in the taxonomy table. Inconsistent tasks will be coded by us for discussion in reaching an agreement. These three phases will take place cyclically and then, if necessary, will ask for advice from experts regarding the presentation of the textbook that has been prepared.

Results

This research uses two analysees, namely the praxis block and the logos block. BI and BM are the names for mathematics textbooks from Indonesia and Malaysia respectively. Before delving into the examination of variations between the praxis and logos components in BI and BM, we initiate the discussion with an introduction to each textbook. The visualization of each BI and BM can be presented in Figure 1 and Figure 2 below.

In our opinion, the introduction of the sequence topic in BI uses a narrative illustration that is less relevant to everyday life, because the illustration may not occur in real life. The context of adding Rp. 1,000 in pocket money aims to interpret Rp. 1,000 as a difference from the previous day's money, but this could potentially cause epistemological obstacles because the construction of knowledge that students have to interpret a difference has been determined previously. The impact of this illustration also has the potential to make it difficult for students to determine the differences/patterns because students think the patterns/differences have been determined beforehand.

In contrast to BI, the topic of sequences in BM is introduced with an activity. This activity can form students' knowledge of line topics. The activities carried out also encourage students to be able to interpret differences that are not predetermined but represent an activity that has been previously designed. Activities at BM also integrate the topic of rows with the culture in Malaysia.
The differences in presenting marching topics become the style of each country. This is a technique for conveying knowledge from each country. In ATD, the knowledge conveyed to students has gone through 4 stages of didactic transposition, namely scholarly knowledge, knowledge to be taught, taught knowledge, and learned knowledge (Bosch & Gascón, 2006, 2014). BI can use the approach used in BM, thereby minimizing the epistemological barriers experienced by students.

**Praxis block analysis**

The types of tasks presented in the textbook are part of the types of tasks \((T)\) in the praxis block. Each textbook has a \((T)\) different number, which in this study is denoted by \(T_1, T_2, T_3, ..., T_n\). The praxis block in BI on the topic of sequences and series is presented in 5 types of tasks \((T_1, T_2, T_3, T_4, T_5)\) (see Table 3) while BM has 7 types of tasks \((T_1, T_2, T_3, T_4, T_5, T_6, T_7)\) (see Table 4). Design BI tasks starting from \(T_1\) using \(T_5\) a contextual problem approach. \(T_1\) and \(T_2\) emphasize students to understand the concept of patterns. Meanwhile, \(T_3\) and \(T_4\) puts more emphasis on students understanding the theory of a sequence. In contrast to BI, the approach used by BM does not use contextual problems. BM uses a simple approach in designing pattern or sequence recognition. \(T_1, T_3, T_4, T_5, \) and \(T_7\) emphasizes fostering students' comprehension of the concept of patterns., while \(T_2, \) and \(T_6\) emphasizes students to understand the concept of sequences.

The tasks \((T)\) in the textbook are completed using \((\tau)\) certain techniques. In a \(T\) particular case at least it can be solved using one technique \((\tau)\). This research uses a type of technique \((\tau)\) adopted by Takeuchi & Shinno, (2020) containing 4 types of techniques, namely perceptual \((\tau_1)\), physical \((\tau_2)\), operational \((\tau_3)\), and algebraic \((\tau_4)\). Perceptual \((\tau_1)\) refers to a way of solving Tusing visuals. Physical \((\tau_2)\) is a solution \(T\) carried out using several physical tools, such as a mirror, compass, ruler, and so on. Operational \((\tau_3)\) is a solution \(T\) that begins by directing students to develop the knowledge they have. Lastly, algebra \((\tau_4)\) is solving \(T\) using mathematical expressions.

<table>
<thead>
<tr>
<th>Table 3. Praxis block in BI</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Type of Task ((T))</strong></td>
</tr>
<tr>
<td>(T_1): Several marbles are grouped and arranged so that each group is arranged in a square shape as follows. The marbles are counted in each group and a sequence is obtained: 1, 4, 9, (\ldots)</td>
</tr>
</tbody>
</table>
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<table>
<thead>
<tr>
<th>Type of Task ( (T) )</th>
<th>Technique ( (\tau) )</th>
<th>Explanation of the methodology utilized in each textbook</th>
</tr>
</thead>
<tbody>
<tr>
<td>Can you find the next number in the sequence? Can you find the sequence pattern? Determine the number of marbles in group 15?</td>
<td>( \tau_1, \tau_3 )</td>
<td>Carry out mental actions to formulate appropriate visualizations and formulate ways to complete tasks using previously owned knowledge.</td>
</tr>
<tr>
<td>Picture. Pile of orange fruit ( T_2 ): the orange collision above. How to determine or estimate the number of oranges in one pile?</td>
<td>( \tau_3, \tau_4 )</td>
<td>Considering the outcomes of perceptual and memory processes to execute mathematical expressions, with the goal of devising suitable problem-solving strategies and taking action to tackle sequence-related problems</td>
</tr>
<tr>
<td>Pay attention to the problem on the side! ( T_3 ): If the height of one step is 20 cm, what is the height of the ladder if there are 15 steps? Determine the sequence pattern!</td>
<td>( \tau_4 )</td>
<td>Perform mathematical expressions to solve sequence applications</td>
</tr>
</tbody>
</table>

\( T_4 \): Lani, a batik craftsman in Gunung Kidul. He can finish 6 pieces of batik cloth measuring 2.4 m \( \times \) 1.5 m in 1 month. The demand for batik cloth continued to increase, so Lani had to provide 9 pieces of batik cloth in the second month, and 12 pieces in the third month. He suspects that the number of batik cloths for the next month will be 3 more than the
| Previous Month | With this work pattern, in what month did Lani finish 63 pieces of batik cloth?

\( T_5: \) Every day Siti saves the rest of her pocket money. The money saved every day for six days follows an arithmetic sequence pattern with the first term \( a = 500 \) and the difference \( b = 500 \). How do you know how much money Siti saved on the 6th day?

\( \tau_4 \) Perform mathematical expressions to solve sequence applications.

| Table 4. Praxis Block in BM

<table>
<thead>
<tr>
<th>Type of Task ((T))</th>
<th>Technique ((\tau))</th>
<th>Explanation of the methodology utilized in each textbook</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 1 \times 1 )</td>
<td>( 1 )</td>
<td>( \tau_3 ) Complete tasks using previously held knowledge</td>
</tr>
<tr>
<td>( 11 \times 11 )</td>
<td>( 121 )</td>
<td>( \tau_3 ) Complete tasks using previously held knowledge</td>
</tr>
<tr>
<td>( 111 \times 111 )</td>
<td>( 12321 )</td>
<td>( \tau_3 ) Complete tasks using previously held knowledge</td>
</tr>
<tr>
<td>( 1111 \times 1111 )</td>
<td>( 1234321 )</td>
<td>( \tau_3 ) Complete tasks using previously held knowledge</td>
</tr>
<tr>
<td>( 11111 \times 11111 )</td>
<td>( 123454321 )</td>
<td>( \tau_3 ) Complete tasks using previously held knowledge</td>
</tr>
</tbody>
</table>

\( T_1: \) Determine the value of the next two terms.

\( \tau_1, \tau_3 \) Carry out mental actions to formulate appropriate visualizations and formulate ways to complete tasks using previously owned knowledge.

\( T_2: \) State the next two numbers.

\( (i) \ 3, 8, 15, 24, 35, \ldots \)

\( (ii) \ 7, 5, 8, 4, 9, 3, \ldots \)

\( (iii) \ 2, 4, 5, 10, 12, 24, 27, \ldots \)

\( (iv) \ 1, 4, 9, 18, 35, \ldots \)

\( \tau_3 \) Complete tasks using previously held knowledge.

\( T_3: \) How will you form the next Fibonacci quadrangle?

\( \tau_1, \tau_3 \) Carry out mental actions to formulate appropriate visualizations and formulate ways to complete tasks using previously owned knowledge.

\( T_4: \) An interior decorator wants to arrange the tiles on the wall like the pattern below.

\( \tau_1, \tau_3 \) Carry out mental actions to formulate appropriate visualizations and formulate ways to complete tasks using previously owned knowledge.
ways to complete tasks using previously owned knowledge.

<table>
<thead>
<tr>
<th>Type of Task($T$)</th>
<th>Technique($\tau$)</th>
<th>Explanation of the methodology utilized in each textbook</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

What's the next pattern?

\[
\begin{align*}
2^2 + (2 + 2 + 1) &= 3^2 \\
3^2 + (3 + 3 + 1) &= 4^2 \\
4^2 + (4 + 4 + 1) &= 5^2 \\
5^2 + (5 + 5 + 1) &= 6^2 \\
\end{align*}
\]

$\tau_3$ Complete tasks using previously held knowledge

$T_5$: (i) State the next two terms
(ii) State the term $n$

$T_6$: What is the pattern for the following propositions?

(i) 1, 4, 9, 18, 35  
(ii) 23, 45, 89, 177 
(iii) 5, 7, 12, 19, 31  
(iv) 0, 4, 2, 6, 4, 8  
(v) 4, 7, 15, 29, 59, 117

$\tau_3$ Complete tasks using previously held knowledge

$T_7$: State the corresponding pairs of numbers in positions A, B, C, D, E.

$\tau_3$ Completing tasks using previously owned knowledge

Overall, the practical blocks in BI start from $T_1, T_2, T_3, T_4,$ and $T_5$ are dominated by $\tau_3$ and $\tau_4$. This indicates that the assignment design on the topic of sequences and series in BI requires students to develop the knowledge they already have and solve it with mathematical expressions. This condition means that students do not have the opportunity to construct their knowledge through observation (Hendriyanto et al., 2023). The learning process should be designed to facilitate the acquisition, processing and imitation of information as well as developing new ideas so as to increase student competence (Rashidov, 2020). Nonetheless, a significant number of Indonesian textbooks solely feature exercises that demand students to
discover solutions without the application of specific methodologies (Hidayah & Forgasz, 2020). In line with this, research by Fuadah et al., (2021) reveals that the problems in Indonesian textbooks are dominated by problems that only require procedural skills.

In BM, knowledge construction of sequences and series is built by carrying out a series of activities. The straightforward activities found in BM consist of organized and continuous learning pathways tailored to the referenced formulation (theory). The activities carried out at BM are representative of the integration of STEM with the education curriculum in Malaysia (Curriculum Development Center (CDC), 2016). This integration is adapted to the curriculum implemented by the Malaysian Ministry of Education, namely the Standard Secondary Curriculum (KSKM). In designing tasks from $T_1$ to $T_7$, the techniques used in BM are dominated $\tau_3$ by occasionally combined with $\tau_1$. This combination of $\tau_1$, and $\tau_3$ cannot be separated from the task design which also accommodates students' perceptual abilities carried out through observation. The design $T$ used in BM is quite simple but requires high cognitive abilities to complete it so that in a task $T$ you can use several types $\tau$. This is in line with the findings of Kul et al., (2018) when comparing textbooks from Turkey and Canada which revealed that the types of tasks given in the two textbooks tended to be more about learning in the cognitive domain. Differences in the presentation of books between countries can be interpreted as a hypothesis that can be formed regarding their effectiveness (Erbaş et al., 2012).

$T_1$, and $T_2$ BI has similarities $\tau$, namely $\tau_1$, and $\tau_3$ to develop a theory (the concept of patterns and sequences) which is then refined in $T_3$, $T_4$, and $T_5$ through $\tau_3$ and $\tau_4$. $T_3$, $T_4$, and $T_5$ puts more emphasis on students to use the understanding they have previously to express through mathematical expressions. The expected formulation of $T_1$, and $T_2$ is $\Theta_1$ temporary $T_3$, $T_4$, and $T_5$ namely $\Theta_2$. Student knowledge about this $\Theta_1$ can be a problem for some students because they are immediately given contextual problems without providing basic knowledge first. As a result, the goal $\Theta_2$ will not be achieved for some students because the students do not understand $\Theta_1$ it well due to differences in students' intelligence levels (Guez et al., 2018). This indicates that it $T_1$, $T_2$, $T_3$, $T_4$, does $T_5$ not accommodate students' knowledge to apply and develop perceptual, memorial and introspective abilities in constructing new knowledge in $\Theta_1$ the world $\Theta_2$. Therefore, the type $T$ used in BI cannot yet form a $\tau$ systematic deep learning trajectory. There is a gap between $T_1$ to $T_5$. The nature of justification in $\tau_1$, and $\tau_3$ on $T_1$, and $T_2$ does not consider students' previous learning experiences which allow for differences in knowledge possessed, students' way of thinking, and students' learning potential. Students' skills cannot be expected to excel if they have not practiced or have experience (Padilla, 1990). Apart from that, the new knowledge he obtains in the form of $\Theta_1$ not yet being able to verify the validity of the new knowledge he has in the form $\Theta_2$ of a true belief that the student discovered from his previous discoveries.

In BM, $T_1$ it $T_7$ has similarities, namely it always uses it $\tau_3$ to introduce a theory (the concept of patterns and sequences). The main focus $T$ in BM is how students get to know the concept of patterns and sequences through $T$ very simple but in-depth types. The expected formulation of $T_1$ finite $T_7$ in BM is $\Theta_1$. Apparently, there is no type $T$ that requires students to
use mathematical expressions. Even though there are types $T$ that use calculations, the type $T$ that is built is how students can do it by building the concept first.

**Logos praxis**

In the logos block there are two components, namely the technology ($\theta$) and theory blocks ($\Theta$). Technology ($\theta$) is used to justify a technique ($\tau$), while theory ($\Theta$) functions to generalize $T$, $\tau$, and $\theta$ the entire process to produce theoretical knowledge.

In BI the formulations produced from several types $T$ are $\theta_1$ and $\theta_2$. $\theta_1$ aims for students to understand the concept of patterns and sequences, while $\theta_2$ it is a development $\theta_1$ that aims to solve contextual problems using mathematical expressions based on previous knowledge. Usage of $\tau_1$, and $\tau_3$ on $T_1$, and $T_2$ and "How to determine or estimate the number of oranges in one pile?" This is possible because the construction of knowledge can be obtained by carrying out mental actions that connect the perception process and develop it. Meanwhile, $\tau_3$, and $\tau_4$ to $T_3$ the extent that $T_5$ it is possible, but it is best to take into account $\theta_1$ what the student has. The qualities $\theta_1$ possessed by students will be the key to obtaining $\theta_2$.

Then, the formulation built on BM from $T_1$ to $T_7$ is $\theta_1$. The type $T$ used focuses on how students understand the concept of patterns and sequences. In BM, everything $T$ focuses on $\theta_1$ construction but $T$ also involves contextual knowledge to familiarize students with solving problems in everyday life. The problems used are quite simple, but still help students build the essence of $\theta_1$ what is expected.

**Discussion**

**Comparison of BI and BM**

In the previous section, researchers have described the praxis block, namely technology ($\theta$) & theory ($\Theta$), and the logos block, namely tasks ($T$) & techniques, ($\tau$) in each BI and BM book. The results of the analysis of two blocks based on two countries (Indonesia and Malaysia) concluded that the $T$ new types could at least be classified into 2, namely: $T_1$: recognizing pattern and sequence topics; and $T_2$: solving contextual problems in pattern and sequence material. These two types $T$ can be specifically divided into 2 major themes, namely: introducing the concept of patterns and sequences and solving contextual problems.

**Table 5. Regional praxeology of sequence and series concepts in BI and BM**

<table>
<thead>
<tr>
<th>Theme</th>
<th>Country</th>
<th>Number $T$</th>
<th>Technique ($\tau$)</th>
<th>Technology ($\theta$)</th>
<th>Theory ($\Theta$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recognize the topic of sequences and series</td>
<td>Indonesia</td>
<td>$T_1, T_2$</td>
<td>$\tau_1, \tau_3$</td>
<td>$\theta_1$: Numbers that are ordered according to certain rules are a number sequence following a certain pattern</td>
<td>$\theta_1$: A number or object arranged</td>
</tr>
<tr>
<td></td>
<td>Malaysia</td>
<td>$T_3, T_4$</td>
<td>$\tau_1, \tau_3$</td>
<td>$\theta_2$: Application of patterns and the</td>
<td>$\theta_2$: Application of the</td>
</tr>
<tr>
<td></td>
<td>Indonesia</td>
<td>$T_3$</td>
<td>$\tau_3, \tau_4$</td>
<td>$\theta_2$: Application of</td>
<td>$\theta_2$: Application of</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$T_4, T_5$</td>
<td>$\tau_4$</td>
<td>of patterns and the</td>
<td>the</td>
</tr>
</tbody>
</table>
Table 5 displays the elements of praxeology used. There are two themes used, namely recognizing sequence and series topics and solving contextual problems. However, in BM there is only one theme used, namely recognizing the topic of rows and series. In BI, the presentation of themes recognizes the topic of sequences and series which are generally presented using $\tau_3$ (operational) with 2 types $T$, namely $T_1$ and $T_2$ also involving $\tau_1$ (perceptual) which respectively refer to $\theta_1$ and $\theta_2$. Meanwhile, BM generally also uses $\tau_3$ (operational) two types $T$, namely $T_3$ and $T_4$ also involves $\tau_1$ (perceptual), each of which also refers to $\theta_1$ and $\theta_2$. This is the similarity of Indonesian and Malaysian textbooks in introducing the concept of ranks and series to students.

The striking difference between the two books is that in BM the main focus is how to build theoretical knowledge to recognize the concepts of patterns and sequences. BM does not provide a type $T$ that requires students to solve contextual problems that refer to $\theta_2$ and $\theta_2$ use mathematical expressions. The knowledge built primarily introduces students to the topic of sequences. For example, look $T_5$ at Figure 3 below.

In Figure 3, students will carry out mental actions by looking at the regularity of the patterns presented. This is in accordance with the didactic situation theory where learning begins with an action situation (Suryadi, 2019). It seems simple, but the process of finding this regularity does not require mathematical expressions, so it helps students construct their knowledge.

In BI, apart from introducing the topic of patterns and sequences, the knowledge built also requires students to understand the theory they have studied to solve contextual problems. In general, what is used in this theme is $\tau_4$ (algebra) which refers to $\theta_2$ and $\theta_2$. The key to success $\tau_4$ is that students can understand $\theta_1$ what they have previously learned well. For example, in the question "With this work pattern, in what month did Lani finish 63 pieces of batik cloth?" There are two things that students must have in solving this problem, namely: students must understand the problems presented in the story, and students must develop strategies to solve these problems. These two things have entered the action and formulation stage in the Theory of Didactical Situations (TDS) (Brousseau, 2002).
The types of tasks in BI range from $T_3$ to $T_5$ are tasks that fall into the *Higher Order Thinking Skills* (HOTS) category. This is because the textbooks in the 2013 curriculum are integrated into *Higher Order Thinking Skills* (HOTS) in the learning process (Pratama & Retnawati, 2018). The consequence that may arise is that students may not be able to do this type of assignment because the student's knowledge about it $\Theta_1$ is not yet able to support them in completing the assignment. This indicates that students have learning obstacles in sequence and series material. Researchers' findings show that many studies have revealed similar things (Hardiyanti, 2016; Maarif et al., 2021; Rachma & Rosjanuardi, 2021). The difficulties experienced by students are caused by errors in delivering the material, giving rise to misconceptions in acquiring knowledge (Tümay, 2016).

**Conclusion**

There are both similarities in the praxis block in Indonesian and Malaysian textbooks. $T_1$, and $T_2$ BI has similarities with $T_3$, and $T_4$ BM. These types $T$ both use $\tau_1$ (perceptual) and $\tau_3$ (operational) in introducing the topic of sequences and series. Other types $T$ have differences in terms of theory ($\Theta$), and techniques ($\tau$) used.

The differences that can be seen from BI and BM are that in BI there are two big themes that are used as objectives, namely recognizing the topic of rows and series; and resolve contextual problems. Meanwhile, in BM, only one theme is used as a goal, namely recognizing the topic of rows and series only. However, the design in BM is very profound because it is given a $T$ simple type but the essence is $\Theta_1$ achieved. The learning trajectory built by BM is more systematic and it is possible that there are no gaps between the types $T$ used. The learning design in BM also allows students to take action because in introducing the concept of rows a series of activities are carried out.

In BI, the design used requires students to think at a higher level. This design can provide the potential for students to experience learning difficulties due to errors in delivering the material. Type $T_3$, $T_4$, and $T_5$ in BI in general it aims to solve contextual problems. This could potentially cause students to have difficulties if $\Theta_1$ they are not yet able to construct new knowledge, namely formulation $\Theta_2$. This is possible because each student has a different level of intelligence. In this way, it appears that the BI design is not too systematic to create a learning trajectory.

This research demonstrates that employing praxeological analysis contributes to elucidating the attributes of knowledge presented in textbooks. Characteristics of the knowledge presented in textbooks, we view textbooks as empirical knowledge that should be taught in the process of didactic transposition in ATD theory. The findings in this research can be used as a reference for educators, policy makers and related parties in considering use of textbooks in the classroom. It is hoped that these findings can become a reference for researchers in designing learning in the future by considering the advantages and disadvantages of each textbook.
Acknowledgment

Thank you to the supervisors, and all parties who have supported this research.

Conflicts of Interest

The authors declare no conflict of interest regarding the publication of this manuscript. In addition, the ethical issues, including plagiarism, misconduct, data fabrication and/or falsification, double publication and/or submission, and redundancies, have been completed by the authors.

Funding Statement

The Author would like to thank LPDP (Lembaga Pengelola Dana Pendidikan/Indonesia Endowment Fun for Educations) which is part of the Indonesia Ministry of Finance for assisting the author in financing the master program and writing this article until it can published, thank you to the supervisors who have guided and provided advice, and all those involved in writing this article.

Author Contributions

Rahmat Kusharyadi: Conceptualization, writing - original draft, editing, and visualization; Siti Fatimah & Kusnandi: reviewing & editing, formal analysis, validation and supervision.

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