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Investigating lower secondary school students' geometric argumentation structure using Toulmin model

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Abstract

Geometric argumentation has an important role in solving mathematical problems in geometric material, so students must have this ability. Each student has different thoughts, including when stating arguments. Each student's arguments will vary and be at different levels. This study aims to determine the levels of lower secondary school students' geometric argumentation. This research was conducted in a lower secondary school involving 20 ninth-grade students. Students participating in this study were asked to work on geometry problems related to proof. Through the proofs carried out, the argumentation structure owned by students is visible. The structures of argumentation given by the students were then analysed using Toulmin's model of argumentation. The components of the Toulmin model used consist of claim, data, warrant, and backing. The results of the analysis of the proof prepared by the students stated that some of the students have been able to reach a high level of geometric argumentation and can compile a series of proofs. But not a few of them also have difficulty compiling the proof, have difficulty providing the components of the Toulmin model, and make some mistakes. Errors made by students include symbol writing errors, calculation errors, and others.

Keywords: geometric argumentation; proofs; Toulmin model

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Introduction

Argumentation is a series of statements used to prove a point. Another understanding of argumentation is the basis for acting, thinking, and communicating through a statement or idea based on strong evidence (Anita et al., 2019). Evidence becomes an important basis in compiling arguments, if there is no evidence then an argument will be weak and easily broken.

Discussing the issue of evidence, one of the subjects that is often related to evidence is mathematics. In mathematics, there is something called mathematical argumentation, which is an expression based on mathematical concepts, rules and principles where predetermined mathematical notation and symbols are not ignored (Resmi et al., 2021). Mathematical argumentation has its role, one of which is as a foundation for expressing ideas accompanied by evidence and theoretical support by mathematical problems (Putra et al., 2022). Various branches of mathematics often require argumentation to solve the problems in it, as expressed by Shamimi and Rosyidi (2021) argumentation helps students in conveying statements appropriately to solve mathematical problems. Soekisno (2015) also argues that developing argumentation can help students improve mathematical problem-solving. One example of a familiar branch is geometry, which is a branch of mathematics that is synonymous with proof.

Through the proof done on geometry material can be seen mathematical argumentation of students or if it is narrowed down to geometric argumentation. Geometric argumentation leads to statements compiled by students using geometry concepts to connect conjectures to conclusions (Lee, 2015). When doing the proof, students do the reasoning process and then they write the answer according to the steps they know, from the answer that can be called student argumentation. Based on this, argumentation is also seen as a basis for improving student reasoning (Sukirwan et al., 2018). The more often students provide arguments in proof, the better students' reasoning will be. Students' proof ability will also be honed if students do it more often because the proof ability that students have affects the argumentation they compile, students who have this ability will be better at presenting their ideas in compiling proofs (Al-Baqie et al., 2022).

It is not uncommon for the argumentation prepared by students to have shortcomings, this can be seen in research by Setiawan (2020) showing that students have not been able to prove it because it consists of various errors, namely concept errors, procedural errors, and carelessness errors. Problems are often found in students in doing mathematical proof, as found in research conducted by Budiarto and Artiono (2019) which states that many students cannot use what is known to prove mathematical problems, not only that but students are also difficult to connect interrelated knowledge. Supporting this incident, research conducted by Basri et al. (2019) also states that some students often do not understand the concept well and have difficulty understanding the entire information about the question, thus preventing students from carrying out proofs. This shows the inability of students to provide arguments and difficulty reaching higher levels of thinking (Shongwe, 2022).

In response to these problems, the cultivation of argumentation as a habit of thinking must be carried out, because argumentation skills begin with reasoning developed through consistent thinking activities (Rosita, 2014). The consistency referred to in this case is getting students used to solving mathematical problems more often. In line with Farhan & Jumardi (2023) opinion that mathematics requires consistent practice to improve their understanding and reasoning about mathematics. Students can learn various methods of proof which can be used to prove mathematical problems. However, it is very unfortunate, that not all students know the proof method, especially for students outside or far from urban areas and with minimal internet signal. Gunur et al. (2018) believe that the environment influences students' ability to solve mathematical problems, this is because students are less motivated due to inadequate facilities such as not having more books, no e-books, and using the internet which is difficult to reach. These limitations result in students only learning what is available and not exploring more widely about mathematical proof. This was also experienced by the students who were the subjects of this research. So researchers want to see the argumentation skills possessed by their students in terms of mathematical proof, of course using various proof methods.

One method that can be recommended is the flow proof method. Flow proof is a method that is presented in the form of a flow chart (Helma, 2019), and is useful for constructing evidence (Scristia et al., 2021). Flow proof can help students fulfill the stages of proof by the required argumentation structure. The argumentation structure becomes a foundation in the formation of argumentation, meaning that every argumentation can be said to be good if it can meet the standards or structures that have been set. Building an argument requires an evaluation aimed at the process of determining whether a mathematical argument can be accepted as valid evidence (İmamoğlu & Toğrol, 2015). It is in this evaluation process that the structure of the argument plays an important role.

The argumentation structure has various models consisting of several components. Argumentation structure models that can be used as a reference include the model according to Toulmin and the model according to Mc. Neill and Krajcik (Sadieda, 2019). However, this research focuses on the Toulmin model, because the Toulmin model is considered a model with complete structural components (Suartha et al., 2020). The Toulmin model is very effective for evaluating an argumentation step (Demiray et al., 2022), this is because the Toulmin argumentation model has advantages in analyzing formal and non-formal arguments (Faizah et al., 2021). The argumentation structure of the Toulmin model consists of data/evidence, claim, warrant, backing, qualifier, and rebuttal (Laamena et al., 2018; Mahdiyyah & Susanah, 2022). These components are the basis of an argumentation, if the argumentation contains all six components, the argumentation is certainly very good (Zulainy et al., 2021).

There is quite a lot of research related to Toulmin model argumentation, one of which is research conducted by Pramesti and Rosyidi (2020) using Toulmin model argumentation to analyze student answers and stating that students can connect data with claims made using a warrant. Another study using the Toulmin model argumentation is research by Umah et al. (2016) which states that in general, the argumentation owned by students consists of data, claim, warrant, and backing components and rarely appears rebuttal or qualifier. As a differentiator, this research aims to analyze students' arguments in geometric proofs using the Toulmin model and then classify students' geometric argumentation abilities into several levels.

Based on this objective, researchers want to see how the flow of proof is prepared by students. The proof prepared by students must contain the various desired components. The questions underlying this research are whether the student's proof flow contains the desired Toulmin model argumentation components, seeing what level of argumentation the student has, and analyzing the deficiencies and problems that exist in the student's mathematical proof structure.

Methods

This research was included in qualitative descriptive research, using an approach that refers theoretically and in the form of content analysis. The content referred to in this study is the proof of geometry performed by students. The proof carried out by students is measured using assessment guidelines that have been created based on the Toulmin model and proof steps. Students who served as the subject of this study as many as 20 students of 4 Lower Secondary School Lalan ninth grade. The selection of subjects was based on smooth communication between the researchers and the teachers at the school. In addition, because this school is far from urban areas, it makes it a challenge to research students from various areas, not just urban areas. The ninth-grade school was chosen as the subject because it was the subject matter chosen by the researchers.

The subject matter used for this research is the similarity of two triangles, consisting of 4 questions, each of which is related to the similarity theorem. Of course, the questions used have been validated by Sriwijaya University Mathematics Education Lecturers based on several indicators. Validity indicators include question content, question construction, and question language. Content indicators assess the suitability of the questions to the selected subject matter, and the questions can be easily understood. Construction indicators include the suitability of questions with evidence, and the abilities being trained, namely communication, representation, reasoning, and argumentation. Language indicators include language conformity with the KBBI, the absence of multiple interpretations, and the use of language that is easy to understand.

Students were asked to answer these four questions which of course take the form of mathematical proof. Before answering the questions given, students were introduced to the flow proof method which is expected to help students in constructing the proof steps. The results of the work done by students will be analyzed and then expressed into several forms of argumentation levels, as follows.

Table 1. Argumentation level

Level	Category
5	Argumentation is very well organized
4	Argumentation is well organized
3	Argumentation is moderately organized
2	Argumentation is badly organized
1	Argumentation is poorly organized

Determining this level is of course based on the Toulmin model which is used when analyzing student evidence. The Toulmin model consists of several components, namely claim, data, warrant, backing, qualifier, and rebuttal, and can be symbolized as C-D-W-B-Q-R

(Purwaningsih, 2019). However, in this study, four components will be used, consisting of three basic components and a backing component. The basic components in the formation of Toulmin's argumentation structure consist of claim, data, and warrant (Faizah et al., 2021). The reason qualifiers and rebuttals are not included in the analysis section is because these two components are difficult to see in the arguments prepared by lower secondary school students, meaning that rebuttals and qualifiers can be the difference between arguments prepared by beginners (lower secondary school students) and arguments prepared by mathematicians (Inglis et al., 2007). The following is a scheme of Toulmin's components that will be used in this study.

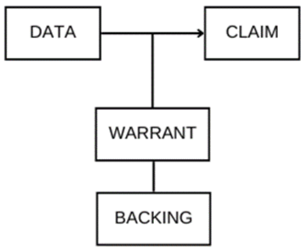

Figure 1. Toulmin's basic components and backing

Figure 1 shows the four components of the Toulmin model of argumentation used. A claim will be proven through an evidence/data process. In achieving a claim, students must use the correct warrant and according to the question given. The warrant used by students must be supported by certain conditions, these conditions or conditions were called backing.

The level of geometric argumentation owned by students will be determined using scoring criteria based on the four components above. In addition to the four components, another important criterion is the structure of the preparation of evidence carried out by students. The scoring criteria are as follows.

Table 2. Scoring criteria

Form of Assessment	Score Criteria	Score		Level
Questions 1-4	• State the claim clearly and correctly (1 Point) • Explaining data/evidence (1 Point) • Proof using warrant (1 point) • There is backing that supports (1 Point) • The flow of evidence is structured (1 Point) Total = 5 points for 1 question, 20 points for 4 questions	17 – 20	5	Argumentation is very well organized
		13 – 16	4	Argumentation is well organized
		9 – 12	3	Argumentation is moderately organized
		5 – 8	2	Argumentation is badly organized
		0 – 4	1	Argumentation is poorly organized

The score criteria in Table 2 were used to assess whether the student's proof contains the four expected Toulmin components, as well as carrying out the proof completely and sequentially. Each student's answer will be dissected by the researcher to see the parts of evidence that are in accordance with the claim, evidence/data, warrant, and backing. Researchers also looked at whether the flow of evidence collected by students was sequential or messy. Each component that is present and appropriate in the student's proof will be awarded one point. All these points were accumulated and seen from the number of points obtained by the student so that the level of the student's argumentation can be determined.

Results

The analysis using the Toulmin model of the proof answers made by 20 students of 4 Lower Secondary School Lalan ninth grade obtained the results listed in the following Table 3.

Table 3. Analysis result

Score	Level	Total of Students	Percentage
17 – 20	Level 5 (Argumentation is very well organized)	0	0%
13 – 16	Level 4 (Argumentation is well organized)	10	50%
9 – 12	Level 3 (Argumentation is moderately organized)	5	25%
5 – 8	Level 2 (Argumentation is badly organized)	5	25%
0 – 4	Level 1 (Argumentation is poorly organized)	0	0%
	Total	20	100%
	Dominant Level	Level 4	

Based on Table 3, it is known that there are no students who fall into the smallest level category of 1, as well as no students whose arguments meet the criteria to reach level 5. It can be seen that the dominant students are in the level 4 argumentation category, namely well-organized arguments. Not a few of them are at level 2, where the arguments they formulate are categorized as not good. In response to this, many factors can reduce points when scoring answers, this can be seen from some of the student answers below.

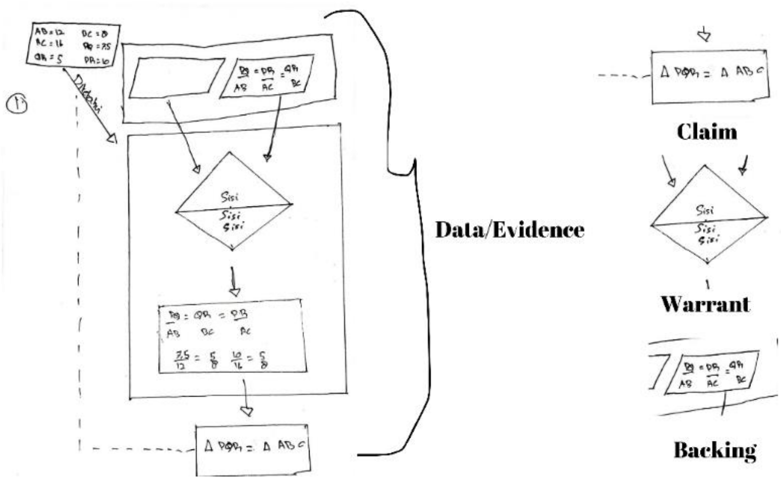

Figure 2. Student answer

The student's answer above shows that the argumentation compiled almost reaches a perfect score, but there is an error that is often taken lightly in learning mathematics. The error in the proof made by the student is in the claim part, namely the wrong use of symbols. The correct answer for the claim part is $\Delta PQR \sim \Delta ABC$ but students write $\Delta PQR = \Delta ABC.$ Different symbols can mean different meanings, if students write $\Delta PQR = \Delta ABC$ it means that the two triangles have the same size, but as we know congruence is not necessarily the same size. This error is included in the factual dimension error, which is an error regarding knowledge of numbers and mathematical symbols (Hidayat & Aripin, 2020).

In addition to the use of symbols that are often wrong, there are several other errors, namely errors in performing calculation operations, and errors in understanding concepts. The following are student answers that contain several errors.

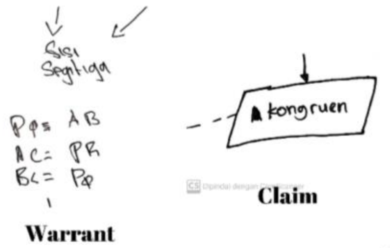

Figure 3. Student answer

Figure 3 shows some student errors in understanding concepts in learning mathematics. The warrant section shows the wrong use of a theory. The theory used should be the side-side-side theorem, not just writing the side of the triangle. As for the claim above, students assume that congruence and equality are the same concepts, even though they are different. Errors regarding concept understanding are included in conceptual dimension errors, namely errors in understanding ideas or ideas related to the properties of an element in mathematics (Suhady et al., 2019).

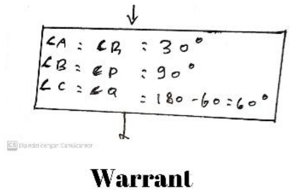

Figure 4. Student answer

Figure 4 shows students' errors in performing calculation operations. Errors in performing calculations often occur in solving math problems. This error is included in the technical dimension error, which is an error that includes counting operations, moving numbers, and steps in counting operations (Ulfa & Kartini, 2021). These errors are often made by students when doing geometric proofs. So that the proof they do becomes less precise and even wrong in some parts of the proof. Of course, this resulted in their points or scores being reduced in the assessment of the argumentation structure. In addition to the above errors, there are still several factors that affect the level of students' argumentation level, namely students who do not carry out structured proofs, and not a few students who have not finished working on problem number 4 due to lack of time. Research conducted by Setiawan (2020) shows that students often make errors in proof procedures. The proof steps prepared by students are not neat and structured. In the assessment criteria, there are separate points for systematic proof, if students do not arrange the proof steps coherently then they will not get these points. Furthermore, Lack of time is a problem that is often experienced by students, this is in line with research conducted by Pramesti and Rini (2019) which states that students often cannot manage time as well as possible in working on math problems. If students have not done a mathematical proof for a particular question, then they will not be given points for that question, therefore time management is very necessary for students so that they are not late in preparing mathematical proofs.

Discussion

Students fulfill 3 levels of argumentation, namely Level 2 (argumentation is badly organized), Level 3 (argumentation is moderately organized), and Level 4 (argumentation is well organized). Each student with a certain level has their own mistakes, the following is a review of student argumentation based on the level they have.

Level 2 (Argumentation is badly organized)

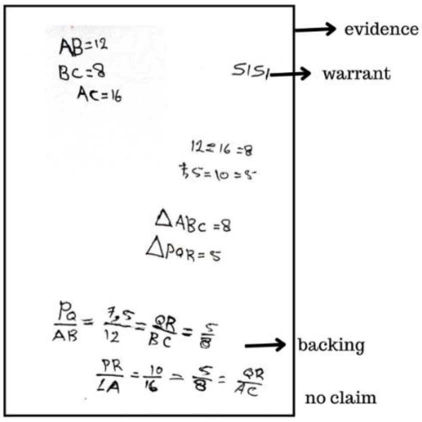

Figure 5. Student answer

Geometric argumentation prepared by students with level 2 tends to be unstructured. This can be seen from the flow of proof written by students there is a less neat flow and there are some steps of proof that are not included. In research conducted by Indrawati & Febrilia (2019), it was stated that students were accustomed to solving problems without completing the proof steps and students could not express ideas. This is what causes some students to work on the proof without completing it. In addition, students with this level of argumentation appear to have difficulty in using warrants, this is because students lack mathematical understanding so conceptual dimension errors occur. Errors in using concepts can also occur if students incorrectly recognize a problem, resulting in inappropriate application of rules, this can also be called a misconception (Al-Mutawah et al., 2019; Mishra, 2020). Research conducted by Abakah and Brijlall (2024) states that misunderstandings in mathematical concepts are more dominant, due to students' inability to remember and understand mathematical concepts. This often happens in the warrant section, students are wrong in using formulas and theories that are not by the prerequisite conditions of the problem (Andriani et al., 2017). In Figure 5, students have incorrectly used the theorem in the warrant part which should be side-side-side.

Level 3 (Argumentation is moderately organized)

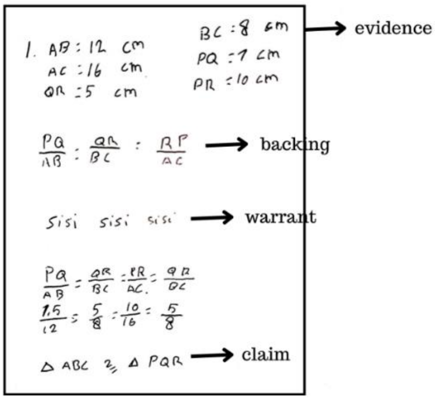

Figure 6. Student answer

At this level, students tend to make mistakes in claims. The error in question is in the form of factual dimension errors, students are often wrong in using symbols. Symbol errors are often encountered, due to several factors including habit and language interpretation errors (Nadjamuddin & Hulukati, 2022). In research conducted by Usodo et al. (2020), symbol errors also occurred, which may seem simple but can affect the entire proof and even affect students' abilities. In Figure 6, the student incorrectly used the symbol in the claim section. The flow of proof compiled by students with this level is good enough to follow the steps of proof. In addition to the use of symbols that are not appropriate, not a few students also incorrectly use the theory in the warrant, as well as proofs that have not been done until completion due to lack of time.

Level 4 (Argumentation is well organized)

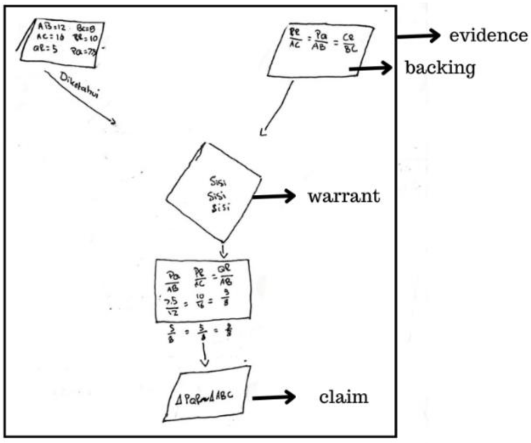

Figure 7. Student answer

At this level, students have been able to compose geometric arguments well. This can be seen from the answers of students with this level tend to be better than students with the level below. In Figure 7, students have been able to compile proof well and correctly. However, several other students made a few mistakes, one of which is an error in calculating which was included in the technical dimension error as in Figure 4. In research conducted by Nawafilah (2019), a similar thing also happened, quite a few students made mistakes in calculating, this was thought to be because the students' focus was disturbed and they were not careful in checking the answers. Some students are slightly wrong in answering, but when viewed as a whole, the flow of proof carried out is good. Not only that, the lack of time is also experienced by students at this level, as seen from some unfinished student answers.

Reviewing what happened to each student, it can be seen that the average student was able to express several components of the Toulmin model of argumentation. The errors that are often seen are only in the claim and warrant sections, where students use symbols incorrectly and theorems or formulas incorrectly. Time limitations are also students' natural enemies because students cannot manage their time well to solve mathematical problems (Pramesti & Rosyidi, 2020). Apart from all that, many students are good at constructing arguments. The results differ from research by Shongwe (2022) which states that students are still categorized as low in constructing arguments because, in the arrangement of students' arguments, there are no rebuttals. Likewise, in research conducted by Andrea (2010), several groups of students and teachers used Toulmin argumentation stages where the results showed that Toulmin argumentation components in the form of claims, data, warrant, and backing could be demonstrated by students, but few could demonstrate rebuttal and qualifier (Soewardini et al., 2020). Based on this, a difference can be seen in this research, as this research does not include rebuttals or qualifiers. To the previous explanation, because these two components are difficult to see in the arguments prepared by lower secondary school students, meaning that rebuttals and qualifiers can be the difference between arguments prepared by beginners (lower secondary school students) and arguments prepared by mathematicians (Inglis et al., 2007).

Furthermore, seeing students' answers that are very creative and not focused on just one method, shows that students have freedom in constructing arguments. In line with research by Sukirwan et al. (2018) students use various ways to express claims, starting from the use of cases, symbol representation, and the use of various student imaginations, of course with notes according to the focus of the question.

Conclusion

Many students were able to formulate arguments. Assessment using Toulmin argumentation for lower secondary school students shows that students are at the level of well-organized argumentation. This conclusion was drawn because the students were dominated by students who were categorized as level 4 argumentation. Based on the argumentation of the Toulmin model, it can be seen that claims and warrants are parts that often contain errors, both errors in the use of theorems and errors in the use of symbols.

Suggestions for future research should pay more attention to the processing time of students because the problem of time is one of the obstacles to students in preparing mathematical proofs. Not only that, understanding of mathematical symbols and language must be emphasized more in learning so that students understand and are not mistaken in using mathematical symbols and language.

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Author Contributions

M. Rizky Ramandani and Yusuf Hartono: Preparation of research instruments and the implementation of data collection. **Cecil Hiltrimartin:** Preparation of theoretical studies related to the research. **Nyimas Aisyah:** Research methodology.

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