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Prospective mathematic teachers' reflective thinking in solving numeracy problems at the critical reflection stage

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Abstract

The low numeracy skills in Indonesia are one factor affecting the quality of mathematics education, so reflective thinking is needed to improve this ability. This study aims to describe prospective mathematics teachers' reflective thinking process in solving numeracy problems at the critical reflection stage based on prior knowledge. This study used a qualitative, descriptive research methodology. The research participants were 34 prospective mathematics teachers at the State University of Malang, East Java. The instruments used are numeracy tests and interview guidelines. Data analysis includes data reduction, presentation, and conclusion drawing. The findings revealed notable differences in reflective thinking based on prior knowledge levels at the critical reflection stage. Subjects with high prior knowledge tried various solutions, were confident in their answers, and accurately drew conclusions and explained their reasons. Subjects with medium prior knowledge tried different methods but needed more confidence in their results. They could conclude but struggled to explain their reasons. Subjects with low prior knowledge used the same method, needed more confidence, and needed help explaining their reasons despite concluding. These findings imply creating a better training program for prospective mathematics teachers, emphasizing numeracy and reflective thinking growth.

Keywords: critical reflection; numeracy; prior knowledge; reflective thinking

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Introduction

Numeracy skills are an essential component in mathematics education and everyday life. These skills enable individuals to perform basic mathematical operations, understand numerical concepts, and apply mathematical knowledge in real-life situations (Ali & Idris, 2013; Saffrina & Baidullah, 2024). Additionally, numeracy skills encompass the ability to access, interpret, and communicate mathematical information effectively, which is crucial for managing the mathematical demands encountered in adult life (Via et al., 2021). Numeracy skills are essential for educational achievement (France et al., 2023; Setiyani et al., 2022). They help individuals analyze, reason, and solve and interpret mathematical problems (Annisavitri et al., 2020; Farida et al., 2021; Linuhung et al., 2023).

Enhancing the standard of mathematics education, particularly numeracy skills, is a pressing issue in Indonesia. In comparison to other Southeast Asian nations, Indonesia is in immediate need of improving its numeracy abilities (Ambarwati & Kurniasih, 2021). The 2022 Program for International Student Assessment (PISA) findings, released by the Organization for Economic Co-Operation and Development (OECD), revealed a concerning trend of declining educational standards in Indonesia. Specifically, the numeracy Indonesia score in 2022 was lower than the assessment in 2018 (OECD, 2023).

Improving numeracy skills in Indonesia is a complex task that faces several challenges. These include the need for effective problem-solving strategies, implementation of solutions, and re-checking of results by students. Additionally, low self-efficacy and student motivation contribute to low numeracy skills (Iswara et al., 2022; Via, 2022). Students often need help organizing their problem-solving strategies, implementing solutions, and reviewing their outcomes, because their numeracy skills and mathematical concept maturity are lacking (Pratiwi, 2021; Via, 2022). Recent education policies like the Minimum Competency Assessment and campus teaching programs aim to enhance numeracy skills in Indonesia. However, further research is crucial to determine the most effective approaches (Marhami et al., 2023; Naitili, 2024; Purnomo et al., 2022).

Developing reflective thinking skills is one efficient approach to improving numeracy skills. Research has shown that developing reflective thinking skills can improve students' achievement in numeracy skills and mathematics (Via et al., 2021). Students' numeracy skills, especially those with field-independent cognitive styles, can be improved by applying a reflective thinking process in solving numerical problems (Setiyani et al., 2023).

Reflective thinking is the mental process of recognizing and solving problems by applying knowledge and experience, starting with confusion and leading to improved understanding and developing new strategies (Kholid et al., 2021; Rodgers, 2002). This process begins with doubt or confusion and involves the individual's effort to identify and address issues by applying pertinent knowledge and experience (Rasyid, 2017; Suharna, 2018). Reflective thinking also entails identifying errors, enhancing understanding, and developing new strategies to solve problems effectively (Anghileri, 2006; Klaczynski, 2014).

Reflective thinking is essential for students and teachers, and incorporating it into teacher education should be a priority (Aldahmash, 2021). This process is crucial for teachers' professional development, as it enhances the standard of education and learning and improves students' educational experiences in classrooms (Choy et al., 2019). Teacher education programs aim to equip students with technical, self-critical, and reflective abilities (Mentari et al., 2018). Reflective thinking is also essential in higher education learning, involving critically considering and evaluating experiences and thoughts (Kember et al., 2000). Consequently, teachers skilled in reflective thinking can significantly improve the standard of learning and better comprehend students' educational experiences (Aldahmash, 2021; Choy et al., 2019).

Various studies on reflective thinking have been conducted, aiming to develop instruments to assess the capacity for reflective thought (Başol & Gencel, 2013; Ghanizadeh, 2017). Research shows that reflective thinking can improve students' academic performance and curiosity (Agustan et al., 2017; Choy et al., 2019). There are three categories of reflective thinking: clarificative, connective, and productive (Suharna, 2018). Prospective mathematics teachers with high mathematical ability exhibit coherent and consistent reflective thought (Sa'dijah et al., 2020). Setiyani et al. (2022) examined how aspiring teachers used their mathematical aptitude to solve numeracy challenges through reflective thinking.

Reflective thinking is essential for developing numeracy skills, and it is positively correlated with prior knowledge. This type of thinking plays a crucial role in enhancing the numeracy skills of prospective elementary school teachers in that it improves their mathematical disposition and problem-solving abilities (Setiyani et al., 2022). Accordingly, students with strong reflective thinking abilities will be better equipped to enhance their numeracy skills. Reflective thinking exercises can further develop these skills (Rakhmawati & Mustadi, 2021; Sellings et al., 2018).

Critical reflection is essential to reflective thinking, pivotal in helping individuals evaluate their actions, identify errors, and enhance problem-solving skills and subject-matter comprehension. Critical reflection involves scrutinizing the assumptions, values, and beliefs underpinning one's actions (Kember et al., 2000). This process enables individuals to critically analyze problem-solving steps, pinpoint mistakes, and adopt new strategies (Setiyani et al., 2023). The research underscores that critical reflection significantly improves mathematics problem-solving skills and deepens students' understanding of the subject matter (Rahayu et al., 2022).

This study aims to bridge the gap in research by explicitly examining the reflective thinking of prospective mathematics teachers at the critical reflection stage when solving numeracy problems, with a focus on their prior knowledge of the inconsistent system of linear equations (Kholid et al., 2020; Setiyani et al., 2023). While many studies have explored reflective thinking and numeracy skills, they have yet to address the inconsistent system of linear equations. The numeracy problem investigated in this study involves the inconsistent system of linear equations with three variables. The findings of this research have the potential to significantly enhance the standards of mathematics education in classrooms and teacher preparation programs in Indonesia.

Methods

This study used a qualitative, descriptive research methodology. This method aims to describe a phenomenon, event, or situation that is currently happening (Creswell, 2015). The research stages include (1) The preparation stage includes the development of research instruments, namely numeracy problem tests and interview guidelines; (2) Data collection: prospective mathematics teachers solve numeracy problems related to the System of Linear Equations Three Variables (SLETV) inconsistent and interview; and (3) Data analysis: reduction data, presenting data, and conclusion.

This study involved 34 sixth-semester prospective mathematics teachers from the Department of Mathematics Education at the State University of Malang, East Java. They had taken the Elementary Linear Algebra course because these students already had an understanding or prior knowledge of Linear Equation Systems. Before collecting data, prior knowledge is categorized based on the results of the problem-solving test for solving the system of linear equations. This category was determined using the mean ($\bar{x} = 69.29$) and standard deviation ($s = 14.19$) of the test scores from this research, following the method outlined by Wijaya et al. (2023). Prospective mathematics teachers were divided into three categories based on their prior knowledge: low, medium, and high.

The data were collected through numeracy problem tests and answer sheet-based interviews with prospective mathematics teachers. Two experts in the fields of mathematics and mathematics education validated the instrument. Content validity was evaluated through the use of a validation form. The results showed that this instrument was feasible to collect research data. Thirty-four prospective mathematics teacher students were given the numeracy test, which was done individually. Figure 1 displays the given numeracy problem test.

Ibu Hanifah berkunjung ke suatu swalayan, dan melihat ada sale paket buah seperti terlihat pada gambar. Ibu Hanifah kemudian membeli ketiga paket buah tersebut. Sesampainya di rumah, Ibu Hanifah penasaran berapa harga perkilogram dari masing-masing buah. Bantulah Ibu Hanifah menentukan harga perkilogram apel, mangga, dan anggur!

Translation

Fruit Promo

Package	Aple	Manggo	Grapes
1	2 kg	2 kg	1 kg
2	3 kg	3 kg	1 kg
3	1 kg	1 kg	

Price:
Package 1: IDR 120,000.00
Package 2: IDR 180,000.00
Package 3: IDR 50,000.00

Mrs. Hanifah visited a supermarket. As shown in the picture, Mrs. Hanifah saw a fruit package sale in the supermarket. Mrs. Hanifah bought all three fruit packages. When she got home, she wondered how much each fruit cost per kilogram. Help Mrs. Hanifah determine the price of apples, mangos, and grapes per kilogram!

Figure 1. Numeracy problem test

The results of the answers of prospective mathematics teachers for each category will be analyzed for their critical reflection stage and reflective thinking process (Kember et al., 2000). Table 1 displays indicators of reflective thinking at the critical reflection stage adapted from Kember et al. (2000) and Setiyani et al. (2023).

Table 1. Indicators of reflective thinking process at critical reflection stage

Reflective Thinking	Indicator
Critical Reflection	- Confusing the solution obtained in solving numerical problems - Looking for other alternative methods. - Being confident about the correctness of the answer obtained. - Being able to conclude and explain why

After completing the test, three prospective mathematics teachers were interviewed. One from each level of prior knowledge who exhibited reflective thinking was selected to explore their reflective thinking process at the critical reflection stage in solving numeracy problems. The researcher interviewed them by considering good communication skills. One subject who demonstrated reflective thinking from each prior knowledge category was selected to be interviewed in-depth. The data from these interviews were analyzed by focusing on the indicators of reflective thinking to identify patterns in their reflective thinking process at the critical reflection stage.

Results

Prospective mathematics teachers solve numeracy problems related to the SLETV. The prospective mathematics teachers' category is based on prior knowledge, as shown in Table 2.

Table 2. Total of prospective mathematics teachers by prior knowledge category and reflective thinking

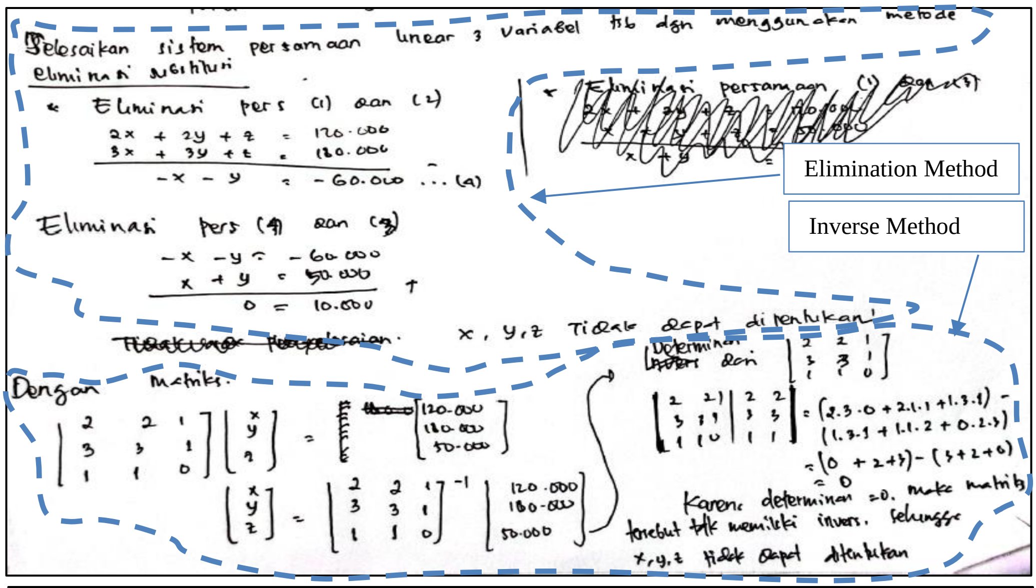
Prior Knowledge Category	Total of Prospective Mathematics Teachers	Total of Prospective Mathematics Teachers Exhibiting Reflective Thinking
High	8	5
Medium	20	15
Low	6	2

Based on Table 2, 34 subjects were categorized based on prior knowledge. Five prospective mathematics teachers with high prior knowledge exhibited reflective thinking. Among the 20 prospective mathematics teachers with medium prior knowledge, 15 exhibited reflective thinking. Meanwhile, out of the six prospective mathematics teachers with low prior knowledge, only two exhibited reflective thinking. Next, one subject who exhibited reflective thinking from each category was selected to be interviewed in-depth based on good communication.

Depending on each category of prior knowledge, the following describes the reflective thinking perspective teachers use to solve numeracy problems at the critical reflection stage.

The reflective thinking process with high prior knowledge subject (HPKS)

Figure 2 displays HPKS's reflective thinking process when solving SLETV numeracy problems at the critical reflection stage.



Translation

Solve the system of linear equations of three variables using the elimination method.

Eliminate equations (1) and (2)

$$\begin{array}{r} 2x + 2y + z = 120{,}000 \\ 3x + 3y + z = 180{,}000 \\ \hline -x - y = -60{,}000 \end{array} \; -$$

x, y, z cannot be determined..

Elimination of equations (4) and (3)

$$\begin{array}{r} -x - y = -60{,}000 \\ x + y = 50{,}000 \\ \hline 0 = 10{,}000 \end{array} \; -$$

With matrix:

$$\begin{bmatrix} 2 & 2 & 1 \\ 3 & 3 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 120{,}000 \\ 180{,}000 \\ 50{,}000 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 & 2 & 1 \\ 3 & 3 & 1 \\ 1 & 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 120{,}000 \\ 180{,}000 \\ 50{,}000 \end{bmatrix}$$

Determinan dari $\begin{vmatrix} 2 & 2 & 1 \\ 3 & 3 & 1 \\ 1 & 1 & 0 \end{vmatrix}$

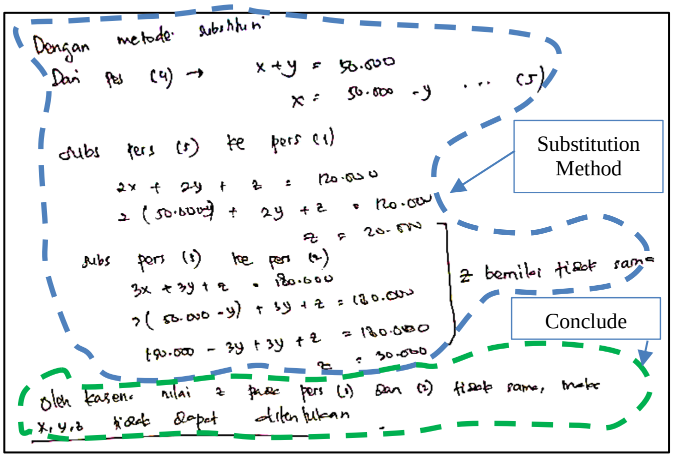
$$\left|\begin{array}{ccc|cc} 2 & 2 & 1 & 2 & 2 \\ 3 & 3 & 1 & 3 & 3 \\ 1 & 1 & 0 & 1 & 1 \end{array}\right| = (2.3.0 + 2.1.1 + 1.3.1) - (1.2.1 + 1.1.2 + 0.2.3) = (0 + 2 + 3) - (3 + 2 + 0) = 0$$

Since determinant $= 0$, the matrix has no inverse.

So, x, y, and z cannot be determined.

Figure 2. HPKS's answer using the elimination substitution and inverse method

Based on Figure 2, HPKS is still confused at the critical reflection stage, as seen from the answer sheet scribbles. Because by using the elimination method, namely by eliminating equations 1 and 2, equation 4 is obtained, namely $-x - y = 60{,}000$. Then HPKS eliminated equations 4 and 3 and obtained the result $0 = 10{,}000$, then HPKS concluded that the values of x, y, and z cannot be determined. HPKS tried to use a different method by using the inverse matrix and found that the determinant value of the coefficient matrix was 0, so the matrix did not have an inverse and concluded that x, y, and z could not be determined. To further confirm the answer, HPKS tried to solve the problem using the substitution method, as displayed in Figure 3.



Translation
By substitution method
From equation (4) → $x + y = 50{,}000$
$x = 50{,}000 - y$ ... (5)
Substitute equation (5) into equation (1)
$2x + 2y + z = 120{,}000$
$2(50{,}000 - y) + 2y + z = 120{,}000$
$z = 20{,}000$
Substitute equation (5) into equation (1)
$3x + 3y + z = 180{,}000$
$3(50{,}000 - y) + 3y + z = 180{,}000$
$150{,}000 - 3y + 3y + z = 180{,}000$
$z = 30{,}000$
Z values are not equal.
Since the values of z in equations (1) and (2) are different x, y, and z cannot be determined.

Figure 3. HPKS's answer using the substitution method

Based on Figure 3, it can be seen that HPKS found that there was a difference in the value found by substituting into equations (1) and (2). Moreover, HPKS got two conflicting results. Therefore, HPKS concluded that the values of and could not be determined.

To support these findings, the researcher then interviewed HPKS to explore the reflective thinking process at the critical reflection stage. The excerpted interview between the Researcher (R) and HPKS is as follows.

R : Was there a moment when you were confused about the solution you found? If so, how did you resolve it?

HPKS : Yes, when I first found the solution, I had doubts. I rechecked my steps from the beginning, trying to find if there were any mistakes and if there were, I corrected them. If I still have doubts, I will look for other methods to see if the results are the same.

R : Can you explain what methods you used to solve this problem?

HPKS : Firstly, I tried to use the elimination method because it is the easiest. Then I eliminated equations 1 and 2, and the result was $-x - y = -60{,}000$, which I formalized into equation 4. Then I eliminated equations 4 and 3, the result in 0=10,000. But I hesitated. After thinking about it, I remembered that this could be solved using a matrix, namely the inverse matrix. I converted the SLETV into matrix form, and then I looked for the inverse value of the coefficient matrix. The result is that the determinant is equal to 0. So, the matrix has no inverse, and it can be said that x, y, and z cannot be determined. Then, to be more convincing, I did a substitution. Previously, from equation 3, it was known that $x + y = 50{,}000$, then $x = 50{,}000 - y$. This is called equation 5. Equation 5 was also substituted into equation 1 so that $z = 20{,}000$ is obtained. Equation 5 was also substituted into equation 2 so that the result is $z = 30{,}000$. Because the result is not the same z value, x, y, and z cannot be determined.

R : Are you sure of the answer? Explain!

HPKS : Sure, because I double-checked every step and used different methods. The result obtained $0 = 10{,}000$ is a contradiction statement, and the determinant of the coefficient matrix is also 0, so the matrix has no inverse. These results are some of the things that I know if the SLE has no solution. This strengthens and convinces me that the SLETV does not have a solution. The SLETV has no solution, so the price of 1 kg for each fruit cannot be determined. The price of each fruit for three packages is different.

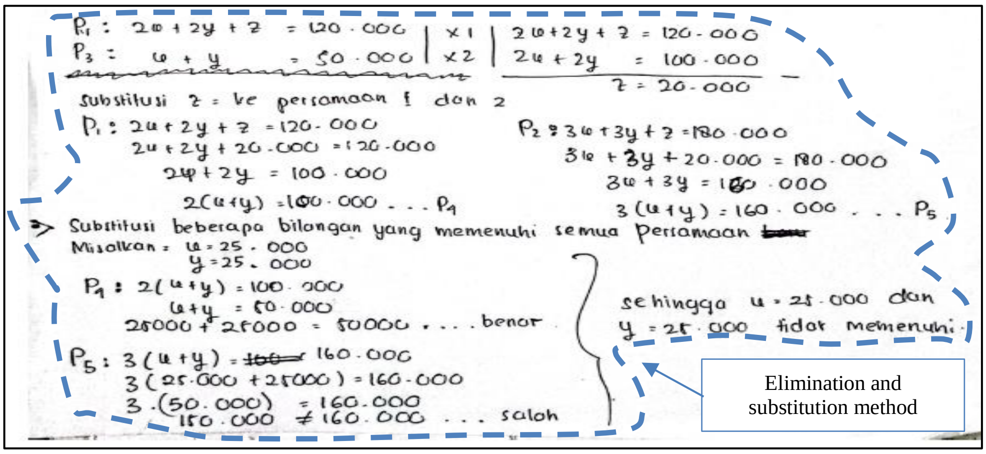
R : Can you explain the conclusion you draw?

HPKS : The conclusion is that the SLETV x, y, and z cannot be determined. SLETV has no solution, so the price of 1 kg for each fruit cannot be determined. So, the price of each fruit for the three packages is different.

Based on the interview with HPKS at the critical reflection stage, he tried to solve the problem using the elimination method because this method is the easiest to use. HPKS was confused when obtaining the results of elimination and substitution methods because the x, y, and z values could not be determined, and it tried to evaluate and correct errors when solving the problem. HPKS tried to solve the problem with the other two methods, namely the inverse and substitution methods because he needed to determine the answer. HPKS became more confident if the value of x, y, and z could not be determined or had no solution. HPKS concluded that the price of each fruit per kilogram in three packages was different.

The reflective thinking process with medium prior knowledge subjects (MPKS)

MPKS's answers in solving SLETV numeracy problems are presented in Figures 4, 5, and 6.



Translation

$P_1: 2x + 2y + z = 120{,}000 \mid \times 1 \mid 2x + 2y + z = 120{,}000$
$P_3: \quad x + y = 50{,}000 \mid \times 2 \mid 2x + 2y = 50{,}000$ —
$z = 20{,}000$

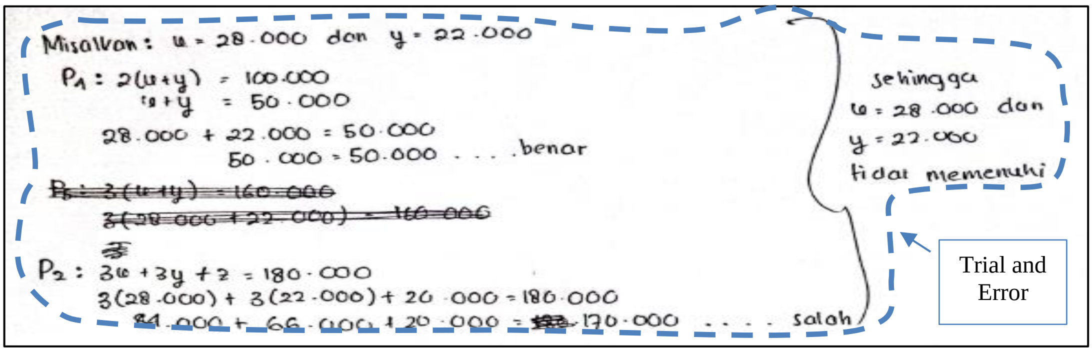
Substitute z into equations (1) and (2)
$P_1: 2x + 2y + z = 120{,}000$
$2x + 2y + 20{,}000 = 120{,}000$
$2x + 2y = 100{,}000$
$2(x + y) = 100{,}000 \ldots P_4$
$P_2: 3x + 3y + z = 180{,}000$
$3x + 3y + 20{,}000 = 180{,}000$
$3x + 3y = 160{,}000$
$3(x + y) = 160{,}000 \ldots P_5$

Substitute some numbers that satisfy all equations
Suppose: $x = 25{,}000$
$y = 25{,}000$
$P_4: 2(x + y) = 100{,}000$
$(x + y) = 50{,}000$
$25{,}000 + 25{,}000 = 50{,}000$... correct
$P_5: 3(x + y) = 160{,}000$
$3(25{,}000 + 25{,}000) = 160{,}000$
$150{,}000 \neq 160{,}000$... wrong

So, x=25,000 and y=25,000 do not fulfill

Figure 4. MPKS's answer using the elimination and substitution method

Based on Figure 4 at the Critical Reflection stage, MPKS used a mixed method of elimination and substitution and obtained two new equations, namely $2(x + y) = 100{,}000$ as equation 4 (P4) and $3(x + y) = 160{,}000$ as equation 5 (P5). Then MPKS tried trial and error by choosing the values of $x = 25{,}000$ and $y = 25{,}000$ and then substituting them into P4. The result was fulfilled, but when substituted into P5, it was not fulfilled. Then MPKS answered, as shown in Figure 5.

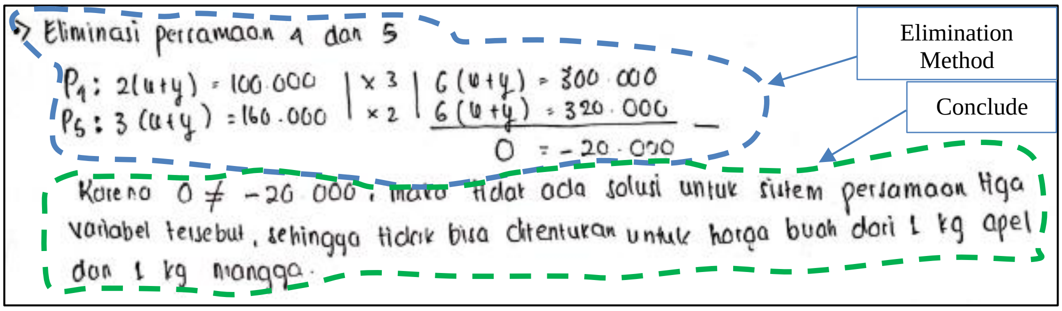


Translation
Suppose: $x = 28{,}000$
$y = 22{,}000$
$P_4: 2(x + y) = 100{,}000$
$(x + y) = 50{,}000$
$28{,}000 + 22{,}000 = 50{,}000$
$50{,}000 = 50{,}000\ldots$ correct
$P_2: 3x + 3y + z = 180{,}000$
$3(28{,}000) + 3(22{,}000) + 20{,}000 = 180{,}000$
$84{,}000 + 66{,}000 + 20{,}000 = 170{,}000\ldots$ wrong

So, x=28,000 and y=22,000 do not fulfill

Figure 5. MPKS's answer using trial and error method

Figure 5 shows that MPKS chose the values $x = 28{,}000$ and $y = 22{,}000$ d and substituted them into P4, and the result was fulfilled. Then MPKS substituted them into P2, and the result was wrong or did not fulfill. Then MPKS substituted them into P2, and the result was wrong or did not fulfill. Next, as shown in Figure 6, MPKS attempted to apply the elimination method.



Translation
Eliminate equation 4 and 5
$$P_4: 2(x + y) = 100{,}000 \mid \times 3 \mid 6(x + y) = 300{,}000$$
$$P_5: 3(x + y) = 160{,}000 \mid \times 2 \mid 6(x + y) = 380{,}000 \quad -$$
$$0 = -20{,}000$$
Since $0 \neq 20{,}000$, there is no solution for SLETV
so the fruit price of 1 kg of apple and 1 kg of mango cannot determined.

Figure 6. MPKS's answer using the elimination method

Figure 6 explains how MPKS attempted to eliminate P4 and P5, and obtained the result $0 = -20{,}000$. Because it was $0 \neq -20{,}000$ MPKS concluded that there was no solution for the SLETV.

To support these findings, the researcher then interviewed MPKS to explore the reflective thinking process at the critical reflection stage. The excerpted interview between the Researcher (R) and MPKS is as follows.

R : Was there a moment when you were confused about the solution you found? If so, how did you resolve it?

MPKS : Yes. The way to overcome it was to try to check again whether there were any mistakes in my work steps. But there wasn't. Then, I thought of trying another way.

R : Can you explain the method you used to solve this problem?

MPKS : Yes, ma'am, I first used the elimination and substitution method. Because I needed clarification, I first tried by choosing the values of $x = 25{,}000$ and $y = 25{,}000$ then substituted them into P4. The result was correct and fulfilled the equation, and then I substituted it into P5, but the result was wrong or was not fulfilled. Then I tried to choose the values x=28,000 and y=22,000 and substituted them into P4, and the result was correct. When I substituted them into P2, the result was wrong or did not satisfy. Then, I tried the elimination method again by eliminating P4 and P5 and obtained the result $0 = -20{,}000$, because $0 \neq -20{,}000$ then there is no solution for the SLETV.

R : Are you sure of the answer? Explain!

MPKS : Actually, I'm not sure, ma'am, because the result of 0=20,000 is impossible.

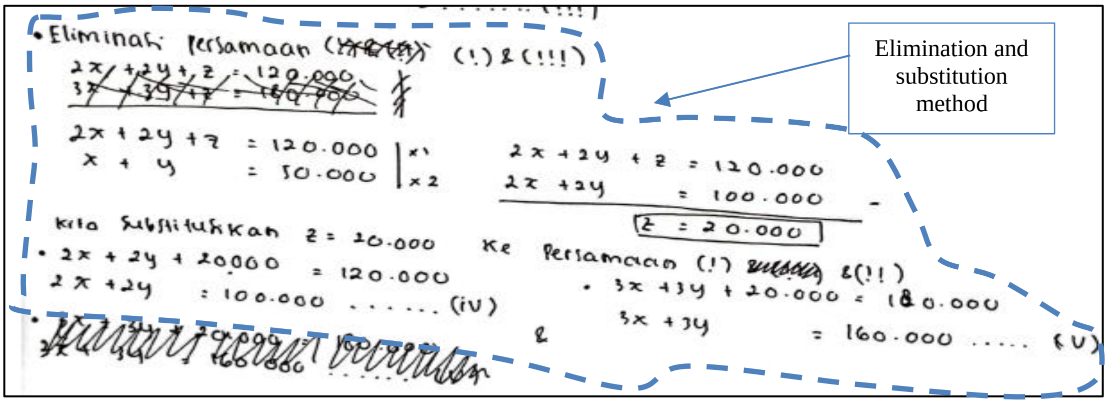
R : What is the conclusion of the solution? Explain!

MPKS : The SLETV has no solution because 0≠20,000, so the price of 1 kg for each fruit cannot be determined..

According to the MPKS interview, the critical reflection stage revealed that MPKS concluded that SLETV has no solution, so the price of 1 kg for each fruit cannot be determined. MPKS used various methods: elimination, substitution, and trial and error. The results obtained were always the same, namely that no solution that fulfilled the given equations. Still confused, MPKS finally tried to eliminate P4 and P5 and came up with $0 = -20{,}000$, but the result contradicted mathematical logic, so MPKS stated that the SLETV had no solution. MPKS felt unsure of his answer because the result was impossible. MPKS concluded that the SLETV has no solution, so the price of 1 kg for each fruit cannot be determined.

The reflective thinking process with low prior knowledge subjects (LPKS)

LPKS's answers in solving SLETV numeracy problems are presented in Figures 7, and 8.



Translation

Eliminate equations I and III

$$\begin{array}{r|l} 2x + 2y + z = 120{,}000 & \times 1 \\ x + y = 50{,}000 & \times 2 \end{array} \quad \begin{array}{rl} 2x + 2y + z &= 120{,}000 \\ 2x + 2y &= 50{,}000 \\ \hline z &= 20{,}000 \end{array} \; -$$

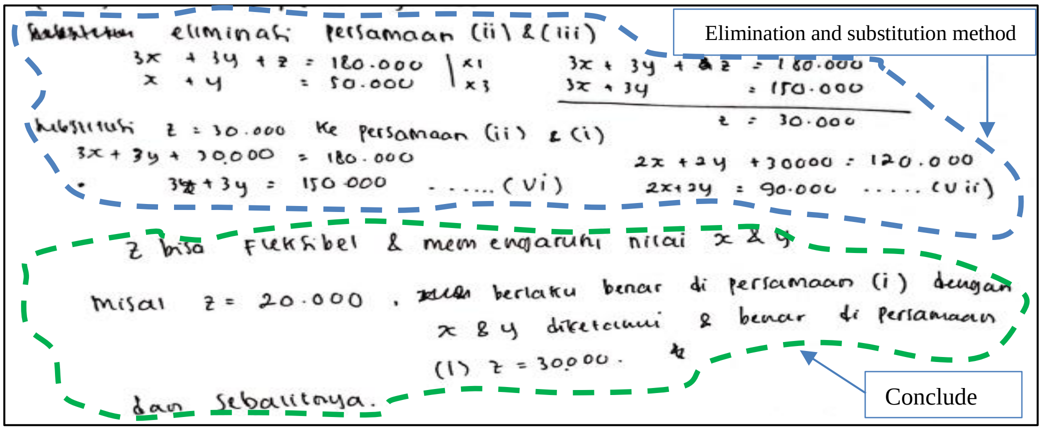
We substitute $z = 20{,}000$ into equations (I) and (II)

$$2x + 2y + 20{,}000 = 120{,}000 \qquad 3x + 3y + 20{,}000 = 180{,}000$$

$$2x + 2y = 100{,}000 \ldots (\text{IV}) \qquad 3x + 3y = 160{,}000 \ldots (\text{V})$$

Figure 7. LPKS's using the elimination and substitution method

Figure 7 shows that at the Critical Reflection stage, LPKS used a mixed method: elimination and substitution. Based on Figure 8, LPKS eliminated equations 1 and 3 first and obtained the value of $z = 20{,}000$; after that, the value of $z = 20{,}000$ to equations 1 and 2 and obtained two new equations, $2x + 2y = 100{,}000$ as equations 4 (IV), $3x + 3y = 160{,}000$ as equation 5 (V). There were scribbles on LPKS's answer sheet, which indicated doubt. Furthermore, LPKS solved the equation using the same method, namely elimination and substitution, but LPKS eliminated equations 2 and 3, as shown in Figure 8.



Translation

Eliminate equation II and III

$$\begin{array}{r|l} 3x + 3y + z = 180{,}000 & \times 1 \\ x + y = 50{,}000 & \times 3 \end{array} \quad \begin{array}{rl} 3x + 3y + z &= 180{,}000 \\ 3x + 3y &= 150{,}000 \\ \hline z &= 30{,}000 \end{array} \; -$$

We substitute $z = 30{,}000$ into equations (II) and (I)

$$3x + 3y + 30{,}000 = 180{,}000 \qquad 2x + 2y + 30{,}000 = 120{,}000$$

$$3x + 3y = 150{,}000 \ldots (\text{VI}) \qquad 2x + 2y = 90{,}000 \ldots (\text{VII})$$

z s flexible and affects the values of x and y.
For example, $z = 20{,}000$ is true in equation (I), with x and y known and trustworthy in equation (I), $z = 30{,}000$, and vice versa.

Figure 8. LPKS's answer using the elimination and substitution method

Based on Figure 8, LPKS eliminated equations II and III to get $z = 30{,}000$, then subtituted $z = 30{,}000$ into equations II and 1 to get two new equations, $3x + 3y = 150{,}000$ as equation VI, $2x + 2y = 90{,}000$ as equation VII. Then, the conclusion drawn by LPKS is that the value of z is flexible, which affects the values of x and y.

To support these findings, the researcher then interviewed LPKS to explore the reflective thinking process at the critical reflection stage. The interview excerpt between the researcher (R) and LPKS is as follows.

R : Was there a moment when you were confused about the solution you found? If so, how did you resolve it?

LPKS : Yes, ma'am. I was confused because of the result I obtained. I rechecked my calculation, but I thought it was correct. Then, I thought of trying to eliminate the other equation.

R : Can you explain how did you solve this problem next?

LPKS : Yes ma'am, I can. Then, I tried eliminating Equations 2 and 3 and obtained the result $z = 30{,}000$. Then, I substituted the z value into equations 2 and 1 to get what I call equation 6 and equation 7. Then, if equation 7 was simplified, this would get the result $x + y = 45{,}000$, whichs does not meet. If equation 6 was simplified, it would get the result $x + y = 50{,}000$, then it meets. The z value was flexible, affecting the x and y values.

R : Are you sure of the answer? Explain!

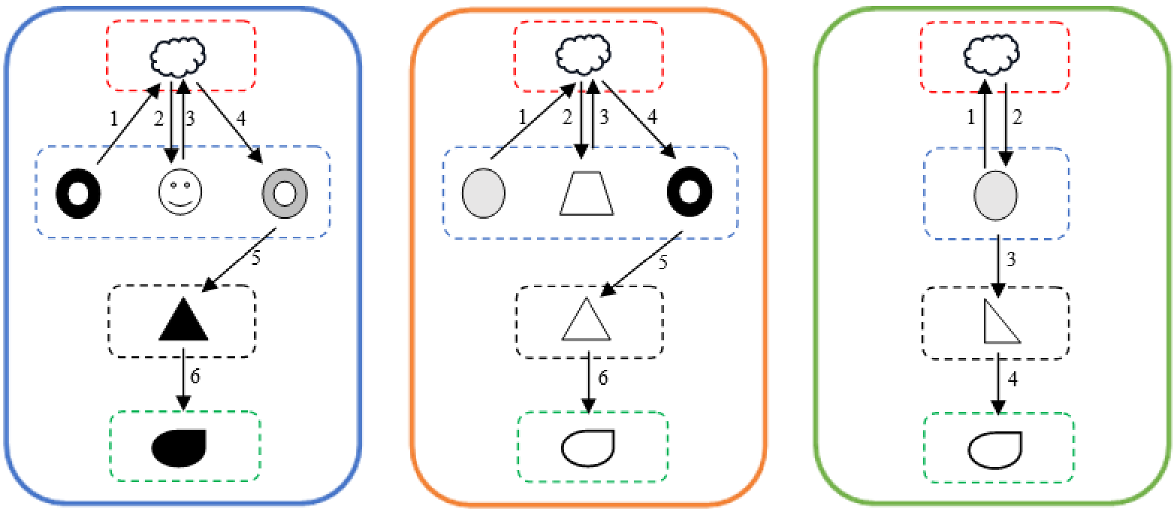
LPKS : Actually, not sure, ma'am.

R : What is the conclusion of this problem? Explain!

LPKS : From this problem, I conclude that z was flexible, so it affects the value of x and y. If the value was $z = 20{,}000$, it fulfills equation 1. Moreover, when $z = 30{,}000$, it fulfills equation 2.

Based on the interview with LPKS at the critical reflection stage, LPKS concluded that z is flexible, thus affecting the values of x and y. Based on various methods used by LPKS, such as elimination and substitution. Because if the value of $z = 20{,}000$, it fulfills equation 1. Moreover, when $z = 30{,}000$, it fulfills equation 2. However, LPKS is still determining the answer obtained.

Figure 9 below presents the reflective thinking structure of HPKS, MPKS, and LPKS in solving numeracy problems at the critical reflection stage.



Descriptions:

- : HPKS Critical Reflection
- : MPKS Critical Reflection
- : LPKS Critical Reflection
- : The Process
- : Confusion
- : Inverse Method
- : Trial and Error
- : Confident about the answer
- : Less Confident about the answer
- : Unconfident about the answer

: Elimination Method
: Conclude and explain
: Substitution Method
: Conclude and cannot explain
: Elimination and substitution method
: Confusing the solution obtained in solving numerical problems
: Looking for other alternative methods
: Being confident about the correctness of the answer obtained.
: Being able to conclude and explain why

Figure 9. Reflective thinking structure of HPKS, MPKS, and LPKS at the critical reflection stage

Discussion

Analysis of the answer sheets and in-depth interviews with HPKS, MPKS, and LPKS showed that all subjects experienced confusion and lack of confidence when solving numerical problems related to the SLETV inconsistency. Students' math achievement correlates with their reflective thinking toward problem-solving (Toraman et al., 2020). After getting the results from the first method, the subject required clarification regarding the answers. Similar to the opinion of Kholid et al. (2021) and Suharna (2018) stated that reflective thinking begins with confusion. The subject's confusion and doubt experienced are a helpful starting point for using a reflective thinking process to solve numeracy problems (Pagano & Roselle, 2009; Setiyani et al., 2023).

The results of HPKS show that the willingness to consider other methods shows flexibility and independence in thinking process. This is in line with Saputri et al. (2020), who stated that students' problem-solving flexibility encourages using various solution methods. Despite initial confusion, HPKS was eventually able to confidently conclude that the $x, y,$ and z values had no solution. HPKS conducts careful analysis and evaluation when making decisions. Through the interview process, HPKS has used a reflective thinking process to understand the problem deeply, evaluate various solutions, and reach a firm conclusion. It is in line with (Chen, 2022), who states that individuals can assess arguments, solve problems, and make decisions based on a thorough evaluation of the available information. HPKS did reflective thinking during the critical reflection stage well in solving numeracy problems, and it also could consider and evaluate various solution strategies.

HPKS, in reflective thinking at the critical reflection stage, can make decisions through careful analysis and evaluation to solve numeracy problems. According to Kember et al. (2000) and Setiyani et al. (2023), the critical reflection stage involves critical analysis of the assumptions, values, and beliefs underlying one's actions, helping to evaluate steps, identify errors, and use new strategies in solving problems. Further to reflective thinking, HPKS shows a deep understanding of various problems, evaluates various solutions, and comes to firm conclusions (Kholid et al., 2021; Sa'dijah et al., 2020). Reflective thinking enhances students' ability to analyze, evaluate, and synthesize mathematical concepts, leading to better problem-solving outcomes (Aquino & Ching, 2022; Simacon & Veloria, 2022). According to the notion that reflection is an integral part of learning from experience and essential for professional development (Funny et al., 2019).

MPKS reflective thinking at the critical reflection stage has tried to solve SLETV numeracy problems using various methods. MPKS used methods such as elimination, substitution, and trying different values. In Kholid et al. (2020) opinion, in solving a system of linear equations, one can connect mathematical concepts such as the concept of elimination or substitution. The conclusion that SLETV has no solution is more due to the difficulty in finding values that satisfy all equations rather than strong mathematical reasoning (Schoenfeld, 2016). The elimination results of P4 and P5 that contradict mathematical logic should have triggered deeper reflection. According to the Aberdein (2023), facing contradictions in mathematical logic can indeed be a catalyst for deeper reflection. Reflective thinking involves contemplating uncertainty or ill-defined problems, which is crucial in problem-solving (Damastuti et al., 2023; Khoshgoftar & Barkhordari-Sharifabad, 2023). Additionally, reflective thinking in mathematics education is essential for developing critical thinking and problem-solving skills (Junaedi & Wahyudin, 2020; Saracoglu, 2022).

LPKS reflective thinking at the critical reflection stage has tried to solve SLETV numeracy problems using mixed methods between elimination and substitution. Although LPKS managed to obtain varying z values from the elimination equation, the conclusion that z values are flexible and affect the values of x and y seems different from mathematical logic. Nevertheless, the steps taken by LPKS showed a reasonable effort in finding a solution, but the final conclusion still seemed more descriptive than analytical. The interview process revealed that LPKS still needed to improve her comprehension of the mathematical ideas that underline problem-solving, and improve her skills in evaluating the results found. Reflective thinking in mathematics relies on a profound grasp of concepts and well-considered decision-making during problem-solving (Gürol, 2011). By engaging in reflective thinking, students can deepen their understanding of mathematical concepts, enhance their problem-solving abilities, and improve their overall mathematical proficiency (Syamsuddin et al., 2023). Students who engage in reflective thinking practices demonstrate improved critical thinking skills and a deeper understanding of mathematical principles (Aldahmash, 2021; Ariany et al., 2021; Saracoglu, 2022).

Prospective mathematics teachers categorized on prior knowledge in solving numeracy problems show differences in reflective thinking at the critical reflection stage. It aligns with the finding that higher prior knowledge positively correlates with increased reflective thinking ability (Muntazhimah, 2021; Toraman et al., 2020). High prior knowledge subjects could use various methods, namely elimination, substitution, and inverse methods. They were able to draw accurate conclusions after considering and evaluating various methods and being able to explain their reasons. It is supported by Fyfe et al. (2012), Pambudi (2022), and Rittle-Johnson et al. (2009), who state that students with high prior knowledge can draw accurate conclusions after considering various methods and can explain their reasons. Medium prior knowledge subjects showed a good understanding of the elimination and substitution methods, used trial and error, and could draw conclusions; however, they needed to be sure about the answer. Low prior knowledge subjects were able to try various methods, namely elimination and substitution, and concluded even though they needed to be sure (Pambudi, 2022). Subjects with higher prior knowledge usually show better reflective ability, while subjects with lower prior knowledge need to improve their confidence and reflective ability (Kholid et al., 2020).

Conclusion

The findings revealed notable differences in reflective thinking based on prior knowledge levels at the critical reflection stage. Subjects with high prior knowledge tried various solutions, were confident in their answers, and accurately drew conclusions and explained their reasons. Subjects with medium prior knowledge tried different methods but needed more confidence in their results. They could conclude but struggled to explain their reasons. Subjects with low prior knowledge used the same method, needed more confidence, and could not explain their reasons despite concluding. This conclusion suggests the need for different learning approaches according to the level of prior knowledge in developing reflective thinking skills in prospective mathematics teachers. Specially designed training programs can help improve confidence, reasoning skills, and choosing appropriate methods to solve numeracy problems. Future research could develop numeracy learning programs specifically designed to meet the needs of students with different categories of prior knowledge. A longitudinal study could show how reflective thinking and numeracy skills develop. Applying learning strategies that help improve students' confidence and reflective thinking skills could also be explored, including scaffolding.

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Conflicts of Interest

The authors declare no conflict of interest regarding the publication of this manuscript. In addition, authors have completed the ethical issues, including plagiarism, misconduct, data fabrication and/or falsification, double publication and/or submission, and redundancies.

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Author Contributions

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