Jurnal Elemen, 10(3), 630-651, September 2024
https://doi.org/10.29408/jel.v10i3.26775

e- ISSN: 2442 - 4226

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Conceptual images and learning obstacles in exponentiation and logarithms: A hermeneutic phenomenological analysis

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Abstract

The concept of exponentials and logarithms is one of the essential concepts in mathematics as part of transcendent functions. However, students' understanding of these two concepts has yet to be fully mastered, so there are still errors in solving problems related to these two concepts. This study aims to describe the meaning and meaning process and explore the potential learning obstacles of students based on the meaning and experience of students in obtaining the meaning of exponents and logarithms in senior high school. This qualitative study uses a hermeneutic phenomenological approach with student participants and high school mathematics teachers. Data was collected using tests, documentation, and interviews with students and teachers. Data were analyzed qualitatively to identify learning obstacles and the meaning of exponentials and logarithms. The results showed the meaning of exponents and logarithms according to students, namely exponents as power numbers and logarithms as the opposite of power numbers. In addition, the results also show the existence of learning obstacles in students, both ontogenic, epistemological, and didactic. Based on the findings, these learning obstacles can be considered when developing an appropriate didactic design.

Keywords: exponential; hermeneutic phenomenological; learning obstacle; logarithm

How to cite: Kusumadewi, Y. A., & Andriatna, R. (2024). Conceptual images and learning obstacles in exponentiation and logarithms: A hermeneutic phenomenological analysis. *Jurnal Elemen*, *10*(3), 630-651. https://doi.org/10.29408/jel.v10i3.26775

Received: 25 June 2024 | Revised: 15 August 2024
Accepted: 25 August 2024 | Published: 1 October 2024

Introduction

Mathematics learning is a complex and continuous process involving interaction among students, teachers, and mathematics (Suryadi, 2019). Teachers face various student characteristics when learning mathematics (Hindi & Muthahharah, 2021). Some students can carry out their learning activities smoothly and successfully without experiencing difficulties. However, not a few students experience difficulties in learning mathematics.

As many as 74% of students out of 60 in Indonesia were categorized as low in problem-solving ability on the material (Hardian, 2019). In addition, students experience difficulties in teaching the material. There are problems with students, namely the need to master the material so that students have difficulty solving exponent and logarithm problems (Gunawan & Fitra, 2021). The interviews in the study explained that students did not master the properties of operations on exponents, especially on division operations. Hence, they had difficulty when working on these problems. Gunawan and Fitra (2021) explained that several factors cause students to experience difficulties in solving exponent problems. These include experience, influential factors (interest, motivation, anxiety pressure, tolerance for ambiguity, resilience, and patience), and cognitive factors. Apart from exponent material, there are other student difficulties in solving logarithm problems. Research by Zuhra (2022) showed that students could use the properties of exponents and logarithms in fractional numbers but did not understand how to convert root forms into rational powers (fractions), so the problem-solving process was wrong.

Apart from the results of previous studies, researchers also conducted a preliminary study related to students' difficulties in understanding the concepts of exponents and logarithms. Based on an interview with one of the mathematics teachers in Indonesia, students need help with the concepts of exponents and logarithms. Students have difficulty understanding the concept of exponents and logarithms because of the many properties that need to be understood. Students can also not construct the correct form of answers themselves, so the teacher only gives questions in the form of multiple choice. Even in multiple choice questions, students still need help. These difficulties indicate problems related to the meaning (hermeneutic) and the meaning of exponents and logarithms related to students' experiences (phenomenology).

Students need better concept images related to exponents and logarithms (Valtoribio et al., 2018). Concept image describes the total cognitive structure associated with a concept and includes all mental images and associated properties and processes (Tall & Vinner, 1981). Concept image refers to conception, as stated by (Sfard, 1991), that concept image is an internal representation that arises in students towards a concept based on experience. Edwards and Ward (2004) stated that concept image could occur due to memorizing formal concept definitions without meaning, thus allowing irrelevance to the formal definition. The concept image will be dynamic, changing according to the experience gained. Thus, concept image can be related to a person's interpretation of a particular concept based on the experience that has been obtained. It is built over many years through various experiences (phenomenology) and can change when individuals encounter new stimuli. In addition, the term concept definition also appears as the language each individual uses to describe a concept specifically. The definition can be expressed in the language accepted by the mathematical community, in the everyday language taught by the everyday language taught by the teacher, or in the students' own words that they understand.

In mathematics learning, the process of constructing the meaning of a mathematical concept by students, in this case, the concept of exponents and logarithms, is also influenced by the meaning from various points of view, such as other students and teachers through the learning design they develop (Suryadi, 2019). This results in a series of unique, meaningful experiences for each student. The meaning by students is an implication of the teacher's meaning, which is not necessarily based on his/her scientific conception. Therefore, this has the potential to cause learning obstacles. In general, learning obstacles (LO) in mathematics can be interpreted as obstacles in the way of thinking and understanding related to mathematical knowledge, such as concepts, theories, and so on. Duroux (Brousseau, 2002) states that LO is a piece of knowledge or conceptions, not a lack of knowledge students receive. Thus, students who experience learning difficulties or obstacles can only be said to be LO if these students have received knowledge. Therefore, LO is also possible due to the teacher's interpretation, which is based on something other than the scientific conception of exponents and logarithms. Brousseau stated that LO identification needs to consider all points of view and their interrelationships (Suryadi, 2019).

According to Brousseau (2002), LO is categorized based on its source into three types, namely ontogenic obstacle, epistemological obstacle, and didactical obstacle. Ontogenic obstacles, namely a mismatch between the learning provided and students' level of thinking, create difficulties in understanding the material. Epistemological obstacles, namely difficulties in the learning process, occur due to students' limited understanding and mastery of something only associated with a specific context. Meanwhile, didactical obstacles are caused by the didactical system, such as the sequence and/or stages of the curriculum, including its presentation. Concerning the three categories of LO, previous research focused more on analyzing student difficulties that could cause LO of exponent and logarithm concepts. In this case, it also still focuses on epistemological LO, such as a lack of mastery of the material (Gunawan & Fitra, 2021).

Meanwhile, the LO experienced by students is not only epistemological but also the possibility of LO of the material concept in the didactic and ontogenic categories. In line with research by Valtoribio et al. (2018), errors made by students indicate that something is wrong in the teaching and learning process. An essential thing for understanding better concepts in mathematics is the learning design used by teachers to teach the subject. In addition, teachers need to reason more deeply about what it takes to support students to learn from mistakes, see the framework of mistakes, and strategically counter them (Alvidrez et al., 2024). Therefore, it is necessary to explore in depth the meaning of students' experience and the possibility of LO of the material concept in the ontogenic, epistemological, and didactic categories.

Several studies related to concept images and learning obstacles have been conducted. Nurwahyu et al. (2020) have investigated students' concept image and how it affects reasoning on derivative concepts. On different materials, Siagian et al. (2021) have also investigated students' concept images in understanding the concept of variables. Meanwhile, at a higher level of education, namely at the university level, several researchers (Nurwahyu & Tinungki, 2020; Ojo & Olanipekun, 2023; Prihandhika et al., 2022) have investigated the concept image on the concept of Calculus, while in the field of Algebra, research has also been conducted on the concept image of undergraduate students by Melhuish et al. (2020). Meanwhile, long before that, Tall and Vinner (1981) had investigated the concept of image on limits and continuity. Vinner and Dreyfus (1989) also did something similar in investigating concept images related to the concept of functions. In addition to concept images, several studies related to LO have also been conducted. Aprizal Bintara and Prabawanto (2024) have analyzed the LO of junior high school students in the concept of a triangle. Another research topic, Unaenah et al. (2024), also analyzed students' LO on the concept of fractions. At the higher education level, Noto et al. (2019) have also investigated learning obstacles in pre-service mathematics teachers. Meanwhile, Beyene (2023) has also conducted a comparative study on obstacles to limits between students in Sweden and Ethiopia. Considering some of the research results above, investigating concept images and LO has become one of the special focuses in mathematics research, so it can be one of the references to develop students' mathematical abilities.

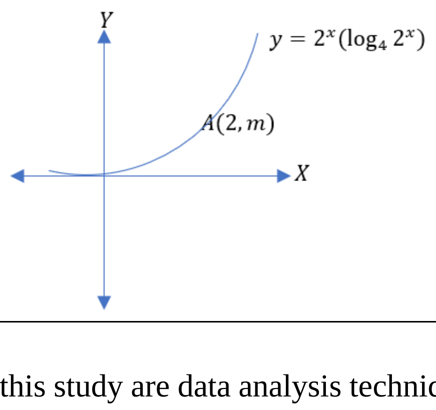
The concept of exponents and logarithms is one of the mathematical concepts that students must master. However, some studies still need help with both concepts. Several studies have been conducted on the concept image of exponents or logarithms (Oktavihari & Priatna, 2023; Oliveira & Lopes, 2024). However, the research needs to relate to the learning obstacles. In fact, according to Tall (1996), obstacles may occur in students due to the weak concept image and concept definition of students. Based on the description of the problems that have been described, there is a need for further review of the concept image of the concept of exponents and logarithms obtained by students and the possibility of LO in students based on their concept image on the material reviewed based on three categories, namely ontogenic obstacle, epistemological obstacle, and didactical obstacle. Therefore, this study is interested in researching students' learning obstacles on exponents and logarithms as a hermeneutic phenomenological study, including the concept image of exponents and logarithms understood by students.

Methods

The research method used in this study is a descriptive qualitative research method with a hermeneutic phenomenological approach. Through hermeneutic phenomenology, researchers aim to describe and interpret students' learning experiences and the meanings and interpretations associated with these experiences (Regan, 2012). The researcher interpreted the interpretations made by the research subjects (students or groups of students) on their own learning experiences related to the meaning of exponents and logarithms. At the end of this research, the reality of the results of hermeneutic phenomenology is linked to relevant theories to identify and categorize LO.

The participants in this study were senior high school students and mathematics teachers in Surakarta city. In this study, data were collected using test tests in the form of an essay and non-test methods with documentation and interviews. The test (Respondent Ability Test/RAT) was given to students to obtain data on students' concept image and learning obstacles, while non-tests were in the form of documentation and in-depth test-based interviews with students and teachers to obtain in-depth data on the concept image of exponents and logarithms and learning obstacles that occur. In this study, the selection of interviewees was carried out by purposive sampling, namely the representation of each category of the meaning of exponents and logarithms expressed by students according to the results of the RAT. Each category of meaning is represented by a student selected based on certain considerations, namely the existence of errors in solving RAT questions so that there is a potential LO. RAT was given to 33 students in class X senior high school students. The meanings of exponents and logarithms were classified into three categories of exponent meanings and four categories of logarithm meanings based on students' answers to RAT questions related to the meanings of exponents and logarithms and their application. From all categories of the meaning of exponents and logarithms, seven students who represented all categories of meaning and indicated errors in solving RAT problems were taken to become sources in student interviews. In the next section, the seven students are stated as student 1 (S1), student 2 (S2), student 3 (S3), student 4 (S4), student 5 (S5), student 6 (S6), and student 7 (S7). In addition to interviews with the seven students, interviews were also conducted with the mathematics teachers of the seven students. Then, the documentation method was used to obtain additional data relevant to the research focus. Documents used in this study include a teaching module, student notebooks, and student handbooks on learning exponents and logarithms.

Table 1. Respondent ability test.

Cognitive Level	Test Questions
C2 (Understanding)	What are exponents and logarithms? Explain!
C3 (Application)	1. Determine the value of x that satisfies $(-2)^{2x-3} = \frac{1}{(-32)^{-x+1}}$! 2. Be known $x = \log_2 5 - \log_2 75 + \log_2 30$, $y = \log_3 2^{-2} + \log_3 54 - \log_3 \sqrt{\sqrt[3]{\frac{1}{64}}}$. Determine the value of $\frac{x}{y}$! 3. Find the coordinate point of A in the following graph! 

The data analysis techniques used in this study are data analysis techniques, according to Sugiyono (2015), which include data reduction, data presentation, and conclusion drawing or verification. In particular, referring to Suryadi (2019), the stages of data analysis in this study use the phenomenological stages by Creswell (2007) and the hermeneutic stages in the application of Ricoeur's interpretation theory (Ghasemi et al., 2011). The data analysis stages are described as follows.

1. Processing and preparing data for analysis in the form of prepare all test results, interviews, and documentation for analysis.
2. Explanantion in the form of recapitulate test results and transcribe interview results including reviewing the results of data recapitulation.
3. Naive understanding in the form of write various significants statements, both related to the finding of the mearning of exponent and logarithms incluidng learning obstacles and compose textural and structural descriptions.
4. In-depth understanding in the form of analyzing relationship based on textural and structural descriptions and create a composite description based on the results of the previous analysis.
5. Appropriation i.e. analyze the relationship of the combined description with significant statements from the research subject, other data sources, and relevant theories so as to identify the research results.

Results

The concept imgae of exponents and logarithms according to students

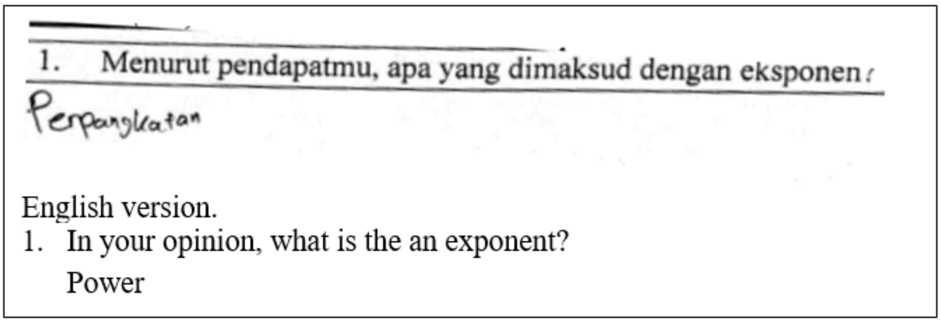
In this study, the concept image of exponents and logarithms represents the meaning of exponents and logarithms. According to students, the concept image of exponents and logarithms means the meaning or definition of exponents and logarithms expressed by students. Based on students' answers to RAT about the meaning of exponents and logarithms, the researcher classified the meaning of exponents and logarithms of all students into several categories. The categories of meaning of exponents according to students based on the RAT results are presented in Table 2.

Table 2. Exponent meaning categories according to students.

Exponent Meaning Category	Definition Description	Number of Students
E1	A power number.	26
E2	The multiplication of n factors where each factor is a, that is $a^n = \underbrace{a \times a \times a \times \dots \times a}_{n\ faktor}$.	4
E3	A mathematical formulas, operations, or equations.	2

Based on Table 2, most students defined exponents as the meaning category (E1), which is exponents as power numbers. In addition, some students wrote the formal definition of exponents in the meaning category (E2), namely n is a positive integer and a is a real number, then a^n is defined as the multiplication of n factors where each factor is a, that is $a^n = (a \times a \times a \times \dots \times a)$. Another student's definition of exponents was the meaning category (E3), which is exponents as mathematical formulas, operations, or equations. There was also one student who did not answer the meaning of exponents.

In Table 2, most students interpret exponents as power numbers. One student, namely student 6 (S6) interpreted exponents as power numbers (see Figure 1). A similar meaning of exponents was also expressed by S6 when answering questions related to his thoughts about the meaning of exponents in the interview.


1. Menurut pendapatmu, apa yang dimaksud dengan eksponen?

Perpangkatan

English version.

1. In your opinion, what is the an exponent?
   Power

Figure 1. Example answer regarding the meaning of exponents

In the interview, S6 revealed that what comes to mind when he hears the word exponent is power. S6 also explained further about exponents by giving an example that 2^3 means 2 multiplied by 2, 3 times. This indicates that S6 interpreted exponents as power numbers. In the other question in RAT, S6 wrote the equivalent form of -32, namely $(-2)^5$. This was also revealed by S6 in the interview. In the interview, S6 revealed that $-32 = (-2)^5$. This shows that S6 interpreted exponents as power numbers. In addition, a similar meaning was also written by S6 in the notebook of exponent and logarithm material during class. S6 wrote the definition of exponent as a^x (a power of x). This shows that S6 interpreted the exponent as a power number in the notebook written when learning exponent and logarithm material in class. Based on the meaning of exponents interpreted by S6 through the answers to RAT questions related to the meaning of exponents, disclosure in the interviews, completion of other RAT questions, and writing related to the definition of exponents in his notebook, S6 can be said to be consistent in interpreting exponents as power numbers. The real meaning of exponents according to students is the meaning included in the meaning category (E1) in Table 2, namely exponents as power numbers. This was obtained based on the research assumptions previously described. The meaning of the exponent becomes the concept image of the exponent that is most widely expressed by students.

Furthermore, the results of students' answers related to the meaning of logarithms are presented in Table 3.

Table 3. Logarithm meaning categories according to students.

Exponent Meaning Category	Definition Description	Number of Students
L1	The opposite of an exponent/powered number	20
L2	The result of a power root.	3
L3	Power	6
L4	Other	3

The meanings of logarithms in Table 3, most students defined logarithm as meaning category (L1), namely logarithm as the opposite of exponent. Other definitions of logarithms by students include meaning category (L2), namely logarithm as the result of the root of a power; meaning category (L3), namely logarithm as a power; and meaning category (L4) others. The miscellaneous categories are logarithms as an easier way to solve exponents and formulas that contain 'log'. In addition, there was also a student who did not answer the meaning of logarithms.

Most of the students interpret logarithms as the opposite of a power number/exponent. One student, namely student 4 (S4) interpreted logarithm as the inverse of exponent. The word 'inverse' in the context of the meaning of logarithm is the opposite, namely $a^c = b$ (exponent), then ${}^{a}\log b = c$ (logarithm). A similar meaning of logarithm was also expressed by S4 when answering questions related to his thinking about the meaning of logarithm in the interview. In the interview, S4 revealed that what he thought of when he heard the word logarithm was the opposite of exponent. This shows that S4 interpreted logarithm as the opposite of exponent as in the following Figure 2.

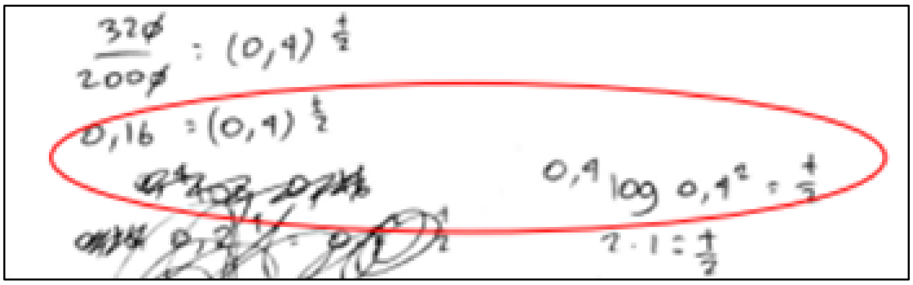


Figure 2. Example answer regarding the meaning of logarithm

The diversity in the concept images that students have in the concept of exponents and logarithms as in Table 2 and Table 3 indicates the existence of different experiences. However, further investigation shows that there are some students' errors on the concept image. The researcher found ambiguities, inconsistency, and incompleteness in the students' concept images. The ambiguity referred to the double meaning understood by students towards exponents and logarithms, where students understand the meaning of exponents or logarithms as in Table 2 or Table 3 but have other meanings when used in solving problems. This has an impact on the inconsistency of students in using their concept image. Meanwhile, the incompleteness that occurs is that students only interpret the concept of exponents limited to positive integers, so that when faced with powers of negative numbers it becomes wrong.

Learning obstacles (LO) on exponent and logarithm

Ontogenic obstacle

Ontogenic obstacles are difficulties related to students' ability to learn. LO is divided into three types, namely ontogenic obstacles caused by psychological factors such as interest and motivation (psychological), technical matters such as notation and properties (instrumental), and mastery of basic concepts, prerequisite materials, and learning demands (conceptual).

Based on the findings, there are ambiguities, inconsistencies, incompleteness, and other findings of the meaning of exponents and logarithms. In addition, the RAT results also showed student errors in solving some RAT problems. Observing these findings, the researcher suspects the existence of LO that are ontogenic or ontogenic obstacles in exponents and logarithms. Therefore, it is necessary to further explore several things that indicate the existence of these LO in exponents and logarithms.

The existence of ambiguity, inconsistency, incompleteness, and other findings of the meaning of exponents and logarithms indicate that students have not fully and definitively interpreted exponents and logarithms. Students do not fully understand the meaning of exponents and logarithms with all their properties and applications. The researcher observed that this is a form of LO that is ontogenic.

Researcher : Okay. Then let's see in number one and two, you answered like this. Are you really not interested in math or what?
Student (S7) : Less. ... Interest, but less. ... Lazy, lazy, I'm a lazy person.
Researcher : Because of what, because there are many formulas?
Student (S7) : Yes, it's hard to memorize.
Researcher : When you do the work, can you still do the math?
Student (S7) : According to memory, sometimes if something comes in, just write it down.
Researcher : You're majoring in science, but you're not interested in math, what about that?
Student (S7) : Try to be interested.

Based on further investigation through interviews, it was found that there were students who had low interest and motivation in exponents and logarithms. The interview results showed that students' interest and motivation in learning exponents and logarithms were not high. The researcher then further explored the reality in this matter through interviews with teachers. Based on the results of interviews with teachers, not all students like math subjects. In addition, students are afraid of exponents and logarithms because of the many formulas. Facing this reality, the teacher also did not have a solution. Therefore, the researcher found the existence of students' psychological ontogenic LO in exponents and logarithms, where students have low interest and motivation in the material.

Researcher : Then what is the response of students when learning in class?
Teacher : Various, if you don't like maths, that's it.
Teacher : There are also those who say they like biology lessons, but when checked with the biology teacher the condition does not show interest.

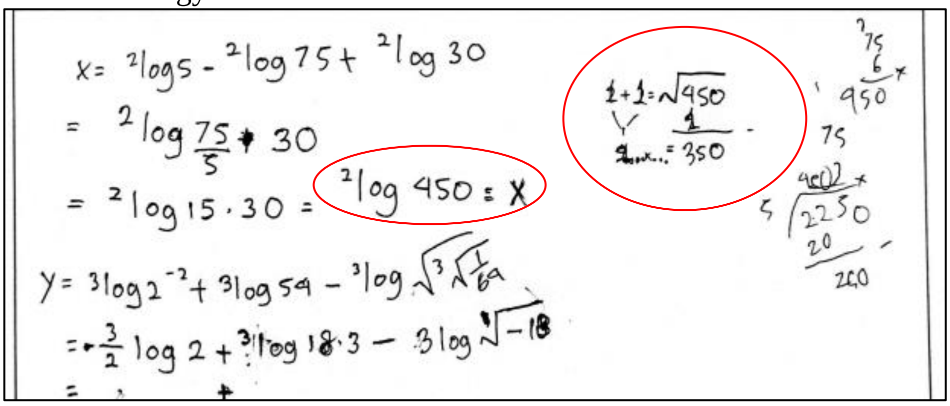


Figure 3. Example answer regarding the ontogenic obstacle

In addition to the existence of students' psychological ontogenic LO, the researcher also observed indications of students' instrumental ontogenic LO. This can be observed from the students' solving process on the RAT problem as in the Figure 3. The interview results show that students do not have technical skills that are key to exponents and logarithms. Students tend to pay less attention to key important things, such as the rules or properties of exponents and logarithms, bases in logarithms, and power numbers in exponents. That is what the researcher observed as a form of students' instrumental ontogenic LO.

Researcher : Well, here (answer to TKR question related number 2 in Table 1) you can 75 per 5, what is that property?
Students (S1) : The logarithm of subtraction.
Researcher : That's subtraction, right? Is it really 75 per 5? Or is it reversed?
Student (S1) : I don't know. But I wrote the big number on top.

In addition to students' psychological and instrumental ontogenic LO, researchers also found indications of students' conceptual ontogenic LO. This can be observed from the students' solving process on RAT problems revealed through interviews. Based on the interview results, S5 did not master the basic concepts of exponents as power numbers and logarithms as the inverse of exponents. In addition, S4 revealed a mismatch in the level of difficulty between the example problems discussed by the teacher in class and the problems that students had to do independently.

Researcher : Do you have any shortcomings related to learning in class?
Student (S4) : This drawback, usually the teachers give examples and problems that are a bit different. ... What is taught is basic, but when there is a problem, it is a little difficult.
Teacher : There are some children who are confused when it comes to powers. For example, 25 to the power of 2. Well, that took a long time to think about.
Researcher : Where is the confusion? 25 to the power of 2. Do they not know the multiplication?
Teacher : The multiplication is confused.

The lack of students' mastery of the prerequisite materials of exponents and logarithms was also revealed by the teacher in the interview. The teacher explained that multiplication is a prerequisite material in order to create a good ability and understanding of the concept of exponents and logarithms. However, some students did not master multiplication, causing learning barriers. Based on some of the things that have been described previously, researchers observe as a form of conceptual student ontogenic LO.

Epistemological obstacles

The existence of ambiguity, inconsistency, incompleteness, and other findings of the meaning of exponents and logarithms also show that students' meaning of exponents and logarithms is still limited (epistemological obstacles). In addition, the RAT results also showed student errors in solving some RAT problems. The researcher suspects the existence of students’ LO that are epistemological in nature on the material of exponents and logarithms. This can be observed from the answers to the RAT questions and interviews with students and teachers. Based on the results of the interview, S4 revealed that S4 was not familiar with problems with multiple root signs such as in RAT problem number 5. Meanwhile, S1 made up the answer to RAT problem number 4 because he did not master the concept of graphs in exponents and logarithms. S4 only mastered exponents and logarithms in the context outside the graph. This shows that S1 and S4 have limited understanding and mastery related to exponents and logarithms.

Researcher : The daily test is the same question model as the example or different?
Student (S3) : Some are the same, some are different.
Researcher : Is there something different? Does that make it difficult for you?
Student (S3) : A little difficult.

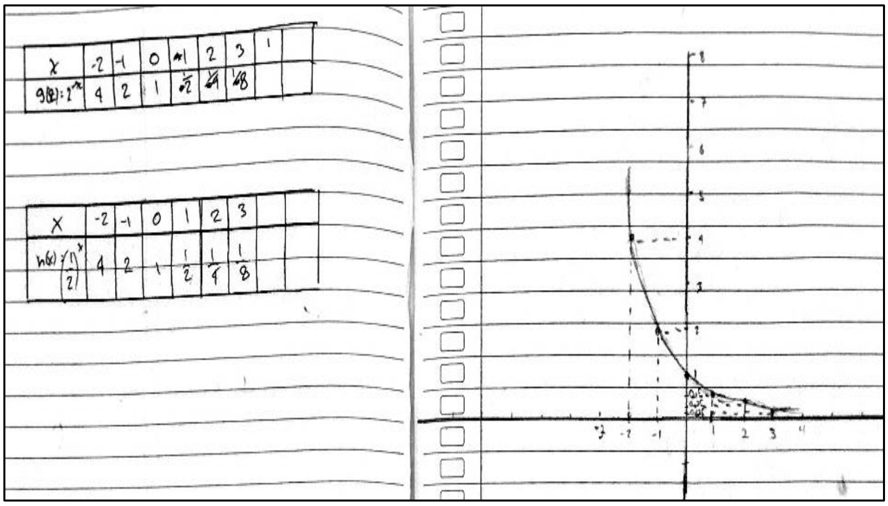


Figure 4. Class example question

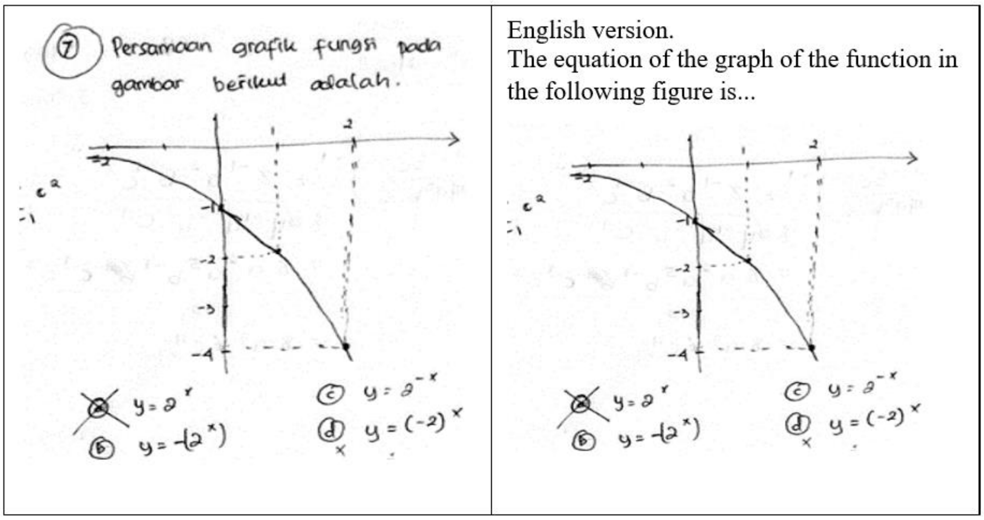


Figure 5. Example daily test

Next, in the interview, S3 revealed a model of daily test questions that differed from the sample problems in class, as in Figure 4 and Figure 5. In an example problem in class, students are asked to draw a graph of an exponent function. Meanwhile, in the daily test problem, students are asked to determine the function from the graph in the problem. It makes it difficult for S3 to solve the problem. This means that students have difficulty solving problems in a way different from the examples given by the teacher or textbook. All the learning barriers described previously refer to one problem: the limited context in interpreting and understanding exponent and logarithm material. The result is that students have difficulty solving a problem that differs in form or model from what students usually receive in class. The researcher observed this as a form of students' epistemological LO in the material of exponents and logarithms.

Didactical obstacles

The reality related to students' LO on exponents and logarithms material can also be influenced by other factors, such as didactic systems such as sequence factors or curriculum stages, including the presentation of material in classroom learning. Related to this, the researcher has a suspicion that there is a didactic LO. Therefore, it is necessary to further explore these LO.

1.1.1 Pangkat Bulat Positif

Dalam pernyataan 2^n, 2 disebut bilangan pokok dan n disebut pangkat. Istilah lain dari pangkat adalah eksponen. Adapun makna dari 2^5 ialah $2 \times 2 \times 2 \times 2 \times 2$.

Definisi

Jika n adalah sebuah bilangan bulat positif dan a bilangan real maka an didefinisikan sebagai perkalian n faktor yang masing- masing faktornya ialah a.

$$a^n = \underbrace{a \times a \times a \times \ldots \times a}_{n \text{ faktor}}$$

Definisi di atas dapat diperluas untuk n bilangan nol dan bulat negatif.

Definisi:

a. Untuk setiap a bilangan real bukan nol, maka $a^0 = 1$.

b. Jika n bilangan bulat positif dan a bilangan real bukan nol maka

$$a^{-n} = \frac{1}{a^n}$$

English version.

1.1.1 Power of Positive Integer In the statement 2^n, 2 is called the base number and n is called the power. Another term of power is exponents. The meaning of 2^5 is $2 \times 2 \times 2 \times 2 \times 2$. Definition If n is a positive integer and a is a real number then an is defined as the multiplication of n factors with each factor being a. $a^n = \underbrace{a \times a \times a \times \ldots \times a}_{n \text{ factors}}$ The above definition can be extended for zero and negative integers.	Definiton. a. For every non-zero real number a, then $a^0 = 1$. b. If n is a positive integer and a is non-zero real number then $a^{-n} = 1/a^n$

Figure 6. The definition of exponent are separated on different pages of the book

Based on further investigation through interviews, some students revealed shortcomings in learning exponents and logarithms in the classroom. S1 felt that the learning in class was incomplete because material not in the book made S1 difficult. Then, S5 revealed that the learning took place too fast, so S5 could not follow the learning well. In addition, S5 also felt that the example problems the teacher gave were still lacking. Meanwhile, S3 talked about the book used in learning. The exponent definition was separated in the book based on the power number on different pages, as in Figure 6. This led to students' incomplete understanding of the meaning of exponents.

Researcher : ...for the properties. Did you give the properties directly?
Teacher : Yes. The problem is that if I want to give the theory first, before I get it like this, there is actually this, this, this, this, this, it runs out of lesson time.
Researcher : ... this is suddenly an exponent function. Even though the function material itself is not yet.
Teacher : No, I haven't gotten it yet. This is also a quadratic function. Even though I haven't. Though not yet. Even though the material for functions and composition functions is in grade 2. How is it? Is it reversed?
Researcher : What do you think? Function and composition function first, right?
Teacher : But power numbers are also important, in grade 10. ... Or even the material is actually reduced. Why not exponent logarithm? And after that, the function.

Through interviews, teachers revealed shortcomings of the didactical system in learning exponents and logarithms. The teacher feels that the learning time is relatively short, so some material discussions are only briefly discussed or not given to students. This condition results in a less detailed presentation of exponent and logarithm material. This has implications for students, namely, students who need more mastery of some material on exponents and logarithms. Based on further investigation through student interviews, some of the concepts that students do not master as a result of the presentation of material that is less detailed include: (1) formal definitions of exponents and logarithms, which are the foundation of meaning; (2) exponent functions and their graphs; (3) root forms; and (4) the application of exponents and logarithms in everyday life. Therefore, it can be observed that the presentation of material that is less detailed has implications for students' ability to solve exponent and logarithm problems.

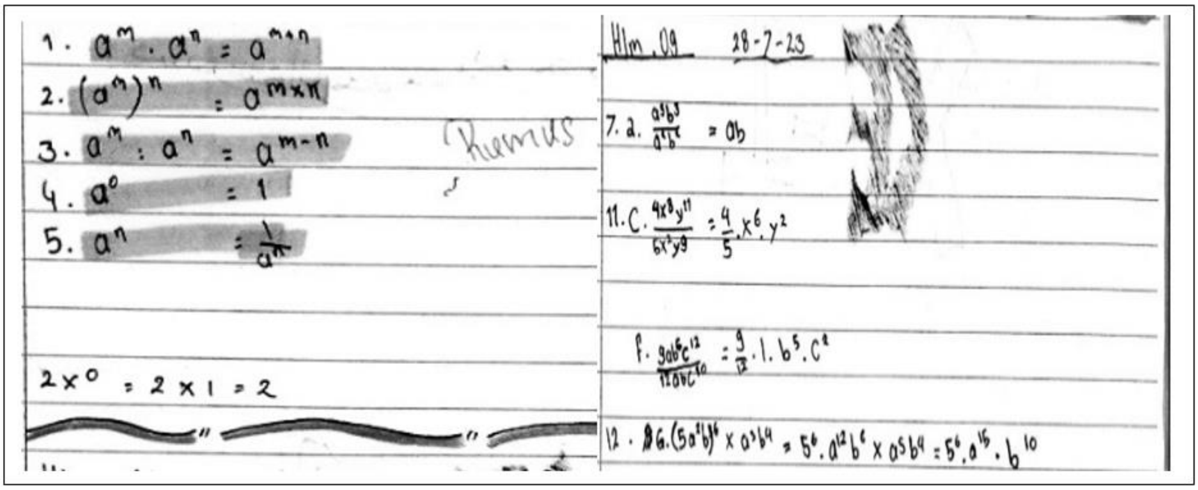


Figure 7. Student notes on exponential learning first meeting

Based on the results of interviews and documentation of teaching modules, teachers do not provide formal definitions of exponents and logarithms in classroom learning. In addition, learning related to the properties of exponents and logarithms is not preceded by proof, as in Figure 7, which shows that students immediately note down the formula or properties. It has implications for students' thinking processes. Thus, there is a functionally inappropriate sequence of materials. Furthermore, material related to exponent functions is considered inappropriate if given to students at the beginning of the odd semester of grade 10. Students have not received material related to the basic concepts of functions, so students often have difficulty when encountering problems related to exponent functions. It has implications for the inappropriate relationship between the concept of exponent functions and functions in general. The teacher also argued that the exponent and logarithm material should be reduced related to the exponent function and continued with the function material, then the exponent function. Thus, an exemplary sequence of material will be created structurally, namely the relationship between concepts.

Daftar Isi

English version.

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Figure 8. Sequence of material in student learning book

Regarding the sequence of grade 10 mathematics learning materials, researchers observed that students should first learn the basic functions concepts. In the student handbook, as in Figure 8, the material is presented sequentially, namely exponents and logarithms, rows and series, trigonometry, systems of linear equations and inequalities, quadratic equations and functions, statistics, rules of enumeration, and probability. The teacher also follows the order of the material in the book. In exponents and logarithms, there is exponent function material. In addition, it is clear that function material will appear in the equation and quadratic function material.

Meanwhile, the function material itself is only presented in grade 10. The formal definition of the function needs to be presented in the exponent and logarithm material. The material directly begins with the definition of the exponential function. Therefore, the researcher observed a sequence of unstructurally appropriate material or presented a good connection between the concepts of exponents and logarithms and other mathematical concepts.

Discussion

The concept image of exponents and logarithms according to students

According to students revealed in this study, the concept image of exponents is exponents as power numbers. Meanwhile, according to students revealed in this study, the concept image of logarithm is the opposite of exponent. Based on the interview results, a mathematics teacher also interpreted exponents as power numbers and logarithms as the opposite of power numbers (finding the power number). Therefore, the researcher observed understanding the conceptions of exponents and logarithms between students and teachers.

In the process of mathematical knowledge construction, a pattern of understanding of information is needed so that the information becomes knowledge (Uriarte, 2008). Meanwhile, in this study, the meaning of exponents and logarithms constructed by students is still at the information stage only, not yet becoming knowledge. As a result, students tend to be unclear and inconsistent in interpreting and applying the meaning of exponents and logarithms. It is revealed in the previous section that there is ambiguity and inconsistency in interpreting exponents and logarithms.

In mathematics, definitions play an important role (Chesler, 2013). Therefore, math teachers and students need to interact flexibly and productively with mathematical definitions in the classroom. Students interpret exponents and logarithms based on their expression or form. This indicates that the approach used by students in interpreting exponents and logarithms is a structural or object conception approach by looking at the visualization of exponents and logarithms (Sfard, 1991; Weber, 2002). The meaning is obtained from the teacher during classroom learning. In other words, the teacher introduces exponents and logarithms using a structural approach, which gives the meaning of exponents and logarithms in accordance with their expression or form. However, researchers have shown that this method leads to a limited conception of the concept of exponents and logarithms (Oktavihari & Priatna, 2023;Confrey & Smith, 1995). This was also revealed in this study, namely the incomplete meaning of exponents. This is in line with the results of research by Oliveira and Lopes (2024) which states that students still do not fully understand the formal definition of an exponent. Students tend to interpret exponents as power numbers with power numbers that are only limited to positive integers. As a result, when encountering power numbers other than positive integers, students cannot interpret them.

Referring to Weber (2002) research, before developing exponent and logarithm expressions as the result of a process, students need to understand the concept of exponential growth as a process. In this case, Weber (2002) used mental action to initiate the construction process of exponents and logarithms. This is in line with the triadic cycle in the meaning of mathematics by Harel (2008) which states that mathematics must be interpreted as a mental act which includes Way of Understanding (WoU) and Way of Thinking (WoT). Weber (2002) used a set of mental constructions that students can make to understand the concept of exponents and logarithms. Weber (2002) provides several activities for students using the idea "a^x represents a number a multiplied by a factor of x". Susanti et al. (2018) also stated that students' representations of the form a^x as a multiple number are expressed as repeated multiplication. Using this definition, students will describe $4^{3.5}$ as a number that is the multiplication of 4 by 3.5 times. This becomes a problem for students because it is not in accordance with their meaning. It is in line with Harel (2008) that the nature of learning mathematics must be preceded by a problem so that students have a goal to obtain a solution to the problem and obtain a certain mathematical meaning. The results of a study by Weber (2002) showed that students who were given a set of mental constructs did better than other students across a range of materials, including performing basic calculations, remembering formulas, explaining why a rule of exponents and logarithms is correct, and answering conceptual questions.

Furthermore, the fact that the meaning of exponents and logarithms is ambiguous, inconsistent, and incomplete indicates that students have not yet reached the situation of institutionalization according to didactic theory in mathematics (Brousseau, 2002). In the didactical theory, students have not been able to use knowledge related to exponents and logarithms to solve various problems. In addition, students are fixated on the knowledge provided by the teacher or book so that they cannot change the knowledge they already have into new knowledge.

Learning obstacles (LO) on exponent and logarithm

Ontogenic obstacles relate to students' learning ability (Suryadi, 2019). According to Suryadi (2019), LO is divided into three types, namely ontogenic obstacles caused by psychological factors such as interest and motivation (psychological), technical matters such as notation and properties (instrumental), and mastery of basic concepts, prerequisite materials, and learning demands (conceptual). Based on the previous discussion about students' meanings and interpretations of exponents and logarithms, there are indications of students' LO on exponents and logarithms. The existence of ambiguity, inconsistency, incompleteness, and the findings of the meanings of exponents and logarithms that have been discussed previously, researchers observed that students need more time to learn exponents and logarithms further. It is revealed from the existing reality. There is a reality of low student motivation and interest in exponents and logarithms. Motivation is essential in learning and mathematics learning outcomes (Schukajlow et al., 2023). The higher the students' interest in learning mathematics, the higher the mathematics learning outcomes students will obtain (Karlina et al., 2021). In addition, students' interest in mathematics partially mediated the relationship between motivation to learn mathematics and mathematics learning outcomes and between the quality of mathematics teaching and mathematics learning outcomes (Arthur et al., 2022). Thus, students' low motivation and interest in exponents and logarithms material causes LO in students. It indicates the presence of LO ontogenic obstacles that are psychological in nature.

In addition to the realities related to student motivation and interest, some students have not mastered the technicalities of the concepts of exponents and logarithms, such as the meaning of notation and bases in logarithms, power numbers in exponents, and their properties. This finding aligns with research by Dintarini (2018), which states that student errors in solving problems are caused by technical matters, namely a lack of understanding and algorithmic skills. This condition refers to errors due to lack of understanding, namely, students using the properties of exponents and or logarithms incorrectly. This reality indicates the existence of ontogenic obstacles that are instrumental in nature.

The next reality is that some students have not mastered prerequisite materials such as basic arithmetic operations, functions, and linear equations. Some students also need to learn the formal definitions of exponents and logarithms fully. As explained earlier, teachers give the meaning of exponents and logarithms through a structural approach, causing a limited conception of the concepts of exponents and logarithms (Confrey & Smith, 1995). This indicates the existence of ontogenic obstacles that are conceptual in nature. The reality described shows the existence of LO, namely ontogenic obstacles that are instrumental, conceptual, and psychological (Brousseau, 2002; Suryadi, 2019).

Epistemological obstacles are difficulties in the learning process due to limited context; students experience limitations in mastering something within a specific context (Dourex in Brousseau, 2002). In the previous discussion, the meaning of exponents and logarithms is at the information stage, so the meaning is still dependent on an independent context (Uriarte, 2008). It is shown from the findings that students have difficulty when solving other problems related to exponents and logarithms, such as graphs, functions, and applications to real problems. It is also observed in students' difficulty when using the meaning of exponents and logarithms to work on problems that differ in form or model from those usually given by teachers and books. This reality indicates that students' meanings of exponents and logarithms are still limited, so students cannot solve problems with specific contexts. This aligns with the research by Confrey and Smith (1995), which was discussed in the previous section. Thus, the reality shows the existence of LO, namely epistemological obstacles in exponents and logarithms (Brousseau, 2002).

Didactical obstacles, namely difficulties that occur due to didactical systems such as sequence factors and or curriculum stages, including the presentation of material in classroom learning (Suryadi, 2019). Factors affecting math learning outcomes include didactic strategies math teachers use in delivering lessons and institutional resources (Mazana et al., 2019). Referring to research by Leikin and Winicki-Landman (2001), teachers do not always realize that it is possible to define a concept differently. Thus, how the concept is defined determines the order of learning, including the order of learning materials and proof procedures. Based on the discussion in the previous section, the nature of learning mathematics must be preceded by a problem so that students can obtain a solution to the problem and specific mathematical meanings (Harel, 2008). Then, it is followed by mental actions that shape students' thinking and produce a good understanding of exponents and logarithms. Based on what was revealed by students and teachers in interviews, teachers do not provide problems and mental actions. Hence, students' ways of thinking are limited and cause a lack of student understanding of the concepts of exponents and logarithms. Whereas teacher instructional practices positively correlate with student learning in mathematics, teachers can continue to motivate and use instructional practices that will further improve student achievement in mathematics (Pulumbarit, 2022).

The teacher precedes the learning of exponents and logarithms by providing information about the concept image and its properties without proof, then giving examples and practice problems. Thus, based on the theory of didactic situations (Brousseau, 2002), students' situation of action and formulation is still less than optimal because a problem does not precede learning. Learning exponents and logarithms preceded by problems and continued with activities to solve problems can develop students' knowledge and more meaningful learning (Borji et al., 2024; Kuper & Carlson, 2020). In addition, some students revealed that the learning felt too fast and incomplete. Therefore, this indicates the existence of didactical obstacles in exponents and logarithms related to the stages of presenting less detailed material (Brousseau, 2002).

Regarding the learning by teachers in the classroom, teachers need to explain the formal definitions and proofs of the properties of exponents and logarithms because they are difficult for students and need more learning time. Providing formal definitions and proofs of the properties of exponents and logarithms is essential for a functionally good sequence of material. In addition, it was revealed that the material in grade 10 is quite a lot, so teachers have difficulty dividing time for mathematics learning. It shows the immaturity of the time allocation in mathematics learning in grade 10, which has implications for the stages of presenting material to teachers in the classroom. Thus, there are didactical obstacles in exponents and logarithms related to the order of material that is not functionally good and the short allocation of learning time.

Furthermore, there is an inappropriate order of material. In the student handbook, the formal definition of exponents is divided based on the power number. It appears on different pages so that students interpret exponents separately, not as a whole. It means that the order of the material could be structurally good (the connection between concepts). In addition, there is exponent function material, while the function material itself, including the formal definition of a function, will only be discussed in grade 10, so the connection between concepts is weak. It indicates a structurally poor sequence of material (Brousseau, 2002). The implication is students' lack of meaning and mastery related to exponent functions, including graphs and their applications.

Our comprehensive analysis confirms the presence of didactical obstacles in the learning of exponents and logarithms. These obstacles are related to the presentation of material that needs more detail, the allocation of short learning time, and the use of a less appropriate sequence of material structurally and functionally. As discussed earlier, these didactical LOs significantly impact the emergence of the other two types of LO in exponents and logarithms.

Conclusion

There were differences in the meaning understood by students of the concepts of exponents and logarithms. Students interpret exponents as power numbers, multiplication of factors, formulas, mathematical operations, or mathematical equations, while students interpret logarithms as the opposite of exponents or power numbers and roots. The diversity of students' meanings of the concepts of exponents and logarithms is based on differences in the process of acquiring the meaning of exponents and logarithms, both directly through the learning process carried out by teachers in the classroom and indirectly through various learning resources in the form of mathematics textbooks.

In addition, the results also show the existence of learning obstacles experienced by students, both ontogenic, epistemological, and didactical. Ontogenic obstacles experienced by students are psychological, instrumental, and conceptual. Epistemological obstacles occur when students experience limitations in applying the concepts of exponents and logarithms to a wider scope or in different contexts, while didactical obstacles that occur to students are due to the presentation of material that is not detailed, including the incompatibility of the order of the material presented, both functionally and structurally. With the obstacles that occur to students, it will be exciting to conduct further research to develop a hypothetical learning trajectory, including didactical design, that has the potential to minimize the occurrence of these learning obstacles.

Conflicts of Interest

The authors declare no conflict of interest regarding the publication of this manuscript. In addition, the authors have completed the ethical issues, including plagiarism, misconduct, data fabrication and/or falsification, double publication and/or submission, and redundancies.

Author Contributions

Yuvita Andriani Kusumadewi: Conceptualization, investigation, writing - original draft, resources, and formal analysis; **Riki Andriatna:** Conceptualization, writing-review & editing, validation, supervision, and methodology.

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