Jurnal Elemen, 11(1), 15-33, January 2025
https://doi.org/10.29408/jel.v11i1.26938

e- ISSN: 2442 - 4226

Check for updates

Abstraction level of van Hiele's theory: Occurrence of side effects in GeoGebra integration

Thesa Kandaga [1*], Idha Novianti [1], Mazlini Adnan [2]

[1] Mathematics Education Department, Universitas Terbuka, Tangerang Selatan, Indonesia

[2] Mathematics Education Department, Universiti Pendidikan Sultan Idris, Tanjong Malim, Perak, Malaysia

[*] Correspondence: thesa.official@ecampus.ut.ac.id

Abstract

One of the obstacles to teaching geometric transformation is the complex procedures that require a broad base of prerequisite knowledge. This complexity often leads students to focus on rote memorization and procedural calculations instead of understanding the underlying context. This study aims to overcome these challenges by developing a hypothetical learning trajectory (HLT) and using GeoGebra to enhance visualization and understanding. The case study involved ten university students in Indonesia who tackled the abstraction level in the Transformation, Isometries, and Reflection topics. The researchers analyzed students' worksheets, activity observations, and learning obstacle tests to extract their geometric thinking. Qualitative analysis of the data revealed that seven out of ten participants met three of the four abstraction level indicators. The use of GeoGebra in HLT helped overcome epistemological obstacles. However, integrating GeoGebra into the HLT introduced a new issue: a GeoGebra-centric habit characterized by an excessive dependence on GeoGebra in solving geometric transformation problems.

Keywords: epistemological obstacle; GeoGebra; geometric thinking; hypothetical learning trajectory; van Hiele's model

How to cite: Kandaga, T., Novianti, I., & Adnan, M. (2025). Abstraction level of van Hiele's theory: Occurrence of side effects in GeoGebra integration. *Jurnal Elemen*, *11*(1), 15-33. https://doi.org/10.29408/jel.v11i1.26938

Received: 8 July 2024 | Revised: 5 December 2024
Accepted: 23 December 2024 | Published: 1 February 2025

Introduction

The van Hiele model of geometric thinking, introduced by Pierre van Hiele and Dina van Hiele-Geldof in the late 1950s, has long been regarded as a cornerstone in understanding how students learn geometry. This model outlines five hierarchical levels of geometric thought: Visualization, Analysis, Abstraction, Deduction, and Rigor (van Hiele, 1957). Each level reflects a distinct way of approaching geometric concepts, starting from recognizing shapes visually to understanding geometry as a formal axiomatic system (Clements & Battista, 1992; Jones, 2001). Moving through these levels is critical for students to develop a strong and comprehensive understanding of geometric principles. Among these levels, abstraction is particularly pivotal, as it requires students to analyze and relate properties of geometric figures in a structured, logical manner (Usiskin, 1982; Burger & Shaughnessy, 1986).

Despite the importance of the abstraction level, many students struggle to progress beyond this stage due to significant epistemological obstacles. As identified by Kandaga, Rosjanuardi, & Juandi (2022a), "Transformation procedures are quite complex and involve various prerequisite knowledge, shifting students' cognitive focus to memorization and procedural calculations, thus resulting in a lack of understanding of the problem context." This obstacle highlights the challenge of balancing conceptual understanding with procedural fluency, where students may become preoccupied with calculation methods at the expense of grasping the broader context and meaning of geometric transformations. Addressing this problem is crucial for ensuring students' success in mastering geometric thinking.

In response to this challenge, this study aims to design a Hypothetical Learning Trajectory (HLT) that specifically targets the abstraction level in geometric transformations. Building upon prior research (Chang & Bhagat, 2015; Molnár & Lukáč, 2015; Noto, Priatna, & Dahlan, 2019), the HLT integrates dynamic geometry software i.e. GeoGebra to bridge the gap between visualization and conceptual understanding. GeoGebra's interactive capabilities facilitate exploration and manipulation of geometric figures, enabling students to construct a deeper understanding of transformations (Pech, 2012; Owusu, Bonyah, & Arthur, 2023). The HLT also includes tasks that encourage analytical reasoning, connecting visual results to procedural steps and problem contexts. By addressing the epistemological obstacle identified by Kandaga et al. (2022a), this research aims to investigate the effectiveness of the HLT in fostering deeper student understanding of transformations.

The implementation of the HLT in this study not only facilitated students' understanding of complex geometric procedures but also highlighted the importance of designing instructional materials that address both conceptual and procedural knowledge. By employing a GeoGebra to Analytical Conversion Table, the study effectively bridged the gap between visual and analytical learning, ensuring that students could transfer their understanding from dynamic software to manual problem-solving contexts.

During the implementation of the HLT, this study uncovered an additional issue: the emergence of GeoGebra-centric behavior among some students. This behavior, characterized by an overreliance on GeoGebra for solving problems, reflected an improper way of thinking that hindered the development of analytical reasoning skills. Students exhibiting this behavior often relied solely on GeoGebra's visual outputs without engaging critically with the underlying mathematical concepts or steps involved. While not entirely incorrect, this behavior mirrors a *didactical obstacle* as defined by Brousseau (2002), where the instructional tool inadvertently becomes a barrier to learning. The identification of GeoGebra-centric behavior provides a new layer of insight into the unintended side effects of integrating technology into mathematics education (Owusu, Bonyah, & Arthur et al., 2023; Laborde, 2006; Mendoza, Nieto-Sánchez, & Vergel-Ortega, 2019).

The discovery of GeoGebra-centric behaviour, alongside the implementation of the HLT, represents a significant contribution to this study's novelty. While the HLT successfully addressed the initial epistemological obstacle by scaffolding student understanding and fostering a balance between visualization and analytical reasoning, it also highlighted the need to carefully integrate technology in a way that avoids dependency and promotes deeper cognitive engagement. These findings underscore the importance of designing balanced instructional strategies that utilize GeoGebra as a tool for exploration without compromising the development of abstract and deductive reasoning (Juandi, Kusumah, Tamur, Perbowo, & Wijaya, 2021; Noto, Priatna, & Dahlan, 2019). This study not only contributes to the field of mathematics education by providing practical solutions for common learning obstacles but also advances discussions on the implications of technology integration in classrooms.

Methods

A case study research was conducted on ten subjects who were students at one of the universities in Indonesia in late 2023. The research subjects consisted of two males and eight females with cumulative grade point averages (academic abilities) ranging from 2.5 to 3.5 on a 4-point scale. All subjects had good proficiency in using digital technology and GeoGebra. The subjects underwent tests for learning obstacles before and after the implementation of the Hypothetical Learning Trajectory (HLT). The tests for learning obstacles were conducted to measure the achievement of indicators of geometric thinking levels.

Data in this study were collected through two primary methods: observation and epistemological obstacle test. Observations were conducted during the implementation of the learning activities designed using the *Hypothetical Learning Trajectory* (HLT). This allowed researchers to closely monitor how students engaged with the HLT, particularly their interactions with GeoGebra and their approaches to problem-solving at the abstraction level. The observations focused on identifying patterns in students' reasoning, their ability to transition between visual and analytical thinking, and any instances of GeoGebra-centric behavior. In addition to the observations, an *epistemological obstacle test* was administered to evaluate students' abilities across the five indicators of mathematical literacy. The test was designed to identify specific challenges faced by students, such as their reliance on procedural memorization and their capacity to relate solutions to the problem context. These complementary methods ensured a comprehensive understanding of how the HLT addressed both the epistemological obstacles and the newly identified GeoGebra-centric behavior.

The main goal of this study was to describe how to overcome epistemological obstacles in the learning of transformation geometry as presented by Kandaga, Rosjanuardi, & Juandi (2022a) within a HLT. To achieve this goal, the data obtained were processed qualitatively using the analysis techniques of Miles, Huberman, & Saldana (2014). These analysis techniques consisted of three stages: data reduction, data display, and conclusion drawing. Additionally, the processing and organization of qualitative data were facilitated using NVIVO 12, which allowed for efficient coding, categorization, and visualization of the observational and test data, ensuring systematic and in-depth analysis.

Data reduction was performed for the indicators that were successfully identified, adopting Harel's (2008) concept of Way of Thinking (WoT). The subjects' responses to the learning obstacles test were grouped based on the similarity of the WoT they presented. Data reduction was then carried out on the WoT that were not suitable for solving the problems. The WoT that were appropriate in addressing the issues were further grouped based on the indicators of the van Hiele levels of geometric thinking they achieved.

The process continued to the data display stage. These groups of WoT, subsequently referred to as categories, were connected to one another with categories at other levels of van Hiele’s model thinking, thus producing a description resulting from these connections. These connections were summarized to answer the research question in this study.

Results

Hypothetical learning trajectory design

The Hypothetical Learning Trajectory (HLT) in this study was specifically designed for the abstraction level, and focused on the material covering the reflection of objects across a straight line. The design of the HLT was intended to address the epistemological obstacles at Level-3 (Abstraction) identified by Kandaga et al. (2022a), which were “Transformation procedures are quite complex and involve various prerequisite knowledge, shifting students' cognitive focus to memorization and procedural calculations, thus resulting in a lack of understanding of the problem context”.

These obstacles were divided into two issues: (1) a shift in cognitive focus towards memorization and procedural tasks, and (2) complex reflection procedures at the abstraction level. The strategies to address these issues were also divided into two parts, as shown in Figure 1.

The Reflection material, which was divided into two Hypothetical Learning Trajectories (HLTs), was due to the different strategies employed to address the epistemological obstacles that emerged. The strategy used to address issue (1) involved using GeoGebra to provide visualization and to assist the understanding of every detail of the reflection procedures. Meanwhile, for issue (2), a conversion table for the Reflection procedures was provided using GeoGebra and the reflection procedures analytically. A more comprehensive HLT for both issues is displayed in Table 1 and Table 2.

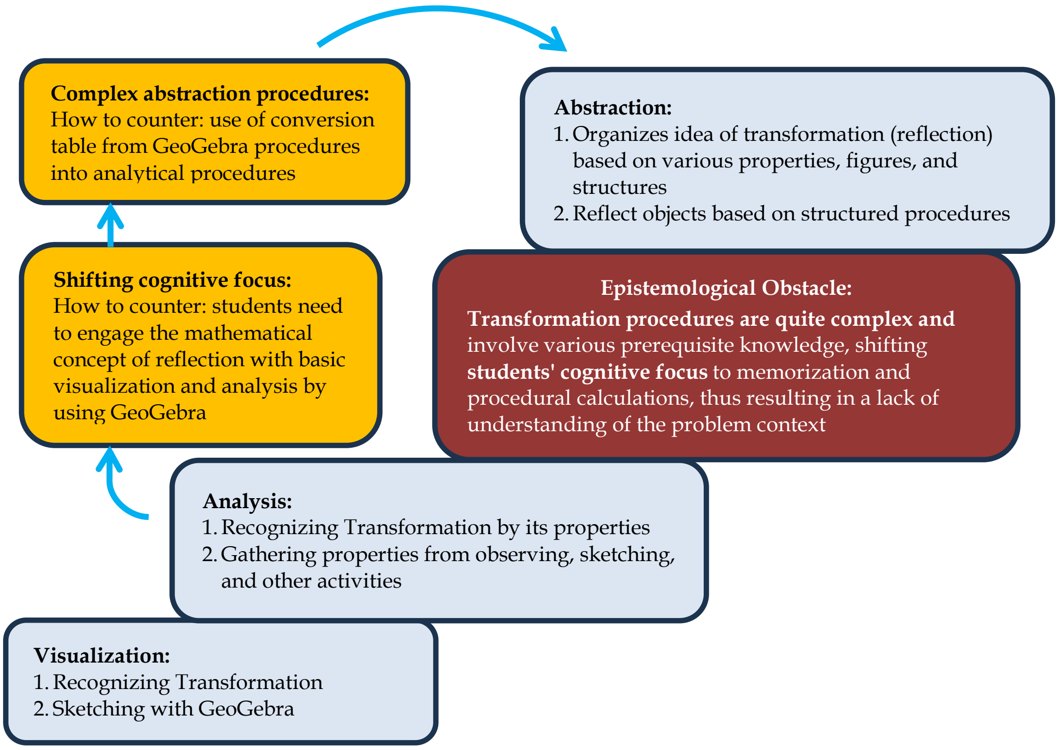


Figure 1. Strategy to overcome epistemological obstacle on the abstraction level

Table 1. HLT for the abstraction level in the first session

Phase	Learning Goals	Hypothetical Learning Process
Information	Recognizes and visualizes reflection into a straight line	Students examines definition of reflection and explores its visualization through GeoGebra pre-made content.
Directed Orientation	Organizes properties of reflection	1) Examining properties of reflection by exploring its visualization through GeoGebra pre-made content
Explicitation		2) Describes identification results of reflection properties 3) Formulates reflection procedures from the activities
Free Orientation	The GeoGebra Procedure of Reflection	Students evaluates to tests the procedures to solve reflection problem by using GeoGebra

Table 1 presents the first session. The instruction emphasized efforts to address the shift in students' cognitive focus that often moved from conceptual understanding to memorization and procedural tasks. For this purpose, effective tools for visualization and calculation were required. Tools such as GeoGebra had been proven capable of providing interactive and intuitive visualizations, which helped students deeply understand the concepts of geometric transformations. By using GeoGebra, students were able to manipulate objects and see the transformation results in real-time, which strengthened their understanding of geometric properties and involved procedures. Recent research by Owusu et al. (2023) showed that the use of visual aids like GeoGebra could enhance students' conceptual understanding and reduce reliance on memorization and procedural calculations.

Besides visualization, accurate calculation was also an essential element in overcoming the obstacle. GeoGebra provided calculation features that allowed students to compute transformation results quickly and accurately, enabling them to mathematically validate their visualizations. This not only improved the accuracy of calculations but also ensured that students remained focused on conceptual understanding rather than getting caught up in manual calculation errors. Research by Mendoza et al. (2019) confirmed that the integration of calculation tools in mathematics instruction helped students better understand abstract concepts and apply them in various contexts. Thus, the use of visualization and calculation tools such as GeoGebra was crucial for maintaining students' cognitive focus on conceptual understanding in the learning of geometric transformations.

Table 2. HLT for the abstraction level in the second session

Phase	Learning Goals	Hypothetical Learning Process
Information	Recalls the GeoGebra reflection procedures and understands current learning goals	Examines The GeoGebra to Analytics Table
Directed Orientation	Excels at converting GeoGebra procedure of reflection into analytical reflection procedure	Filling the worksheet contained with The GeoGebra to Analytics Table
Explicitation	Formulates analytical procedure of reflection	Describes their findings about the reflection procedures
Free Orientation	Understands every step of the analytical procedure	Implements and evaluates the analytical procedure of reflection into various problems which design to tests detailed understanding
Integration	Understands the essence of abstract concepet of reflection	Summarizes the procedure of reflection

The second session, presented in Table 2, aimed to transform the conceptual understanding of reflection acquired in the first session into more detailed and complex analytical procedures. As previously mentioned, reflection procedures are complex and require various prerequisite materials. This complexity often leads to students not understanding the bigger picture of reflection itself (Kandaga et al., 2022a). Students merely follow each procedural step without comprehending the main concept and the benefits of what they are doing. Therefore, scaffolding is needed to convert the conceptual understanding of reflection into a more analytical process while maintaining the procedural direction in line with the students' concepts. The study used a GeoGebra to Analytical Conversion Table, which served as scaffolding to overcome this learning obstacle.

The GeoGebra to Analytical Conversion Table divided into three main sections: 1) GeoGebra Instructions, this section provides step-by-step instructions on how to use GeoGebra for performing specific geometric transformations; 2) Manual Analytical Procedures, this section details the procedural steps for performing the same task manually, ensuring students understand the underlying mathematical concepts; 3) Manual/Hand Sketches, this section is for students to manually sketch the geometric transformations. It helps reinforce their understanding by visually drawing the steps and outcomes, such as plotting points and reflecting them across a line. The GeoGebra to Analytical Conversion Table presented in Figure 2.

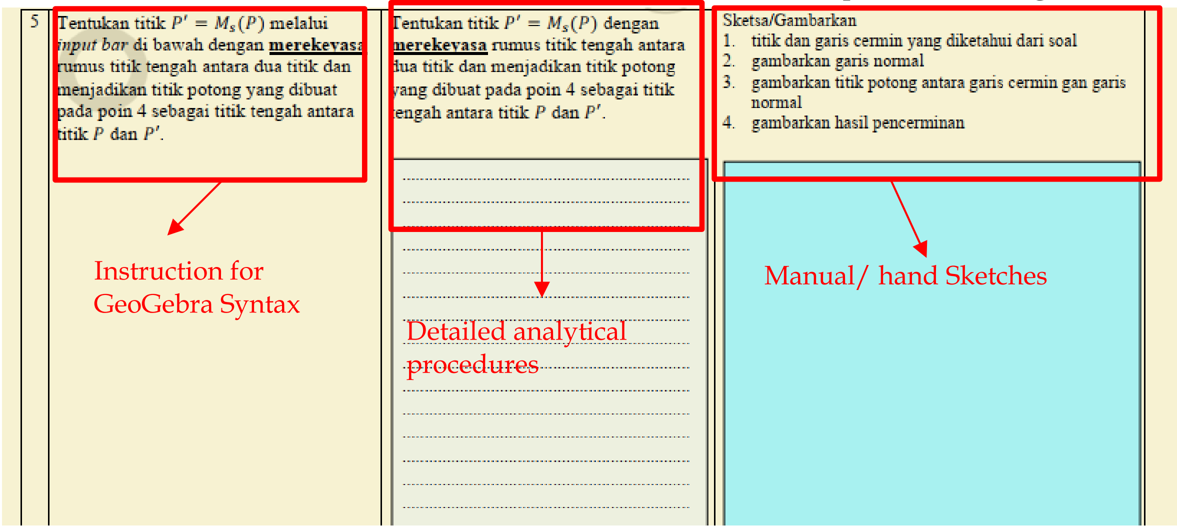


Figure 2. The GeoGebra to analytical conversion table

The structured format helps in converting procedural knowledge gained from GeoGebra into a deeper conceptual understanding of geometric transformations. This method ensures that students are not only able to perform the transformations but also understand the 'why' and 'how' behind them. By combining digital instructions, analytical procedures, and manual sketches, the worksheet addresses different learning styles, catering to visual, analytical, and kinesthetic learners. This multi-modal approach is supported by educational research, which indicates that using various methods can enhance understanding and retention of mathematical concepts (Owusu et al., 2023; Confrey et al., 2014).

Students' performance

The results of the HLT implementation in both sessions indicated that students were able to understand the reflection procedure well and create accurate visualizations of the reflection concept with structured procedures using GeoGebra. The data analysis was conducted with the assistance of NVivo, coding processes were applied to address the three abstraction indicators in the visual-GeoGebra aspect, namely: (1) *descriptive coding* to capture explanations of the transformation based on its properties, (2) *pattern coding* to analyze reasoning using the properties and structure of the transformation, and (3) *thematic coding* to organize ideas or concepts based on the relationships among various properties, images, and transformation structures. Out of ten subjects, seven met all these indicators. The coding results for these three indicators presented in Figure 3 below.

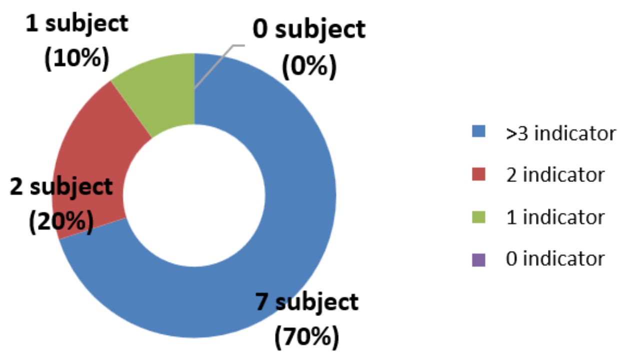


Figure 3. The number of subjects that meet the three abstraction indicators

The concept of reflection was then extended to the analytical aspect. There was only one indicator in this aspect, which was constructing reflections using mathematical procedures. This indicator was the primary indicator at the abstraction level, whereas the visual aspect provided by GeoGebra and all its indicators served merely as scaffolding to achieve this indicator. To reach this indicator, subjects were asked to complete the conversion table from GeoGebra procedures to analytical procedures. Their proficient understanding of reflection procedures using GeoGebra was then converted into various mathematical calculations. Through this conversion process, the subjects did not lose direction in performing the analytical reflection procedures and gained a deeper understanding of the reasons behind these procedures. Thus, the scaffolding involved presenting the outer shell of reflection through visualization and procedural framework, followed by detailed analytical procedures with the previously understood procedural framework.

The implementation results for the analytical aspect of the reflection material showed that seven out of ten subjects successfully met the above-mentioned indicator. These seven subjects managed to convert their understanding of visual reflection procedures in GeoGebra into analytical reflection procedures. Some of the subjects' work was very detailed, even providing visualizations by replicating the images they created in GeoGebra. The use of the conversion table also served as one of the scaffolding methods in achieving the indicator at the abstraction level. With several scaffolding methods in place, it was proven that the subjects could meet the indicators and overcome epistemological obstacles at the abstraction level. Figure 4 displays the best response during the HLT implementation.

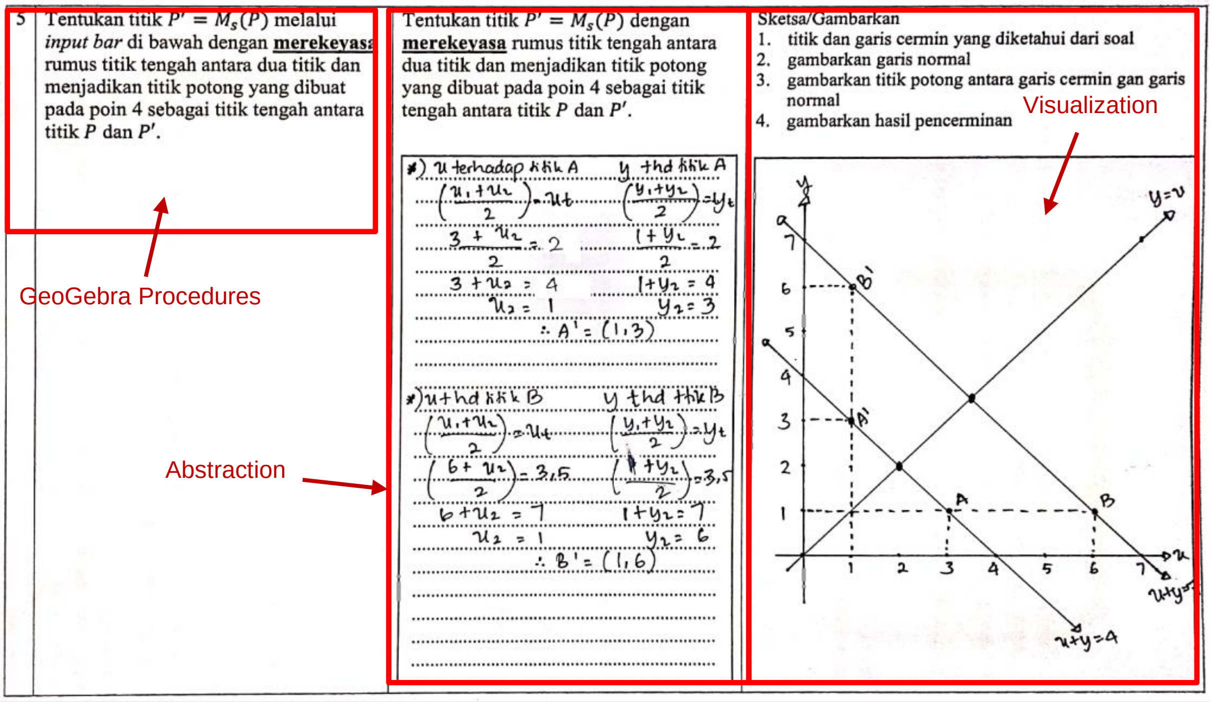


Figure 4. Best response during the HLT implementation on filling the GeoGebra conversion table and overcoming the epistemological obstacles

Observing the students' positive responses, demonstrated by their good performance using the GeoGebra conversion table, a learning obstacle test was then conducted. In order to assess the achievement of the fourth abstraction indicator, which involves constructing reflection through a systematic procedure, students were asked to describe and calculate mathematically based on the elements they successfully identified. This method would connect their understanding of the elements in reflection with the procedures they needed to perform. Students would transform geometric shapes into an abstract mathematical model according to its elements, solve it mathematically, and then convert it back into geometric form. The instructions provided for the Manual or Analytical Calculation section were as presented in Figure 5.

Tentukan titik $P' = M_s(P)$ dengan **merekeyasa** rumus titik tengah antara dua titik dan menjadikan titik potong yang dibuat pada poin 4 sebagai titik tengah antara titik P dan P'.	Translation: Determine the point $\boldsymbol{P'} = \boldsymbol{M_s}(\boldsymbol{P})$ by constructing the midpoint formula between two points and using the intersection point created in step 4 as the midpoint between point P and P'

Figure 5. Test instructions for the manual or analytical calculation section at the abstraction level

Students were also asked to manually sketch the problem to provide a connection between the visualization of the elements of reflection and their abstraction. In this way, students fully understood each step constructing the reflection.

Sketsa/Gambarkan
1. titik dan garis cermin yang diketahui dari soal
2. gambarkan garis normal
3. gambarkan titik potong antara garis cermin gan garis normal
4. gambarkan hasil pencerminan

Translation:
Sketch:
1. Point and the mirror line
2. Normal line
3. Intersection point between the mirror line and the normal line
4. The reflection result

Figure 6. Test instructions for the manual sketch section

The test instructions were presented in Figure 6, and these instructions were made in the same conversion table as the analytical calculation instructions. The conversion table included instructions to write the GeoGebra syntax; however, this part was deemed unnecessary for the abstraction level test instrument.

Based on the analysis of the learning obstacles test, out of the 7 subjects who met the previous three indicators, all succeeded in meeting the fourth indicator. The results presented in Figure7 clearly demonstrate the effectiveness of the Hypothetical Learning Trajectory (HLT) designed to address the epistemological obstacles encountered by subjects in the topic of reflections in geometric transformations. The blue-colored scores, representing the group guided by the HLT, show consistent and significant improvement across levels. This progression indicates that the carefully constructed learning path successfully supported students in overcoming their initial conceptual difficulties. For instance, the highest scores in the blue group increased steadily from 2 at Level 1 to 14 at Level 5, while even the lowest scores showed a marked improvement, rising from 1 at Level 1 to 8 at Level 5. These results validate that the HLT provided not only a structured framework for students to engage with complex concepts but also scaffolded their understanding, enabling them to transition from basic comprehension to higher-order problem-solving in this domain.

In contrast, the red group, which did not follow the HLT, exhibits stagnation and inconsistency in their performance. Their scores peaked at Level 3 and then declined, with some subjects failing to produce correct solutions by Level 5. This stark difference reinforces the value of a well-structured HLT in addressing epistemological obstacles that often hinder students' ability to connect abstract mathematical concepts with practical applications. The HLT's intentional sequencing of tasks and emphasis on gradual conceptual development effectively bridged gaps in students' prior knowledge, allowing them to overcome the common misunderstandings in geometric reflections. These findings strongly suggest that integrating a well-designed HLT into mathematics education can significantly enhance learning outcomes, especially in areas where epistemological challenges are prevalent.

If a broader analysis is conducted involving several other levels of geometric thinking in the van Hiele model, it would be found that the abstraction level has the steepest learning curve and is a key determinant of proficiency at other levels. Figure 7 shows the achievements of several subjects against the indicators at each level of geometric thinking.

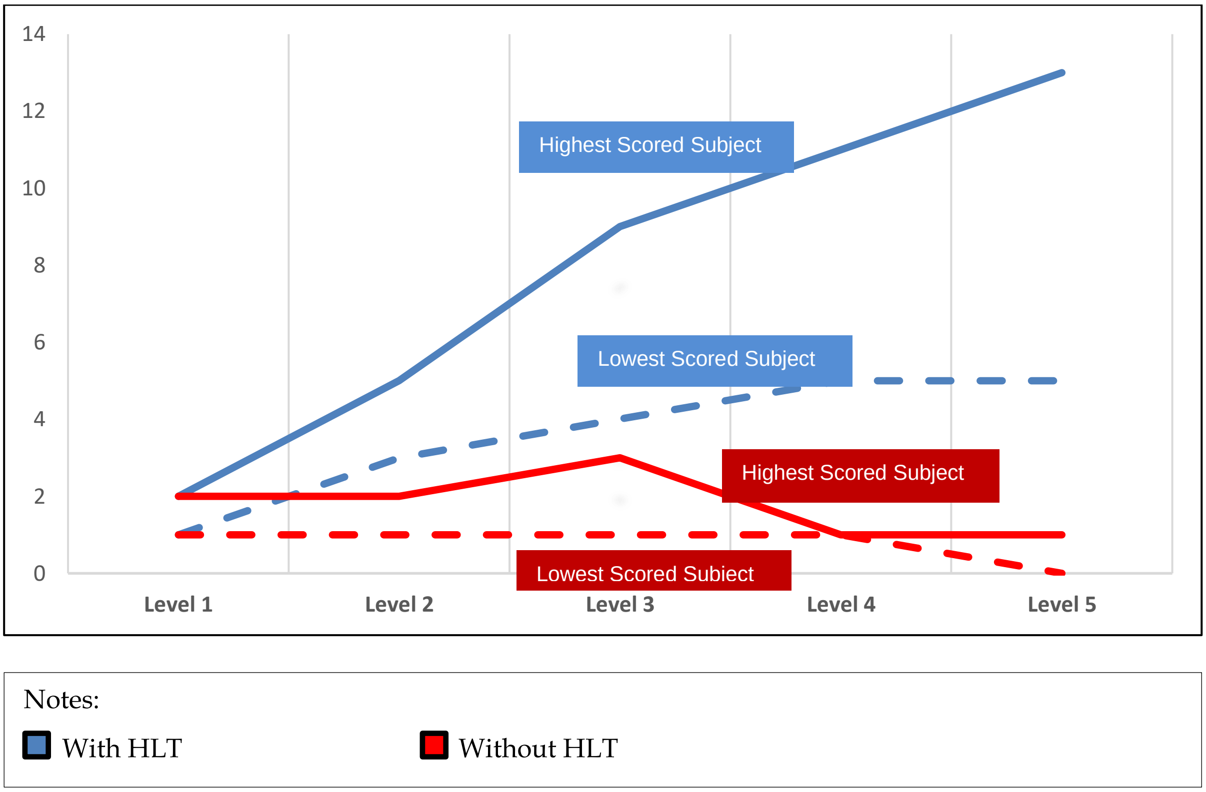


Figure 7. Wider review of students' performance in the van Hiele's levels of geometric thinking

In Figure 7, the blue and red graphs each represent two subjects with the best and worst indicator fulfillment, respectively. Based on the graph, it is evident that Level-3 (abstraction) had the steepest learning curve. Subjects who underwent learning without using the HLT showed poor results. The red line graph indicates that both subjects failed to meet the indicators for levels beyond abstraction. In contrast, subjects who underwent learning with the implementation of the HLT were able to follow the indicators at subsequent levels successfully. The connection between these ability levels is crucial because each level develops its own terminology and way of thinking (van Hiele-Geldof & van Hiele, 1984).

In the designed HLT, although the focus was on the abstraction level indicators, it also included the use of GeoGebra to provide visualization of the problems. This visualization process was also one of the scaffolding methods in constructing participants' conceptual understanding. As believed by many researchers, such as Jones (2005), Clements & Battista (1992), and van Hiele-Geldof (1957), visualization is the initial perception in the concept construction process. Therefore, the HLT design was intended to integrate GeoGebra into learning activities that directed participants to perform a series of deductions. Moreover, the designed HLT also required participants to analyze, reason, and conclude the concepts being constructed.

Discussion

HLT and students' achievement

The findings from this study strongly affirm that the Hypothetical Learning Trajectory (HLT) successfully addressed the epistemological obstacle previously identified in the topic of reflections in geometric transformations. The obstacle, as described by Kandaga, et al. (2022a) and Noto, Priatna, & Dahlan (2018, 2019), involves the complexity of transformation procedures that require various layers of prerequisite knowledge. This complexity often shifts students' cognitive focus to memorization and procedural calculations, resulting in a lack of understanding of the problem's context. The HLT, designed with a clear focus on scaffolding learning and integrating visual tools like GeoGebra, enabled students to overcome these challenges by fostering a deeper conceptual understanding.

The analysis of the test results showed that seven subjects who met the first three indicators successfully fulfilled the fourth indicator. This progression validates that the HLT not only provided a structured pathway but also effectively addressed the cognitive obstacle students faced when engaging with geometric transformations. These findings align with prior research emphasizing the importance of structured learning trajectories in overcoming epistemological obstacles (Kandaga et al., 2022a; Noto, Priatna, & Dahlan, 2018; Karso, 2016).

The HLT's design, particularly its focus on the abstraction level (Level 3), was critical in overcoming these obstacles. Without the HLT, as evidenced by the red group's performance, students struggled to progress beyond Level 3, with their scores stagnating or declining at higher levels. In contrast, subjects guided by the HLT successfully navigated through abstraction and demonstrated proficiency at subsequent levels. This aligns with van Hiele-Geldof & van Hiele's (1984) theory, which highlights the importance of structured cognitive transitions between levels. The integration of GeoGebra within the HLT provided dynamic visualizations of the problems, which helped students connect abstract concepts with their practical implications, as supported by Jones (2005) and Clements & Battista (1992).

The visualization process, as highlighted in the HLT, played a pivotal role in addressing the focus on memorization and procedural calculations. GeoGebra served as a scaffold that allowed students to explore geometric reflections dynamically, fostering an intuitive understanding of the transformations. This approach is supported by previous findings, which argue that visualization is a fundamental component in constructing mathematical concepts (Jones, 2001; 2005; Mendoza, Nieto-Sánchez, & Vergel-Ortega, 2019). Moreover, students' varied use of GeoGebra—ranging from replicating the outputs, creating manual sketches, to using GeoGebra-generated answers—demonstrated that the tool was highly effective in addressing obstacles at both the visualization and abstraction levels (Noto, Priatna, & Dahlan, 2019; Kurniawati & Mahmudi, 2019).

The HLT also emphasized reasoning, deduction, and contextual analysis, which helped students shift from procedural problem-solving to a deeper understanding of the problem context. This was evident in the subjects' ability to analyze patterns, deduce mathematical relationships, and relate their solutions to the real-world context. These findings resonate with research from Juandi et al. (2021), Chang & Bhagat (2015), and Molnár & Lukáč (2015), which highlight the importance of reasoning and critical thinking in overcoming cognitive obstacles. By integrating both visual and analytical components, the HLT helped students construct solutions that were not only mathematically accurate but also meaningful in the context of the original problem.

To summarize, the findings underscore that the HLT successfully overcame the epistemological obstacle by shifting students' cognitive focus from rote memorization to a meaningful understanding of reflections in geometric transformations. This achievement supports prior research (Pech, 2012; Noto & Priatna, 2018) and highlights the value of integrating visualization tools and carefully sequenced tasks into learning trajectories. The success of this HLT further reinforces its potential as a model for addressing similar epistemological challenges in other mathematical topics.

Side effects of using GeoGebra at the abstraction level

Despite the various advantages of using GeoGebra in learning, it also introduced new learning obstacles, particularly at the abstraction level of geometric transformations. In certain problems, participants exhibited a GeoGebra-centric behavior, where they focused excessively on solving problems using GeoGebra without critically analyzing the mathematical reasoning behind their solutions. In certain problems, participants appeared to focus excessively on solving problems using GeoGebra. This GeoGebra-centric behavior constituted an improper way of thinking. Such a condition could be considered a learning obstacle stemming from didactics. Like other improper ways of thinking, GeoGebra-centric was not entirely wrong, but it could be disadvantageous for subsequent thought processes.

In addition to the reflection problems that were directed to be solved analytically, the learning obstacle test also included similar problems but allowed students freedom in their solution process. Initially, this was intended to measure the attachment of the analytical procedures they had understood with the visualization and concept of reflection. Figure 8 shows the problem given in the test. The test instrument is also available on Appendix 1.

2. Misalkan garis r adalah cermin dengan persamaan $r: 2x + 7y = 0$. Suatu garis $g: x + y = 5$ kemudian dicerminkan terhadap garis r, sedemikian sehingga didapatkan $M_r(g)$.
   a) Lukislah garis r (cermin), g (objek yang dicerminkan), dan $M_r(g)$ (hasil pencerminan)
   b) Uraikanlah setiap langkah yang digunakan untuk menentukan pencerminan tersebut.

Figure 8. Problem given to measure the attachment of analytical procedure

Translation of Figure 8:
Suppose the line r is a mirror with the equation $r: 2x + 7y = 0$. A line $g: x + y = 5$ is then reflected over the line r, resulting in $M_r(g)$.

a) Draw the line r (mirror), g (object being reflected), and $M_r(g)$ (reflection result)
b) Describe each step you used to determine the reflection.

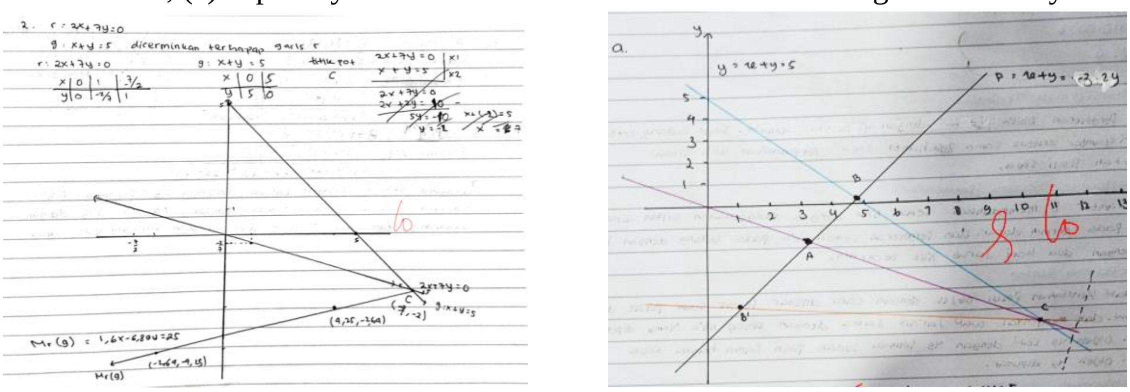
GeoGebra-centric behavior was particularly evident in the task where students were asked to reflect the line $g: x + y = 5$ over the line $r: 2x + 7y = 0$. Instead of using mathematical reasoning to describe each step in determining the reflection, many participants relied entirely on GeoGebra's output to generate the result, bypassing the need to engage with the underlying mathematical concepts. This GeoGebra-centric approach reflects what Brousseau (2002) describes as a didactical obstacle, which arises from how the tool was integrated into the learning process.

Such behavior does not necessarily indicate that the students’ way of thinking was entirely wrong. GeoGebra-centric tendencies stem from the efficiency and accessibility of the software, which aligns with its purpose as a tool for visualization and exploration (Shadaan & Leong, 2013; Juandi et al., 2021). However, this reliance can hinder deeper conceptual understanding and abstract reasoning, which are essential at higher levels of mathematical thinking, such as those described in van Hiele-Geldof's levels of geometric thought (van Hiele-Geldof & van Hiele, 1984). For example, in the task above, while students were able to draw the reflected line $M_r(g)$ accurately using GeoGebra, many failed to articulate the steps they followed mathematically, such as finding the perpendicular from a point to the line or solving for the intersection points. This over-reliance on GeoGebra resulted in incomplete explanations and a lack of procedural fluency, which are critical for constructing mathematical arguments and proofs.

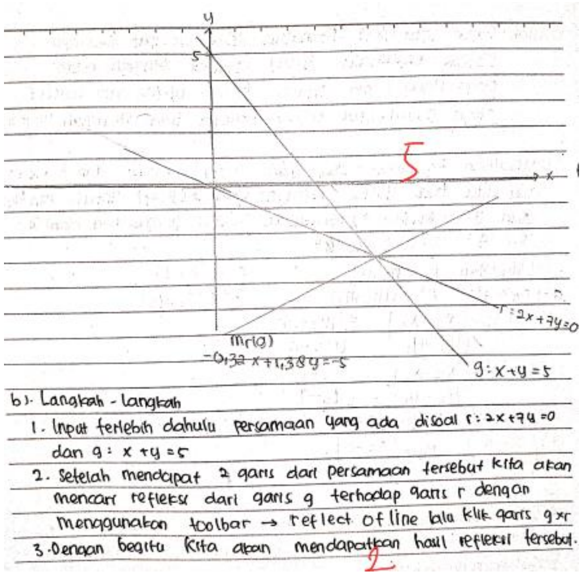
The analysis of the students' answers revealed a strong dependence on GeoGebra. As shown in Figure 9, students sketched the reflection result without including several key elements required for manually drawing lines, suggesting that the answers were likely created using GeoGebra. The absence of some reflection elements in the students' sketches also indicated that the sketches were made with the aid of GeoGebra. Instead of providing manual calculations or specific reflection procedures, some students included the GeoGebra procedures in their sketching process.

One of the main negative impacts of excessive use of GeoGebra is students' dependence on the visualization tool. This dependence can hinder their ability to deeply understand geometric concepts without technological assistance. Students may become less skilled in performing manual analysis and calculations, which are crucial for understanding the fundamentals of mathematics (Kandaga, 2022b).

Upon examining the responses of some subjects who did not fully meet the fourth abstraction indicator in Figure 9, two of the subjects (a) and (c) provided correct reflection results but did not include supporting elements of the manual reflection procedure, such as constructing the normal line, its intersection point, or testing points to create the straight line. Meanwhile, (c) explicitly stated that the results were obtained using GeoGebra syntax.



(a) (b)



(c)

Translation:
b). Steps:
1. Input the equation line $r:\ 2x + 7y = 0$ and $g: x + y = 5$
2. After obtaining the two lines from the equations, we will find the reflection of line g over line r by using the toolbar -> reflect of line, then click on lines g and r.
3. In this way, we will obtain the result of the reflection

Figure 9. Student responses indicating GeoGebra-centric characteristics

The excessive use of GeoGebra could lead students to rely on instant visualization rather than developing abstract and deductive thinking skills. The ability to think abstractly and perform deductions is a key skill in mathematics, necessary for solving more complex problems. When students become accustomed to the visualizations provided by GeoGebra, they may become less trained in developing logical and coherent mathematical arguments (Laborde et al., 2002). As shown in Figure 9 (b), where the student attempted to sketch manually but then failed due to several errors in determining the intersection points between the mirror line and the normal line.

Research on the integration of technology in mathematics education highlights similar risks. According to Garcia (2023), while digital tools like GeoGebra enhance visualization and problem-solving, they can also create new obstacles, such as a lack of balance between procedural and conceptual understanding. Similarly, Clements & Battista (1992) emphasized that while technology can scaffold learning, it may inadvertently encourage students to prioritize results over the reasoning process. In this case, GeoGebra’s ability to provide immediate and accurate visualizations of reflections may have led students to focus more on the output rather than engaging with the reflection procedure itself. As a result, the abstraction level—where students should be developing formal reasoning and constructing arguments—was not fully achieved by some participants.

The findings regarding the side effects of GeoGebra in learning geometric transformations contribute to how we should design integrated learning with digital technology. Although GeoGebra is a highly useful tool in learning geometry, it is important for educators to ensure that its use is not excessive. Educators must balance the use of technology with the development of students' analytical and deductive skills. In this way, students can develop a more holistic and in-depth understanding of geometric concepts and the ability to apply them in various contexts.

Conclusion

The implementation of HLT for the geometric thinking model at the abstraction level in teaching geometric transformations demonstrated significant positive impacts, successfully addressing the research question of overcoming epistemological obstacles. Based on the findings, seven out of ten participants met three out of the four indicators at the abstraction level. With the support of GeoGebra, the HLT facilitated students' understanding of geometric transformations and enabled all participants to complete the abstraction process and overcome obstacles. These results indicate that HLT effectively supported students' progression in geometric thinking and addressed challenges at the abstraction level. However, the study also revealed the emergence of GeoGebra-centric behavior, in which participants overly relied on GeoGebra to solve problems and understand transformation concepts. This dependence on GeoGebra can be problematic, as it may hinder the development of the deeper conceptual understanding and critical thinking skills necessary for mastering geometric transformations. The emergence of these side effects represents a didactic obstacle that arises from the teaching methods or tools used rather than from the content itself. When the necessary didactic prerequisites, such as balancing technology use with traditional problem-solving skills, are not met, students may develop over-reliance on digital tools. This overreliance can limit their ability to engage with and understand mathematical concepts independently. Educators must ensure that the use of such tools enhances rather than detracts from the development of essential mathematical reasoning and problem-solving skills.

In terms of limitations, the small sample size may restrict the generalizability of the findings, and the focus on the abstraction level means the results are specific to this stage of geometric thinking. Future research should refine the HLT to explicitly address GeoGebra-centric behaviour and explore its implementation across larger and more diverse participant groups to improve its scalability and applicability.

Acknowledgment

The authors would like to express their deepest gratitude to Universitas Terbuka for providing the facilities and support necessary to conduct this research. We also extend our sincere thanks to Universitas Pasundan for their collaboration as a research partner and for granting us the opportunity to carry out this study. Our heartfelt appreciation goes to the validators who generously contributed their time and expertise to ensure the quality of the research instruments. Lastly, we are profoundly grateful to the research participants, whose willingness and cooperation as study subjects were invaluable to the success of this research.

Conflicts of Interest

The authors declare no conflict of interest regarding the publication of this manuscript. In addition, the ethical issues, including plagiarism, misconduct, data fabrication and/or falsification, double publication and/or submission, and redundancies, have been completed by the authors.

Funding Statement

This work received no specific grant from any public, commercial, or not-for-profit funding agency.

Author Contributions

Thesa Kandaga: Conceptualization, writing original draft, editing, submission; **Idha Novianti:** editing, tabulations, and visualizations; **Mazlini Adnan:** Translation, proofreading, templating.

References

Bakker, A., & van Eerde, H. A. A. (2015). An introduction to design-based research with an example from statistics education. In L. D. English & D. Kirshner (Eds.), *Handbook of International Research in Mathematics Education* (3rd ed., pp.373-396). Routledge. https://doi.org/10.1007/978-94-017-9181-6_16

Brousseau, G. (2002). *Theory of didactical situations in mathematics*. Springer Dordrecht: Kluwer Academic Publishers. https://doi.org/10.1007/0-306-47211-2

Burger, W. F., & Shaughnessy, J. M. (1986). Characterizing the van Hiele levels of development in geometry. *Journal for Research in Mathematics Education*, *17*(1), 31–48. https://doi.org/10.2307/749317

Chang, C-Y & Bhagat, K. (2015). Incorporating GeoGebra into geometry learning - a lesson from India. *Eurasia Journal of Mathematics, Science and Technology Education*, *11*(1), 77-86. https://doi.org/10.12973/eurasia.2015.1307a

Clements, D. H., & Battista, M. T. (1992). Geometry and spatial reasoning. In D. A. Grouws (Eds.), *Handbook of Research on Mathematics Teaching and Learning* (pp. 420-464). Macmillan Publishing Co, Inc.

Clements, D. & Sarama, J. (2004). Learning trajectories in mathematics education. *Mathematical Thinking and Learning*, *6*(2), 81-89. https://doi.org/10.1207/s15327833mtl0602_1

Confrey, J., Maloney, A. P., & Corley, A. (2014). Learning trajectories: A framework for connecting standards with curriculum. *ZDM Mathematics Education*, *46*(5), 719–733. https://doi.org/10.1007/s11858-014-0598-7

CPRE Research Report (2011). Learning trajectories in mathematics: A foundation for standards, curriculum, assessment, and instruction. *https://files.eric.ed.gov/fulltext/ED519792.pdf*

Garcia, A. (2023). *Technology might be making education worse*. Stanford Report. https://news.stanford.edu/stories/2023/04/technology-might-be-making-education-worse

Gravemeijer, K., & Van Eerde, D. (2009). Design research as a means for building a knowledge base for teachers and teaching in mathematics education. *The Elementary School Journal*, *109*(5), 510–524. https://doi.org/10.1086/596999

Harel, G. (2008). What is mathematics? A Pedagogical answer to a philosophical question. In B. Gold & R. Simons (Eds.), *Proof and Other Dilemmas: Mathematics and Philosophy (Mathematical Association of America* (pp. 265-290)

Jones, K. (2005). Implications for the classroom: Research on the use of dynamic software. In J. Edwards & D. Wright (Eds.), *Integrating ICT into The Mathematics Classroom.* Association of Teachers of Mathematics (pp. 27-29)

Jones, K. (2001), Learning geometrical concepts using dynamic geometry software. In K. Irwin (Eds.), *Mathematics Education Research: A catalyst for change*, (pp. 50-58). University of Auckland.

Juandi, D, Kusumah, Y. S., Tamur, M., Perbowo, K. S., Wijaya, T. T. (2021). A Meta-analysis of GeoGebra software decade of assisted mathematics learning: What to learn and where to go?. *Heliyon*, *7*(5). https://doi.org/10.1016/j.heliyon.2021.e06953

Kandaga, T., Rosjanuardi, R., & Juandi, D. (2022a). Epistemological obstacle in transformation geometry based on van Hiele's Level. *Eurasia Journal of Mathematics, Science and Technology Education*, *18*(4), 1-12. https://doi.org/10.29333/ejmste/11914

Kandaga, T. (2022b). *Analisis proses berpikir geometri mahasiswa calon guru matematika dalam perspektif way of thinking dan way of understanding pada implementasi desain didaktis berbantuan software GeoGebra. [Analysis of the geometric thinking process of prospective mathematics teachers from the perspective of way of thinking and way of understanding in the implementation of a didactic design assisted by GeoGebra Software]*. [Unpublished Doctoral Dissertation]. Universitas Pendidikan Indonesia.

Laborde, C. (2002). Integration of technology in the design of geometry tasks with Cabri-Geometry. *International Journal of Computers for Mathematical Learning*, *6*, 283-317. https://doi.org/10.1023/A:1013309728825

Martinez, M. V., Castro-Superfine, A., & Stoelinga, T. (2022). A Curriculum-based approach to learning trajectories in middle school algebra. *Journal of Research in Mathematics Education*, *11*(1), 5-32, https://doi.org/10.17583/redimat.5539

Mendoza, D., Nieto-Sánchez, Z.C. & Vergel-Ortega, M. (2019). Technology and mathematics as a cognitive component technology and mathematics as a cognitive component. *IOP Conference Series Earth and Environmental Science*, *1414*(1), 012007. https://doi.com/10.1088/1742-6596/1414/1/012007

Miles, M.B., Huberman, A.M, & Saldana, J. (2014). *Qualitative data analysis, a methods sourcebook* (3rd ed.). Sage Publications.

Molnár, P., & Lukac, S. (2015). Dynamic geometry systems in mathematics education: attitudes of teachers. *ICTE Journal*, *4*(19), 19-33. https://doi.org/10.1515/ijicte-2015-0017

Noto, M. S., Priatna, N., & Dahlan, J. A. (2019). The learning obstacles of pre-service mathematics teachers on transformation geometry. *Journal on Mathematics Education*, *10*(1), 117–126. https://doi.org/10.22342/jme.10.1.5379.117-126

Noto, M. S., Priatna, N., & Dahlan, J. A. (2018). Analysis of learning obstacles on transformation geometry. *International Conference on Mathematics and Science Education (ICMScE 2018), Journal of Physics: Conf. Series 1157, 042100*.

Owusu, R., Bonyah, E., & Arthur, Y. D. (2023). The effect of GeoGebra on university students' understanding of polar coordinates. *Cogent Education*, *10*(1), 2905-2927. https://doi.org/10.1080/2331186X.2023.2177050

Pech, P. (2012). How integration of DGS and CAS helps to solve problems in geometry. *17th Asian Technology Conference in Mathematics*, Bangkok, Thailand, pp. 16–20

Shadaan, P. & Leong, K. (2013). Effectiveness of using GeoGebra on students' understanding in learning circles. *The Malaysian Online Journal of Educational Technology*, *1*. 1-11.

Simon, M. A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education, 26*(2), 114–145.

Usiskin, Z. & Senk, S. (1982). *Van Hiele levels and achievement in secondary school geometry*. The University of Chicago.

van Hiele-Geldof, D. & Van Hiele, P. M. (1984). *English translation of selected writings of Dina van Hiele-geldof and Pierre M. van Hiele*. In D. Fuys, D. Geddes, & R. Tischler, (Trans.), Monograph. Brooklyn College.

van Hiele-Geldof, D. (1957). *De didaktiek van de meetkunde in de eersteklas vanhet V.H.M.O. [Didaktik geometri di sekolah dasar V.H.M.O.]*. [Unpublished Doctoral Dissertation], University of Utrecht.

Wilson, P., Lee, H. & Hollebrands, K. (2011). Understanding prospective mathematics teachers' processes for making sense of students' work with technology. *Journal for Research in Mathematics Education*, *42*, 39-64. https://doi.org/10.5951/jresematheduc.42.1. 0039