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Learning obstacle of proportion learning based on proposional reasoning level: A case study pre-service mathematics teachers

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Abstract

The proportion has emerged as an important mathematical topic in high school because of its foundation in other mathematics topics. This study aims to identify the learning barriers of pre-service mathematics teachers (PMT) based on the material proportion and level of reasoning proportion. This study implemented Didactical Design Research (DDR) and used qualitative research methodologies within an interpretive paradigm. The research participants were 25 PMTs from a seventh-semester candidate teacher at a private college in Cianjur, Indonesia. The data collected from the test results and interviews were analyzed using procedures such as identification, clarification, reduction, and verification. We then present the findings in a narrative format. Langrall and Swafford categorized the results of written tests at the proportional-reasoning level. Based on the results, the PMT at the varied proportional level but at the nonproportional level 0 identified learning obstacles, which include ontogenetic obstacles, epistimology obstacles, and didactic obstacles. The results of this study are expected to be used as a basis for designing hypothetical learning for school mathematical research in future PMT.

Keywords: learning obtacles; pre-service mathematics teacher; proportion; proportion reasoning level

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Introduction

Proportion, a mathematical concept, has become part of the mathematics curriculum in secondary schools. Proportion is an important foundational topic of school mathematics for students, as it will be used in discussing more advanced mathematical topics (Bintara & Suhendra, 2021; Lamon, 2007, 2020a). Proportion is known as a way of saying that two ratios are equivalent. The Oxford Concise Dictionary of Mathematics (Clements & Sarama, 2014) explains that there are two types of proportion, namely direct proportion and indirect proportion. A straight proportion relationship is represented by the equation $y = k \cdot x$, where y and x are quantities in a proportional relationship, and k is the constant of proportionality. In contrast, an inversely proportional relationship is represented by the equation $y \cdot x = k$, where the product of corresponding values is a constant number. (Arican, 2019a; Arican, 2020).

The proportional reasoning includes fraction equivalence, division, place value, percentage calculation, and measurement conversion (Scheibling-Sève et al., 2022). So to understand the concept of proportion, one must use proportional reasoning (Burgos et al., 2022). Proportional reasoning requires some understanding of scale relationships and appears in everyday problems (Boyer & Levine, 2012). The capacity of students to engage in proportional reasoning significantly impacts their comprehension of fractions, ratios, and proportions during their primary school years. Furthermore, it serves as a foundation for their understanding of equations, growth and relative size, dilation, scaling, constant rate of change, slope, speed, rate, percent, trigonometric ratios, probability, relative frequency, density, and straight lines (Abramovich & Connell, 2021; Burgos et al., 2022; Scheibling-Sève et al., 2022).

Proportional reasoning belongs to the component of mathematical ability, which is logical thinking, and is an important thing to be mastered by both students and teachers (Pişkin Tunç & Çakıroğlu, 2022). However, learning proportion is not easy. There are several learning obstacles that can arise in the learning process. Brousseau (2002) identifies three types of learning obstacles: ontogenic, didactical, and epistemological. Ontogenic obstacles are psychological, instrumental, and conceptual. Didactical obstacles arise from a teacher's method, like inefficient material skipping. Epistemological obstacles arise from students' limited knowledge in a specific context, such as lack of conceptual understanding in prerequisite materials or previous learning experiences.

Many studies have documented and identified barriers to learning about proportion, especially in Indonesia. For example, students experience learning obstacles on the concept of proportion ontogenically, didactically, and epistemologically (Bintara & Suhendra, 2021; Wahyuningrum et al., 2019), epistemological obstacles in concept understanding, use of problem solving procedures, and operational techniques for solving proportion problems (Biori et al., 2022), and obstacles in understanding and contextual application (Andini & Jupri, 2017). Some barriers students have difficulty determining strategies in proportion relationships (Özen Yılmaz, 2019; Riehl & Steinthorsdottir, 2019; Wahyuningrum et al., 2022). The problems presented in proportion are missing value problems and ratio comparison problems (Carraher et al., 2018). In the presentation of these problems, students cannot distinguish indirect proportion and invese proportion (Karli & Yildiz, 2022), and cannot distinguish additive and multiplication relationship situations (Bintara & Suhendra, 2021; Karli & Yildiz, 2022).

Several strategies for solving proportions are used, such as the unit rate strategy, factor of change strategy, fraction strategy, building-up strategy, and cross-product strategy (Cramer et al., 1993; Lamon, 2020b). Some students use cross-multiplication strategies without understanding the purpose of the solution procedure, relying solely on formulas and procedures without understanding the purpose (Arican, 2024). They only memorize map formulas and speed with distance per time formulas, only understanding problems exemplified by the teacher (Arican, 2024).

Proportional reasoning describes different types of reasoning that focus on the relationship between two ratios and require complex ideas. When a teacher presents different proportional quantities, they usually also use standard phrases, such as "When one quantity increases, the other also increases, and if one decreases, the other also decreases"; or in the case of inverse value comparisons, "When one increases, the other decreases" (Cabero-Fayos et al., 2020).

This failure provides evidence that students and teachers do not understand the term equivalent ratio in proportion. Higher education as a forum for producing pre-service mathematics teachers (PMT) is important for identifying learning obstacles for future PMT in school mathematics, especially proportion. In general, there has been a lot of research related to the learning obstacle proportion among PMT. Among them are the findings of Arican, (2018) and Osana & Royea (2011), who found that PMT experienced obstacles in providing representations of ratios and gave inaccurate arguments in explaining the solutions they made. In addition, PMT have difficulty distinguishing proportional relationships from non-proportional relationships even after being given instructions about these relationships (Arican, 2019b; Valverde & Castro, 2012). One of the causes of the lack of proportional reasoning abilities of pre-service mathematics teacher (PMT) is that, when they were at school, they were used to focusing and memorising the steps to get the solution to a problem (Valverde & Castro, 2012).

This condition does not only occur in Indonesia but also in other countries. PMT in the United States, for example, have problems when applying the proportion strategy to constant difference problems because they use an inappropriate strategy (Lim, 2020). While Irfan et al, (2019) say that future teachers have trouble picking the right knowledge to solve proportion problems, which causes them to use the wrong strategies. Other research on learning obstacles relates to how using representations when teaching proportional reasoning can help PMT tell the difference between proportional and non-proportional, understand the part-by-part relationships in ratios, and support their multiplicative reasoning (Arican, 2015; Arican & Özçakır, 2020; Johnson, 2017).

This research examines what makes it difficult for students to learn more about the proportions of their thinking processes and learning experiences in the case of PMT. Barriers to student learning can be identified by using questions that measure professional reasoning. It is possible to find out what students find hard about learning proportional ideas by giving them questions that test their reasoning on Langrall and Swafford's (2020) four levels of reasoning:

informal reasoning, formal proportional reasoning, non-proportional reasoning, and quantitative reasoning. In this case, proportional reasoning was chosen for reasons of urgency, as previously explained. The novelty of this research lies in identifying learning obstacles among PMTs in proportion learning, specifically rooted in different levels of proportional reasoning. A key research gap is found in the interplay between the use of representations and strategy selection: Arican highlights the role of representations in fostering reasoning development, while Johnson emphasizes strategic misapplications in particular problem contexts. This creates an opportunity to investigate how certain representations may influence PMTs' strategy choices, especially when differentiating between proportional and non-proportional problems. Therefore, this study seeks to identify specific learning challenges faced by PMTs in proportion learning based on their levels of proportional reasoning.

Methods

This study was a qualitative research effort utilizing the Didactical Design Research (DDR) method, which provided intellectual tools for designing and analyzing learning phenomena. DDR was based on two important paradigms (Suryadi, 2010), interpretive and critical, examining the reality of phenomena in relation to the impact of didactical design on ways of thinking. A key focus in DDR was that the interpretive paradigm served as the starting point, in this case, through the implementation of learning obstacle identification. The formation of meaningful knowledge and perspectives, whether individual or collective, was central to interpretative paradigms (Creswell, 2017; Suryadi, 2019). In this study, this knowledge resulted from a didactic situation. The interpretative paradigm was used to identify learning obstacles on the topic of proportion, marking the initial step in didactic design research.

The subjects in this study were 25 prospective mathematics teachers (PMT) from a private university in Cianjur, Indonesia, selected based on two criteria: they possessed heterogeneous abilities within one class, and they had completed school mathematics courses on the material of proportion. PMT were selected for interviews based on indications of learning obstacles. This study used a qualitative approach grounded in constructivist theory, employing two primary and two supporting instruments to gain an in-depth understanding of the research context. The researcher acted as the main instrument, emphasizing the importance of interaction between researcher and participants to gather rich and meaningful data. Two supporting instruments were utilized: test instruments and non-test instruments. The test instruments were validated by five experts through a Focus Group Discussion (FGD) to ensure validity and enhance data quality. Meanwhile, the non-test instruments supported the accuracy and depth of qualitative research results (Kalu, 2017).. Method triangulation was applied to increase the validity and credibility of findings, using various data collection techniques, including observation, tests, interviews, and documentation studies.

The data obtained from the collection process were then processed and analyzed (Figure 1) through several stages. Data analysis in this study was based on hermeneutic phenomenology, emphasizing subjective understanding and meaning within the social context, aligned with the interpretive paradigm framework. The analysis began with the data identification stage, which included checking scanned test results, followed by the clarification stage, in which the Atlas.ti application was used to assist in data source coding based on levels of proportional reasoning (Langrall & Swafford, 2000, 2020). Next, in the data verification stage, learning obstacles were further confirmed through answer analysis, teaching material document checks, and interviews with selected participants to explore the obstacles they experienced. Data verification was also conducted through data source triangulation and member checking to ensure the validity of findings. The results were ultimately presented in a narrative format, providing a comprehensive view of the identified learning obstacles (Creswell, 2017).

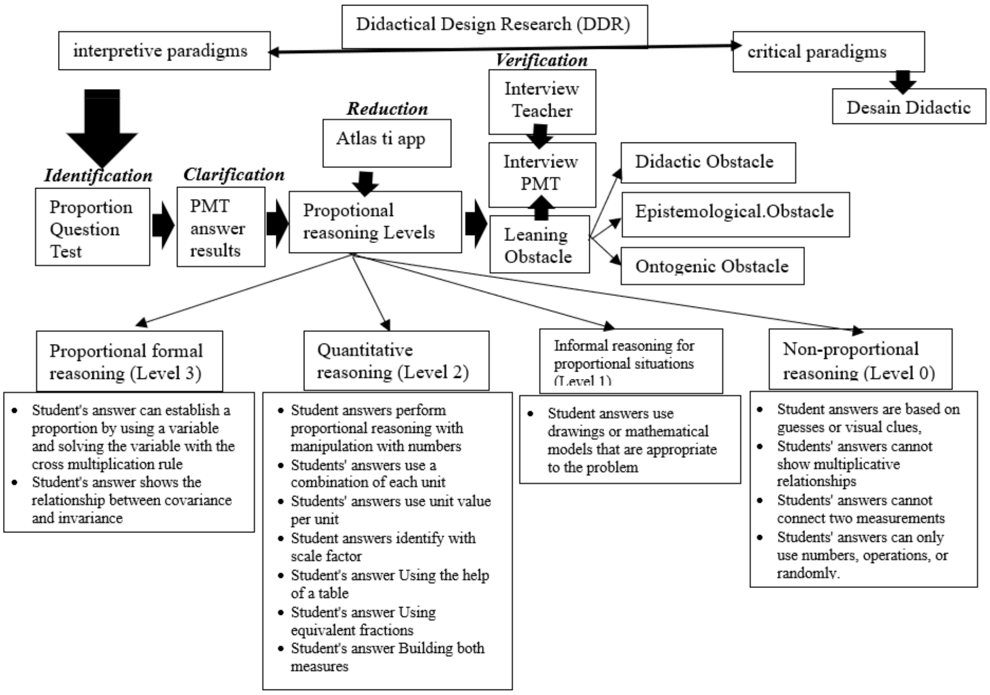


Figure 1. Data analysis framework

Results

The results of PMT answers were then categorized based on the level of proportional reasoning developed by categorizing PMT answers based on the proportion level based on Langrall and Swafford (Langrall & Swafford, 2000, 2020) precisely Table 2, namely non-propositional reasoning, informal reasoning for proportional situations, quantitative reasoning, and formal proportional reasoning.

Analysis of the level of proportional reasoning of pre-service mathematics teacher

The results in Table 1 show the proportional reasoning level of PMT. Each piece of data is explained and given a unique code. For example, the label code M8-5 shows the results of the 5th subject's answer, which is from M8-1 to M8-25.

Table 1. Results level of proportional reasoning of PMT

Task Proportional Reasoning	Proportional reasoning Levels	Result of Answer
Determining an Equivalent Ratio	Level 0	M8-5, M8-6, M8-9, M8-11, M8-13, M8-16, M8-17, M8-18 (8 subject)
	Level 1	M8-12, M8-14 (2 subject)
	Level 2	M8-10, M8-21 (2 subject)
	Level 3	M8-1, M8-2, M8-3, M8-4, M8-7, M8-8, M8-15, M8-19, M8-20, M8-22, M8-23, M8-24, M8-25 (13 subjek)
Solving problems in everyday life related to comparative values	Level 0	M8-23, M8-24, M8-12, M8-11 (4 subject)
	Level 2	M8-3, M8-6, M8-10 M8-14, M8-15, M8-18, M8-21 (7 subject)
	Level 3	M8-1, M8-2, M8-4, M8-5, M8-7, M8-8, M8-9, M8-13, M8-16, M8-17, M8-19, M8-20, M8-22, M8-25 (14 subject)
Solving a scale problem	Level 0	M8-1, M8-2, M8-3, M8-4, M8-5, M8-6, M8-7, M8-8, M8-9, M8-10, M8-11, M8-12, M8-13, M8-14, M8-15, M8-16, M8-17, M8-18, M8-19, M8-20, M8-23, M8-24 (22 subject)
	Level 3	M8-21, M8-22, M8-25 (3 subject)
Solving problems in everyday life associated with inversed comparison values using a variety of strategies	Level 0	M8-11, M8-12, M8-23 (3 subject)
	Level 2	M8-3, M8-6, M8-14, M8-15 (4 subject)
	Level 3	M8-1, M8-2, M8-4, M8-5, M8-7, M8-8, M8-9, M8-10, M8-13, M8-16, M8-17, M8-18, M8-19, M8-20, M8-21, M8-22, M8-24, M8-25 (18 subject)
Solving Changing Speed (Speed and Debit) Problems Related to Contextual Problems	Level 0	M8-2, M8-3, M8-4, M8-5, M8-6, M8-7, M8-8, M8-9, M8-10, M8-11, M8-12, M8-13, M8-14, M8-17, M8-18, M8-19, M8-21, M8-23, M8-25 (19 subjek)
	Level 3	M8-1, M8-15, M8-16, M8-20, M8-22, M8-24 (6 subject)

Based on the results of the analysis of the level of proportional reasoning of PMT in Table 1, students at level 2 quantitative reasoning with the characteristics of the answers of PMT using unit values per unit, identifying with scale factors, table assistance, and fractions worth in the case of equivalent ratios and proportion problems. It was found that there were pre-service mathematics teacher students who used the building-up strategy at level 1 of informal reasoning towards proportional situations in solving proportional problems. Pre-service mathematics teacher students are already at level 3 proportional formal reasoning using cross multiplication in solving direct proportion and inverse proportion problems. This is in line with the findings of Arican et al. (2023) that PMT often use the first-time cross strategy as the first strategy in solving the problem of the comparison of equal and inverse values.

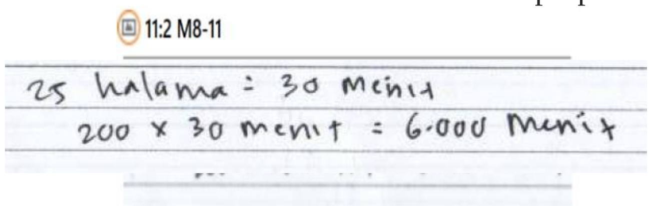
Most PMT students are at Level 0, which involves non-propotional reasoning using cross-multiplication strategies without paying attention to rules. They have a weak understanding of propotional reasoning at nonpropotional levels. In proportion problems, they rely on guessing or visual clues, cannot show multiplicative relationships, cannot connect two measurements, and can only use numbers, operations, or random numbers. These learning obstacles are identified through the analysis of their answers. We present the results of this analysis in relation to the learning obstacles PMT students face when solving proportion problems.

Learning obstacle in problem 1: Equivalent ratio

Problem No. 1 is an equivalent ratio problem that aims to measure the propotional reasoning ability of PMT in solving missing value problems. In this case, the speed context was chosen because it is a familiar context for PMT. However, it is still found that PMT at the lowest stage, namely non-proportional reasoning, experience learning obstacles (for example Figure 2).

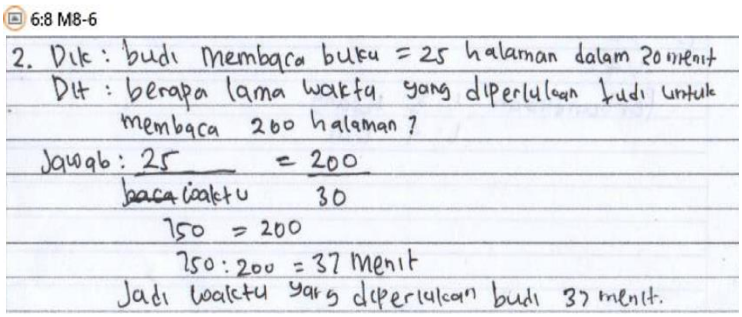
Problem 1: Budi reads 25 pages of a book in 30 minutes. If his reading speed remains as before, how long will it take him to read 200 pages?

Level 0: non proportion reasoning.



(a)

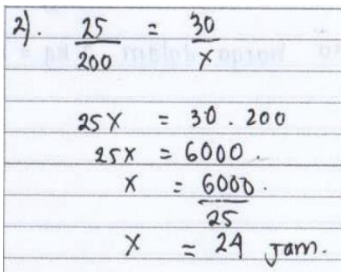
Translation:
25 pages = 30 minutes
200 x 30 minutes = 6,000 minutes



(b)

Translation:
Unknown: Budi reads a book= 25 pages in 20 minutes
Ask: How long does it take Budi to read 200 pages?
Answer: $\frac{\mathbf{25}}{Time} = \frac{\mathbf{200}}{30}$
$750 = \mathbf{200}$
$750 : 200 = 37\ minutes$
So the time required by Budi is 37 minutes.

(c)



Translation:
$\frac{\mathbf{25}}{\mathbf{200}} = \frac{\mathbf{30}}{\boldsymbol{x}}$
$\mathbf{25}\,\boldsymbol{x} = 30{,}200$
$25\,x = 6{,}000$
$x = \frac{6{,}000}{25}$
$\boldsymbol{x} = \mathbf{24}\ hours$

Figure 2. Identification of PTM answers based on proportional reasoning for question 1: Parts (d,e,f) show non-proportional reasoning.

The learning obstacle identified in the equivalent ratio problem is that the propotional reasoning level student miscalculated (for example, M8-17 in Figure 2a). The following is the transcript of the translation of the researcher (P) who interviewed the propotional reasoning level student (M8-17) on the answer:

P : Can you explain how to answer the equivalent ratio question?
M8-17 : Using the ratio of the number of pages to the time, cross-product to get 24 minutes.
P : Is the calculation correct?
M8-17 : Yes, ma'am, eh...iah, ma'am, less zero should be 240 minutes.

The results of PMT answers to the question: in what way to solve the equivalent ratio problem? M8–17 answered by comparing what is already known and then using cross-tabulation to find the answer. The M8–17 times study revealed errors in basic arithmetic calculations, with propotional reasoning level PMT incorrectly interpreting their answers based on their calculations, despite their attention to the results.This is in line with the findings of Burgos & Godino (2022) on the difficulty of PMT in understanding the prerequisites for the concept of propotional problem solving.

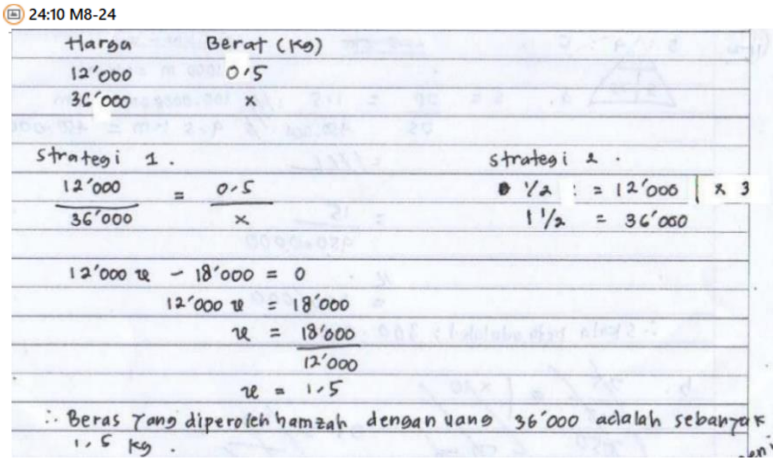
Based on observations, documentation of test results and student interviews, one of the causes of weakness in student unpreparedness related to technical matters of a learning process. At level 0, namely non proportional reasoning, obstacles were found related to the knowledge of PMT. These obstacles can be seen in the characteristics of the answer documents of PMT who still use the wrong strategy by randomly using addition or difference (For example M8-11 in Figure 2b), unfinished work (For example M8-9 in Figure 2c), and Not filled in (For example M8-18). This can be revealed through student responses and completion errors in the learning process, namely in the arithmetic calculation process. Brosseau (2002) and (Suryadi, 2019) stated that this condition shows that students experience instrumental ontogenic barriers. Identify student unpreparedness related to previous learning experiences, for example, a lack of understanding of concepts in the prerequisite material. Students could not connect the two measures (e.g., M8–6). This condition shows that students experience epistemological obstacles (Brosseau, 2002; Suryadi, 2019a).

Learning obstacle in problem 2: Direct proportion problems

Problem No. 2 (as presented in Figure 3) is a problem of solving proportion-type problems, namely direct proportion problems, in everyday life in the context of shopping. In solving this type of problem, level non-propotional identification of learning obstacles (Figure 3).

Problem 2: The price of 1/2 kilogram of rice is Rp. 6,000. If Hamzah paid Rp 36,000 for the rice, how many kilograms of rice did Hamzah get?

Level 0: non proportion reasoning.



(a)

Translation:
Price Weight
12,000 0,5
36,000 x

$$\frac{12{,}000}{36{,}000} = \frac{0.5}{x}$$

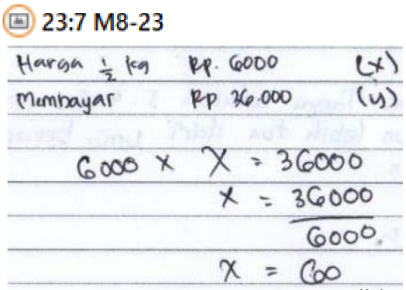
12,000x – 18,000 =0

12,000 x = 18,000

$$x=\frac{18{,}000}{12{,}000}$$

x=1.5

The rice obtained by Hamzah with the 36,000 is 1.5 kilograms.



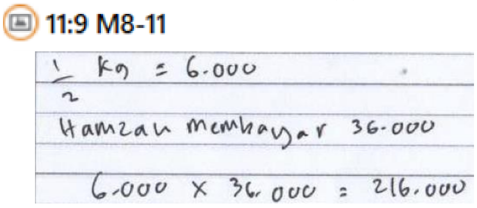
(b)

Translation:
Price ½ kilogram Rp. 6,000 (x)
Pay Rp. 36,000 (y)

6,000 x x = 36,000

$$x = \frac{36{,}000}{6{,}000}$$

x = 60



(c)

Translation:
½ kilogram = 6,000
Hamzah paid 36,000
6,000 x 36,000 = 216,000

Figure 3. Examples of PTM answers based on proportional reasoning for problem 2: Part (a,b,c) shows non-proportion reasoning.

Learning obstacles identified in the problem of value comparison problems in the context of the purchase price of rice, namely PMT miscalculated (for example, M8-17 in Figure 3a), based on the translation transcript of the researcher (P) interviewing PMT (M8-24):

P	:	Can you explain how to answer the question?
M8-24	:	With cross product and equation, eh... I wrote it wrong, mom; it should be 6,000, not 12,000. Half of it is 6000; we equate x to make it 36000, so we multiply 6 x 6000 to make it 36000, so the result is 30 kilograms.
P	:	Is the calculation result correct?
M8-24	:	Yes,ma'am. Uh, h my mistake should be3 kilogramss.

Based on the interview transcripts, students were identified as experiencing instrumental ontogeny obstacles, which were revealed through student responses during the interview, which found errors in the completion of the learning process, namely in the arithmetic calculation process. prospective teacher students at level 0, namely non-proportional reasoning, who experience learning obstacles (for example, Figures 3) on the problem of equal comparison.

The characteristics of conceptual ontogeny obstacles are determining the difference that is less precise, for example, M8-24 in Figure 3a, while M8-11 randomly uses operations with the wrong multiplier. The dependency of PMT is found in the results of research (Ölmez, 2016)using additive or multiplication relationships in determining proportion problems. Meanwhile, epistemological obstacles are unable to connect the two measures, for example, M8-23 and M8-12 in Figure 3b, 3c.

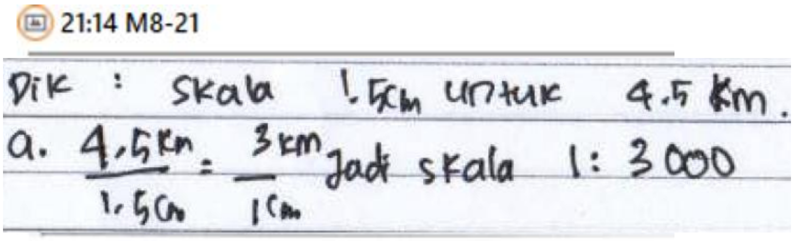
Learning obstacle on problem 3: Scale

Problem number 3 (as presented in Figure 4) is a problem-solving problem of the proportion type, namely inderct proportion about scale in the context of maps. The problem given to PMT is to determine the scale of the map, the actual distance, and the map distance identification learning obstacle pada level non-propotional reasoning (Figure 4).

Problem 3: On a map 1.5 centimeters represents 4.5 kilometers.

a. Determine the scale used on the map?
b. If on the map, the distance between place A and place B is 75 centimeters, how many kilometers is the actual distance between place A and place B?
c. If the actual distance from place A to place B is 18 kilometers, how many centimeters is the distance between the two places on the map?

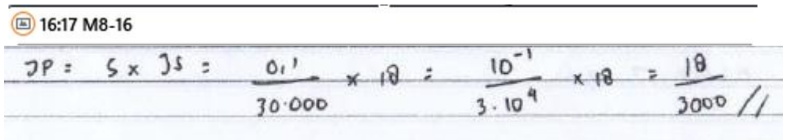
Level 0: non proportion reasoning.



Translation:
M8-21
Known: 1.5 centimeter scale for 4.5 kilometers

$\frac{4.5\ kilometer}{1.5\ centimeter} = \frac{3\ kilometer}{15\ centimeter}$

so scala 1:3,000

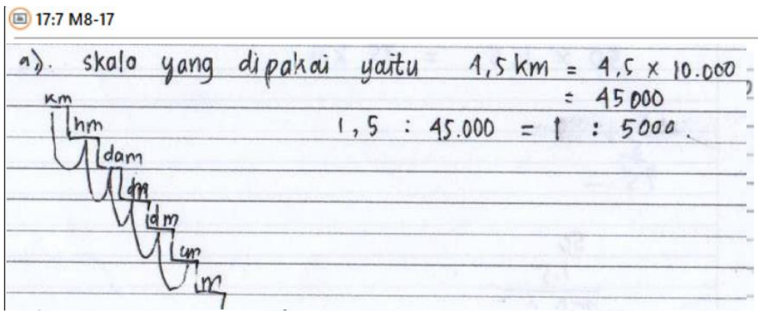


Translation:
M8-16

$JP = S \ X \ JS = \frac{0.1}{30{,}000} x\ 18$

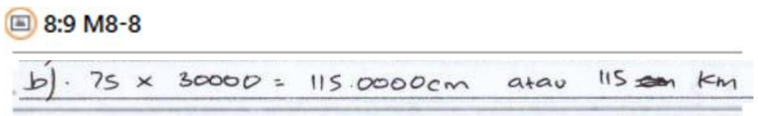
$= \frac{10^{-1}}{3 \,.\, 10^4} x\ 18 = \frac{18}{3{,}000}$

(a)



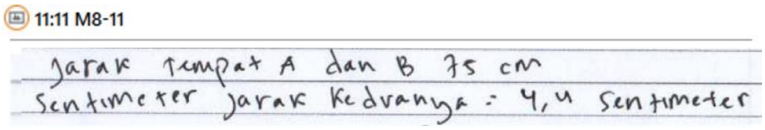
Translation:
The scale used is 4.5 = 4.5 x 10,000 = 45,000
1.5 : 45,000 = 1:5,000

(b)



Translation: M8-8
75 x 30,000 = 115,0000 cm or 115 km

Translation: M8-11
The distance between A and B is 75 centimeters



Centimeter distance between them = 4.4 centimeters

(c)

Figure 4. Examples of PTM answers based on proportional reasoning for problem 3: Part (a,b,c) shows non-proportion reasoning.

Learning obstacles that were identified at level 0 of non-proportional reasoning about value comparison problems in the context of scale were that prospective mathematics teacher students incorrectly determined the scale (for example, M8-21 and M8-16 in Figure 4a) and could not convert units from centimetres to kilometers. The following is the translation transcript of the researcher (P) interviewing the student (M8–17):

P : Look at the unit ladder from colometer to centimetre.. Down how many stairs?
M8-17 : 5 mom..eh mom, so 450,000, so 1.5 centimetres: 450,000 centimetres,
P : simplified to 1:50,000
M8-17 : How do I simplify it? Can it be 1:50,000?
P : Divided by 5, ma'am,
M8-17 : so that's it.Let's see if it's true that 1.5:450,000 divided by 5 results in 1:50,000.
P : Wrong mom, Hehe
M8-17 : What is the right one?
It's hard, mom, to divide by a comma.

Based on the results of student answer documents and interviews, for example, M8-19 at level 0 non-proportional reasoning has not been able to calculate the actual distance so that it is identified as experiencing instrumental ontogencial obstacles revealed through student responses and completion errors in the learning process, namely in the process of converting units, and has not been able to determine the relationship on a scale by multiplying without a clear basis so that conceptual ontogenical obstacles are identified (for example, M8-8 and M8-11 in Figure 4c).The identification results of other instrumental ontogenical obstacles are that students have correctly solved the problem but made mistakes in the final calculation (for example, M8-19 and M8-8 in Figure 4b).

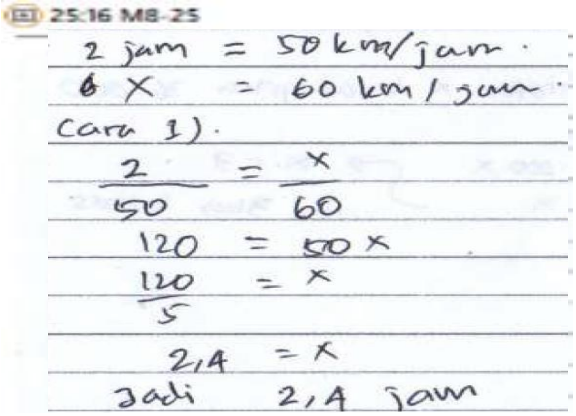
The identification results in conceptual ontogenical obstacles with the characteristics of determining the actual distance using the formula but being unable to solve (for example, M8-21, M8-16 in Figure 4a, and M8-17), and students write that the answer is not correct by multiplying the actual distance by the wrong scale (for example, M8-11 and M8-8 in Figure 4c).

Learning obstacles in problem 4: The inverse proportion problem rate of change problem in the context of speed

Problem number 4 (as presented in Figure 5) is a problem of solving proportion-type problems, namely inverse proportion in everyday life in the context of speed. We identify that in solving proportion-type problems, namely inverse proportion, most students have a formal level of proportion reasoning, namely, they are able to learning obstacles at a non-propotional level (Figure 5).

Problem 4: A car travels from city A to city B in 2 hours, with an average speed of 50 kilometers/hour. If the average speed of the car is 60 kilometers/hour, how much time does it take to cover the distance from city A to city B?

Level 0: non proportion reasoning.

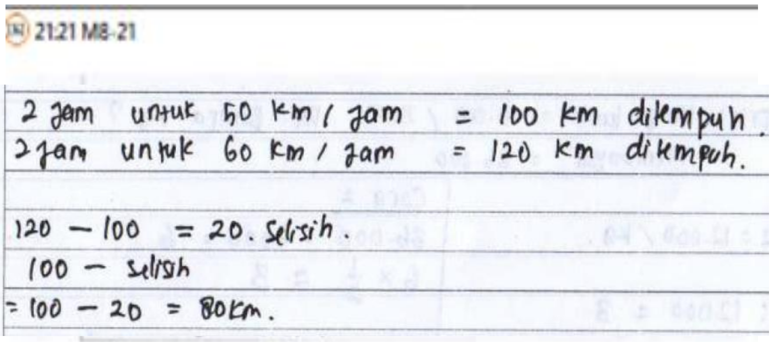


Translation:
2 = 50 kilometers/hour
6x = 60 kilometers/hour
Method 1

$$\frac{\mathbf{2}}{\mathbf{50}} = \frac{\boldsymbol{x}}{\mathbf{60}}$$
$$\mathbf{120}\,\boldsymbol{x} = \mathbf{500}\,\boldsymbol{x}$$
$$\frac{\mathbf{120}}{\mathbf{5}} = \boldsymbol{x}$$
$$\mathbf{2.4} = \boldsymbol{x}$$

So, 2.4 hours

(a)



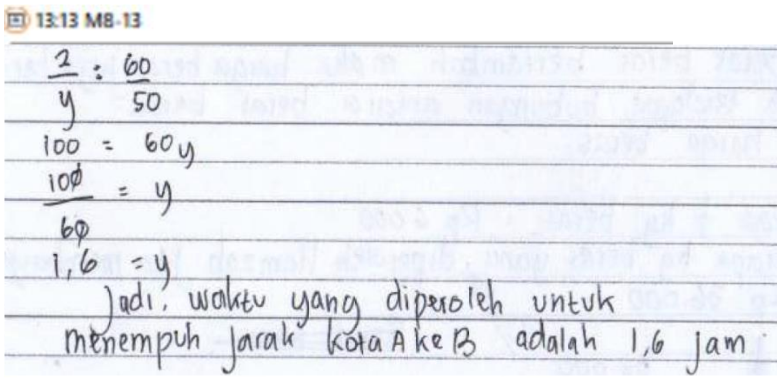
Translation:

Unknown: Budi reads a book= 25 pages in 20 minutes
Ask: How long does it take Budi to read 200 pages?
Answer: $\frac{25}{Time} = \frac{200}{30}$

$$750 = 200$$
$$750{:}\,200 = 37\ minutes$$

So, the time required by Budi is 37 minutes.

(b)



Translation:

$$\frac{2}{y} = \frac{60}{50}$$
$$100 = 60\,y$$
$$\frac{100}{60} = y$$
$$1.6 = y$$

So, the time required to cover the distance from city A to B is 1.6 hours

(c)

Figure 5. Examples of PTM answers based on proportional reasoning for problem 4: Part (a,b,c) shows non-proportion reasoning.

PMT at the non-proportional reasoning stage identified learning obstacles in solving, namely instrumental ontogeny obstacles, namely PMT has correctly solved the problem but is mistaken in the final calculation of miscalculation (for example, M8-13 in Figure 5c), the work has not been completed (for example, M8-21 in Figure 5b), and randomly using operations (for example, M8-10) and cannot connect the two measurements correctly (for example, M8-25 in Figure 5a) so that the second identification of conceptual ontogeny obstacles is identified. Based on the interview with PMT, she was unable to develop her concept due to a lack of understanding of the concept in the material. The following is the translation transcript of the researcher (P) interviewing PMT (M8-25), who is suspected of experiencing conceptual ontogenical obstacles:

P Can you explain how to answer the speed question?
M8-25 : Yes, ma'am, it means hours with hours 2/50; hemm 2 is from hours 50
: kilometres from the distance, then x/60 directly cross times to get 2.4 the x
P : value.
M8-25 : So what type of comparison is the solution?
Direct proportion, ma'am

The interview results clearly show ontogenical conceptual obstacles where PMT is still confused in finding the relationship between worth and inverse value by using the sialng multiplication strategy.

Learning obstacles in problem 5: Understanding the rate of change in inverse proportion problems

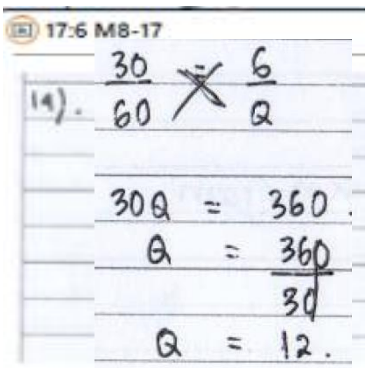
Problems in the contextof water Problem number 5 (as presented in Figure 6) is a problem ofsolving proportion-typee problems, namely inverse proportion in everyday life in the context of water discharge presented in the table. We identified that in solving proportion type problems they are able to learning obstacles at a non-propotional level (Figure 6).

Problem 5: Consider the following data on rice field water flow!

Discharge (x)	Time (y)
20	9
30	6
60	Q

The table shows the ratio between the discharge and the time taken to fill the rice field flow, state the equation that models the problem? What is the correct value of Q?

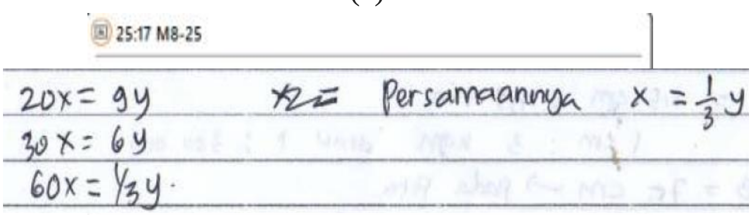
Level 0: non proportion reasoning.



Translation:

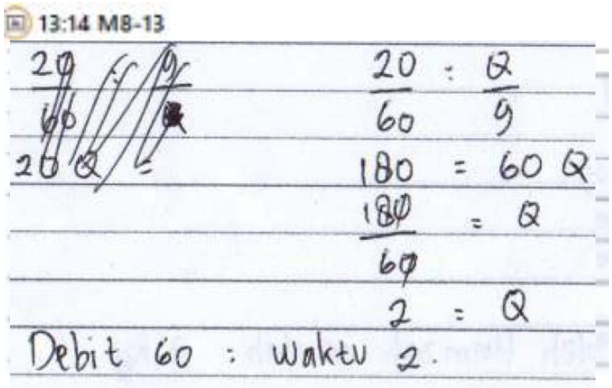
$$\frac{30}{60} = \frac{6}{Q}$$
$$30\,Q = 360$$
$$Q = \frac{360}{30}$$
$$Q = 12$$

(a)



Translation:
20x = 9y
30 x = 6y
60x = 1/3y
Equation x = $\frac{1}{3}y$

(b)



Translation:

$$\frac{20}{60} = \frac{q}{9}$$

$$180\,x = 60\,Q$$

$$\frac{180}{60} = Q$$

$$2 = Q$$

Discharge 60 : Time 2

(c)

Figure 6. Examples of PTM answers based on proportional reasoning for problem 3: Part (a,b,c) shows non-proportion reasoning.

There are 12 PMTs who have not been able to solve the rate of change (speed and discharge) problems related to contextual problems, thus identifying learning obstacles. PMTs at the first non-proportional reasoning stage experienced instrumental learning obstacles, namely not being able to calculate well (for example, M8-13 in Figure 6c), and most of them could not connect the two measures of inverse value comparison well, so they identified conceptual learning obstacles, which were revealed from the results of the interview with the researcher (P) who interviewed PMT (M8-17), namely.

P : Can you explain how to answer this question?
M8-17 : Yes, ma'am, just cross 30/60 = 6/Q, so the value of Q is 12.
P : Why does the discharge increase as the time increases to 60? Why is the answer
M8-17 : 12?
P : I don't know, ma'am.
M8-17 : When asked the equation of the discharge and time problem,?
Hemmm, y=...x=.....multiplied...don't know, mom

PMT can solve cross multiplication without knowing the comparison relationship is correct; this is also revealed from the results of interviews, which cannot explain the solution in the form of equations. In line with the findings of Joshua & Lee (2022), PMT had difficulty interpreting the answer results, and Arican (2019b) found that PMT had difficulty representing and interpreting proportional and non-proportional relationships and relied on the cross-multiplication strategy. PMT only focuses on procedural steps to reach the correct answer (Zulkarnain et al., 2020). In line with the findings of Cabero-Fayos et al (2020), PMT solves problems only through formulas without understanding the meaning of the formula.

Based on questions 1, 2, 3, 4, 5 identified ontogenic learning, conceptual ontogenical obstacles and instrumental ontogenical obstacles. In the presentation of proportion in the design of teaching materials, it was identified fromPMT notes and lecturers' teaching materials that there was no presentation of equivalent ratio material, so that didactic obstacles were identified as difficulties in developing their concepts due to a lack of understanding of concepts in prerequisite material or skipping material in https://bit.ly/teaching-material-before from the definition of comparison to proportion, where the ratio was only briefly defined without any explanation of the equivalent ratio. In fact, the ratio of ratio understanding is the main key needed in teaching materials as a provision for PMT to solve propotional problems derived from two equivalent ratios. In line with the findings of Ölmez (2022), students experienced difficulties in forming ratios, which resulted in not being able to form proportional relationships. In https://bit.ly/Notes-PMT, one of the notes of PMT shows the flow of material in line with the teaching materials prepared by lecturers, so that PMT have difficulty developing the concept of equivalent ratio in problem no. 1.

The challenges observed in the lecturer's teaching material and PMT notes, particularly in the context of formulas such as a1 / b1 = a2 / b2 and a1 / b2 = a2 / b1, point to a broader issue of conceptual development among students. The suspicion arises that students struggle due to a lack of understanding of the material, leading to a rigid application of solutions. This issue extends to the scale formula, where a 1: n ratio is involved, potentially hindering students from developing correct concepts. Moreover, there is a concern that insufficient understanding of prerequisite units further complicates the learning process.

In examining these challenges, it is noteworthy that the use of the inverse value comparison formula a1 / b2 = a2 / b1 is identified as a contributing factor. This formula is applied without students fully grasping the relationship of the comparison. These difficulties align with the findings of Cabero-Fayos et al.(2020) on PMTs, where obstacles are identified in distinguishing between direct proportion and inverse proportion using the cross-product strategy. This limitation results in weaknesses in solving proportion problems among PMTs. Building on these insights, Arican (2018) emphasizes the importance of determining directly and inversely proportional relationships by considering qualitative relationships and rates of change. The findings suggest that PMTs tend to rely on fixed strategies, such as the cross-product strategy, derived from prior experiences. However, diversifying methods in solving proportion problems is crucial for enhancing proportional reasoning and providing a deeper understanding in various proportional situations, as highlighted by (Buforn et al., 2018). The uniqueness of this finding lies in its professional competence to enhance the understanding of the level of proportional reasoning in PMTs and to identify the learning obstacles faced by PMTs, serving as a foundation for designing future proportional learning programs.

Discussion

This study explores the learning obstacles faced by prospective mathematics teachers (PMTs) in understanding proportional concepts based on their levels of proportional reasoning. According to didactic theory, learning obstacles can be categorized into ontogenic, epistemological, and didactical obstacles (Brousseau, 2002). Additionally, the proportional reasoning ability of PMTs can be understood at various levels. Lamon (2007) categorizes proportional reasoning into levels ranging from level 0 (non-proportional), where students cannot correctly relate two quantities, to higher levels where they are capable of understanding more complex proportional relationships. This study aims to identify the obstacles encountered by PMTs at different levels of proportional reasoning when dealing with proportion problems such as equivalent ratios, direct proportions, inverse proportions, and rates of change. The findings are consistent with and extend the existing literature on mathematical learning difficulties, providing nuanced insights into how PMTs' reasoning levels impact their ability to navigate proportional problems.

Instrumental ontogenic obstacles are often found in PMTs with proportional reasoning at level 0 (non-proportional). For example, when solving problems involving equivalent ratios, PMTs at this level tend to make basic arithmetic errors or use inappropriate strategies, such as random addition or multiplication without a clear basis. These obstacles indicate a lack of procedural understanding or incorrect application of strategies, which are instrumental skills, as seen in arithmetic errors and irrelevant use of operations. According to Burgos and Godino (2022), these errors likely stem from a lack of foundational knowledge and understanding of necessary concepts, indicating epistemological obstacles, where learners apply procedures without adequate conceptual grounding. This finding supports the hypothesis that students' foundational knowledge significantly impacts their problem-solving strategies, reinforcing the need for stronger foundational instruction. Additionally, Suryadi (2019) highlights that students at early stages of proportional reasoning often struggle to grasp fundamental proportional concepts, indicating the need for improvement through targeted curriculum design and pedagogy. To address this, the curriculum can be structured in a gradual manner, introducing ratio and proportion concepts in a simple way before progressing to more complex calculations. This staged learning module will strengthen students' conceptual understanding through visual aids, such as graphs and diagrams, as well as the use of relevant real-world examples, such as recipes or maps (Arican & Özçakır, 2020). Additionally, teachers should receive training on the stages of proportional reasoning development so they can adjust their teaching methods according to students' comprehension levels, including diagnostic assessments to identify and address specific learning obstacles (Burgos & Godino, 2022). Implementing problem-based learning (PBL) is also recommended to enhance students' ability to apply proportional concepts in various practical contexts, thereby strengthening their foundational knowledge overall (Irfan et al., 2019). These recommendations aim to bridge the gap in understanding proportional concepts at the early stages of students' reasoning development.

This sentence indicates the presence of epistemological obstacles in PMTs, where they do not fully grasp the connection between proportional concepts and real-world contexts. As a result, they tend to rely on learned procedures without a deep understanding of the underlying conceptual relationships. These epistemological obstacles arise from a lack of fundamental understanding of necessary mathematical concepts or principles, leading PMTs to apply procedures or rules mechanically without comprehending the rationale behind them. Moreover, the lack of nuanced teaching materials identified in this study points to a didactical obstacle that affects the overall learning process.

Didactical obstacles are also identified in the design of instructional materials used to teach proportions to PMTs, particularly for those at mid to higher levels of proportional reasoning. The study found that topics such as equivalent ratios and proportions are often presented superficially without in-depth explanations or adequate contextual exercises (Ölmez, 2022). This causes PMTs at intermediate levels to struggle to develop a deeper understanding and tend to rely on non-varied problem-solving strategies, such as cross-multiplication. This finding not only aligns with but also extends the work of Zulkarnain et al. (2020), who emphasize that a lack of variety in teaching approaches can limit the development of proportional reasoning at higher levels. By proposing that more diverse teaching strategies could mitigate these obstacles, the current study contributes to the existing dialogue on improving mathematics education for prospective teachers.

The study also shows that PMTs at different levels of proportional reasoning tend to use the same strategy, such as cross-multiplication, for both direct and inverse proportions without understanding the fundamental differences between them. Buforn et al. (2018) stress that diversifying teaching methods is crucial for developing more flexible proportional reasoning skills. Here, PMTs should be encouraged to employ various problem-solving strategies, not just to obtain the correct answers but also to deeply understand proportional concepts. This finding suggests a possible future research direction: investigating the effectiveness of varied instructional strategies on developing deeper conceptual understanding among PMTs. Joshua & Lee (2022) also note the importance of understanding direct and inverse proportional relationships by considering qualitative relationships and rates of change.

Overall, this study highlights the importance of enhancing PMTs' understanding of proportional reasoning at various levels. To achieve this, improvements in instructional design and teaching strategies are needed that not only focus on procedures but also integrate deeper and more contextual conceptual understanding. By diversifying teaching approaches and emphasizing conceptual understanding, PMTs are expected to develop stronger and more effective proportional reasoning skills for future teaching practice. These findings provide valuable insights for the development of more comprehensive and adaptive proportional learning programs in the education of prospective mathematics teachers. The novelty of this study lies in its detailed exploration of the specific types of obstacles encountered at various levels of reasoning, offering a more targeted understanding compared to previous research, and suggesting practical applications for future curriculum and instructional design. Future research could focus on testing these proposed instructional strategies in classroom settings to determine their effectiveness in overcoming the identified learning obstacles.

Conclusion

Overall, the findings of this study illustrate the distinct challenges encountered by PMT at different levels of proportional reasoning. PMT at the level of informal reasoning towards proportional situations, quantitative reasoning, and proportional formal reasoning tend not to experience learning obstacles. However, the non-propositional reasoning level is dominated by conceptual and instrumental ontological learning obstacles. Didactical obstacles are actually seen in the variety of PMT answers that are prioritized in the cross-product strategy. The limitation of this research is to find learning obstacles in proportion to learning.

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Conflicts of Interest

The authors declare no conflict of interest regarding the publication of this manuscript.

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Author Contributions

Rani Sugiarni: Conceptualization, writing - original draft, editing, visualization, review, editing, formal analysis, and methodology; **Tatang Herman**: Validation and monitoring; **Didi Suryadi**: Validation and monitoring; **Sufyani Prabawanto**: Validation and monitoring; **Nia Jusniani**: Validation.

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