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Students' misconceptions in algebraic concepts: A four-tier diagnostic test approach

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Abstract

Misconceptions in algebraic thinking refer to misunderstandings in grasping and manipulating abstract concepts such as variables and mathematical relations. These misconceptions are often consistently applied by students in certain situations even though they deviate from the correct concept. This qualitative study used a case study approach focused on analyzing students' misconceptions in understanding algebra, specifically in the topic system of linear equations in two variables (SLETV). This study utilized a four-tier diagnostic test combined with the Certainty of Response Index (CRI) to identify students' misconceptions. The test consisted of 12 questions covering four aspects of algebraic thinking: problem-solving, mathematical modeling, generalization, and analytical thinking. The subjects were 118 junior high school students in Ponorogo who had studied SLETV materials. The results showed that students experienced misconceptions, such as errors in interpreting mathematical statements, simplifying concepts, and organizing ideas. Although their understanding was incorrect, the students exhibited a high level of confidence in their answers. Teachers should use more explicit approaches to convey abstract concepts and provide appropriate feedback. Diagnostic tools such as the CRI can help teachers identify students' misconceptions and offer more effective remedial teaching, thereby significantly enhancing students' understanding of algebra.

Keywords: algebraic thinking; diagnostic test; four-tier; misconception

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Introduction

PISA 2023 data indicates that Indonesian students' mathematics literacy scores have dropped by 13 points compared to 2018, with only 18% of students reaching Level 2 and almost none achieving Levels 5 or 6 (Esti, Hersulastuti, Indiyah, & Kun, 2023). This position is far below the global average. Similar results are evident in the 2019 TIMSS report, where Indonesia ranked 46th out of 51 countries (Mullis, Martin, Foy, Kelly, & Fishbein, 2020). These reports highlight the low level of mathematical literacy in Indonesia, which is partly due to students' weak algebraic thinking skills. This is because Algebraic concepts are a fundamental foundation in mathematics because they support the development of mathematical literacy and problem-solving skills (Kaput, Carraher, & Blanton, 2017; Kieran, 2004; Stacey, 2011).

Algebraic thinking is the ability to generalize, model, and solve mathematical problems, both with and without using symbols (Farida & Hakim, 2021). This skill encompasses the understanding of patterns, relationships, and the manipulation of abstract concepts like variables and equations (Hee-Chan, 2004). However, many students struggle with algebra due to its abstract nature and the challenging transition from arithmetic to algebraic thinking (Breiteig & Grevholm, 2006). For instance, students often make mistakes in simplifying equations, such as transforming $4y + 20 = 8y + 5$ into $y + 20 = 2y + 5$, indicating an inability to grasp basic algebraic concepts.

To better understand students' difficulties in algebra, teachers need to identify whether students do not comprehend the concept or are experiencing misconceptions (Parandrengi & Hiltrimartin, 2023). Misconceptions are incorrect understandings that contradict the correct concept but are applied consistently in certain situations (Clarkson, 2006). These are different from random errors because misconceptions are systematic and recurring (Leinhardt, Zaslavsky, & Stein, 1990).

Misconceptions in algebraic thinking can manifest in various forms, such as errors in algebraic operations, incomplete or incorrect understanding, and a lack of interest in learning mathematics. Concrete examples of misconceptions in algebra include errors in subtracting two algebraic expressions, where students mistakenly believe that the negative sign only affects the first term of the expression. Additionally, misconceptions can occur in the operations of addition, subtraction, and simplification of algebraic forms.

Identifying misconceptions is crucial in mathematics education (Moosapoor, 2023). Identifying misconceptions ensures that all students have the same opportunity to deeply understand mathematics. Moreover, teachers can address these errors specifically and help students develop a correct conceptual understanding (Boaler, 2002). However, commonly used diagnostic instruments, such as multiple-choice tests, have limitations. These tests only assess students' final answers without revealing the underlying thought processes (Aristiawan & Istiyono, 2020). Moreover, multiple-choice tests cannot distinguish between students who truly understand the material and those who guessed the correct answer (Simkin & Kuechler, 2005). To overcome these limitations, this study used a four-tier diagnostic test combined with the Certainty of Response Index (CRI).

The four-tier diagnostic test is a form of multiple-choice test that allows students to choose more than one answer at different levels, thus better capturing their thought processes (Fariyani & Rusilowati, 2015). CRI adds a new dimension by measuring students' confidence in their answers. A low CRI score (0–2.5) indicates guessing, while a high CRI score (2.5–5) indicates high confidence. If an incorrect answer is accompanied by a high CRI score, it suggests a deep-seated misconception.

Previous research on misconceptions in mathematics has been conducted extensively (Latifah, Wakhyudin, & Cahyadi, 2020; Lumbantoruan & Male, 2020; Luneta, 2015; Mohyuddin & Khalil, 2016; Rosyidah, Maulyda, & Oktaviyanti, 2020). However, in the field of algebra, studies using the four-tier diagnostic test are still limited. Using the four-tier diagnostic test and CRI, this research aims to identify students' misconceptions in algebraic thinking, providing more accurate insights for developing more effective teaching strategies.

Methods

This study employed a qualitative approach using a case study design. The case investigated was the students' misconceptions in algebraic thinking. Data on students' misconceptions were obtained using a four-tier diagnostic test instrument. The instrument consisted of 12 questions representing four indicators of algebraic thinking: problem-solving, mathematical modeling, generalization, and analytical thinking (Cai & Hwang, 2002; Kaput et al., 2017; Kieran, 2004; Radford, 2011). The test was administered to 118 consists of 45 males and 73 females junior high school students in Ponorogo, East Java, Indonesia, who had learned about SLETV. Because in solving systems of equations, students perform algebraic manipulations such as simplifying expressions and substituting variables, which are essential skills in algebra.

The instrument used in this study was a four-tier diagnostic test. This test was designed to reveal not only students' answers but also the reasons behind their answers, allowing the researchers to identify whether students experienced misconceptions or merely made calculation errors. The first tier consisted of multiple-choice questions that assessed understanding of algebraic concepts. The second tier required students to explain the reasoning behind their choices in the first tier. This helped to identify students' thought processes, such as whether they applied incorrect logic when solving algebraic problems.

The four-tier diagnostic test instrument used in this study underwent a development phase and was validated by both expert validators and post-trial analysis. The instrument was deemed valid, reliable, and had an appropriate level of difficulty. All test items have an Aiken's validity value above 0.92, and the overall reliability of the items is 0.759. This demonstrates that all items are suitable for use in research.

The data was obtained through tests administered to students using a four-tier format. Students' answers on the four-tier diagnostic test were analyzed to identify items that indicated misconceptions. Misconception identification was based on adopting a misconception decision matrix for both answers and reasons (Tables 1 and 2) (Hasan, Bagayoko, & Kelley, 1999).

Table 1. Misconception decision matrix for answers

Low CRI (< 2.5)	**High CRI (> 2.5)**
Correct answer and low CRI (CL): Does not understand the concept (lucky guess)	Correct answer and high CRI (CH): Correct understanding
Wrong answer and low CRI (WL): Does not understand the concept	Wrong answer and high CRI (WH): Misconception

Table 2. Misconception decision matrix for reasons

Low CRI (< 2.5)	**High CRI (> 2.5)**
Correct reason and low CRI (CL): Does not understand the concept (lucky guess)	Correct reason and high CRI (CH): Correct understanding
Wrong reason and low CRI (WL): Does not understand the concept	Wrong reason and high CRI (WH): Misconception

Identification of answers that contained misconceptions was further analyzed qualitatively to understand the types of misconceptions experienced. The analysis focused on wrong answers with high CRI scores and incorrect reasoning with high CRI scores. This was done to identify the thought patterns that led students to choose incorrect answers and reasons. The research procedure followed is illustrated in Figure 2.

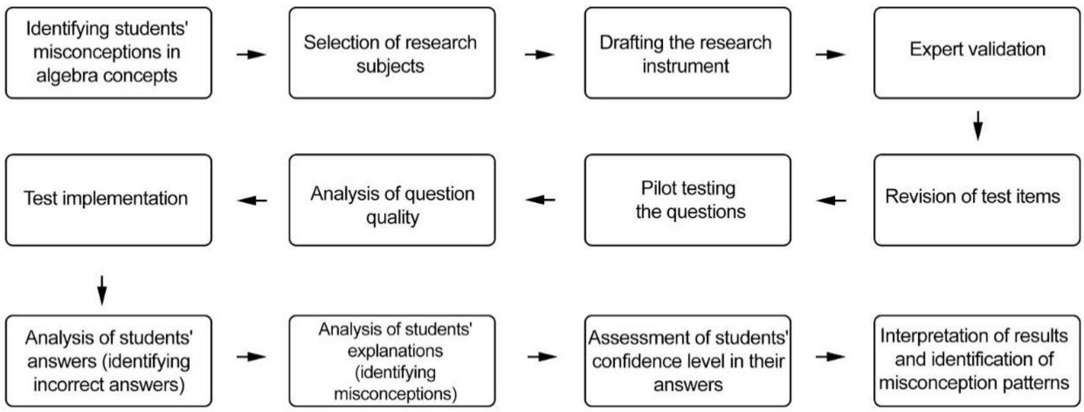

Figure 2. Research prosedur

Results

Misconceptions in algebra occur when students are unable to correctly manipulate or interpret abstract concepts such as variables and mathematical equations. These errors are typically consistent and recur in various situations. A high CRI score on wrong answers indicates the presence of misconceptions. Based on the analysis, the average CRI values for answers and reasons across all test items were identified (Figures 3 and 4).

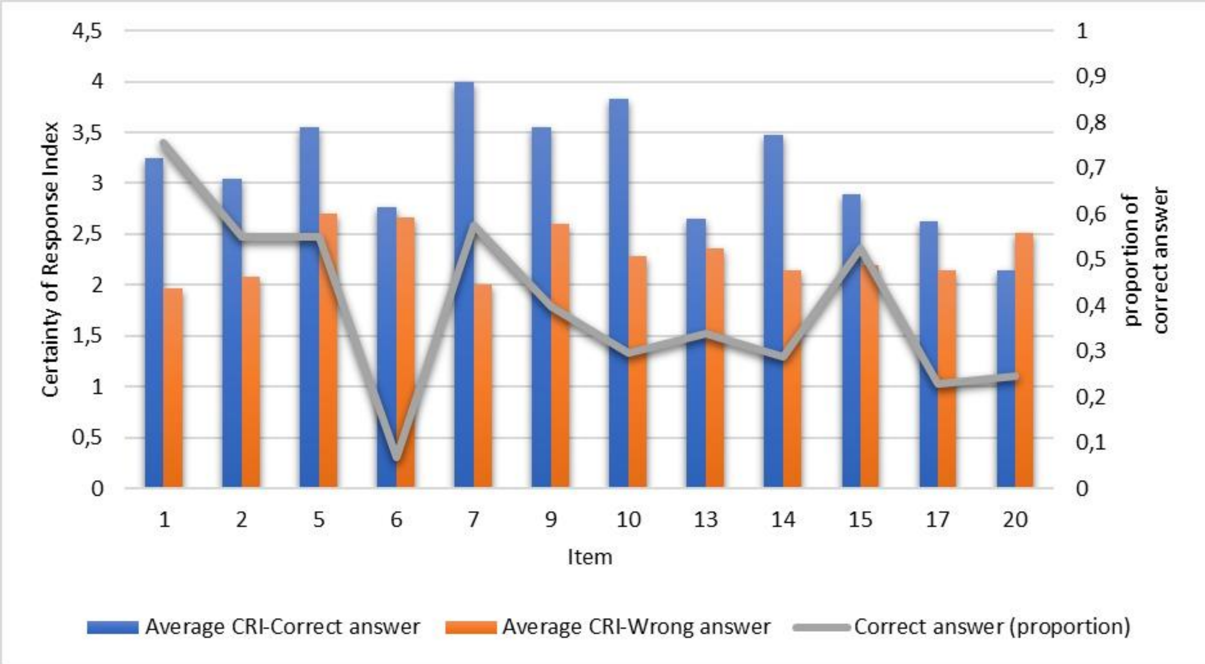

Figure 3. Average CRI values for answers on the four-tier diagnostic test

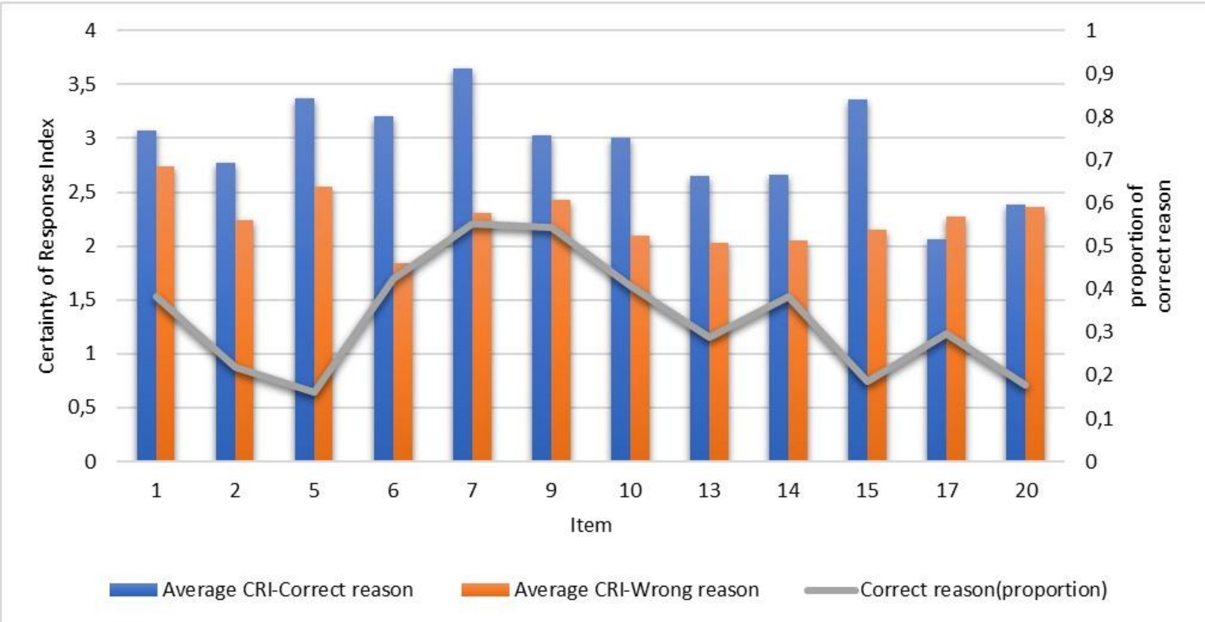

Figure 4. Average CRI values for reasons on the four-tier diagnostic test

Based on Figure 3, students demonstrated misconceptions in their answers to questions 3 (analytical thinking), 4 (generalization), 6 (generalization), and 12 (mathematical modeling). Students were considered to have misconceptions if they answered incorrectly with a high CRI score above 2.5. Detailed information about students' understanding of all test items can be seen in Table 3.

Table 3. Identification of students' answer errors on each question

Indicator	CRI	Description
Problem Solving	2	Does not understand the concept
Problem Solving	2.1	Does not understand the concept
Analytical Thinking	2.7	Misconception
Generalization	2.6	Misconception
Mathematical Modeling	2	Does not understand the concept
Generalization	2.6	Misconception
Generalization	2.3	Does not understand the concept
Analytical Thinking	2.4	Does not understand the concept
Analytical Thinking	2.1	Does not understand the concept
Mathematical Modeling	2.2	Does not understand the concept
Problem Solving	2.1	Does not understand the concept
Mathematical Modeling	2.5	Misconception

Based on Table 3, misconceptions on students' answer occurred in question number 3 (analytical thinking), question number 4 (generalization), question number 6 (generalization), and question number 12 (mathematical modeling). Similarly, misconceptions in reasoning occurred on questions 1 (problem-solving) and 3 (analytical thinking). Detailed information about students' reasoning errors on all test items can be seen in Table 4.

Table 4. Identification of students' reasoning errors on each question

Indicator	CRI	Description
Problem Solving	2.7	Misconception
Problem Solving	2.3	Does not understand the concept
Analytical Thinking	2.6	Misconception
Generalization	1.8	Does not understand the concept
Mathematical Modeling	2.3	Does not understand the concept
Generalization	2.4	Does not understand the concept
Generalization	2.1	Does not understand the concept
Analytical Thinking	2	Does not understand the concept
Analytical Thinking	2.1	Does not understand the concept
Mathematical Modeling	2.2	Does not understand the concept
Problem Solving	2.3	Does not understand the concept
Mathematical Modeling	2.4	Does not understand the concept

Based on Table 4, misconceptions on students' reasoning occurred in question number 1 (problem-solving) and question number 3 (analytical thinking). Based on the data shown in Tables 3 and 4, misconceptions in both students' answers and reasoning occurred in question number 3. Question number 3 is a problem that requires students to think analytically. In this question, data is presented in the form of the difference and sum of the ages of two individuals. Students are asked to determine the age of each individual. The complete question can be seen in Figure 5.

3. Mega is 7 years older than Sarma. The sum of their ages is 43 years. What are their respective ages?

a.	Mega is 24 years old and Sarma is 19 years old	c.	Mega is 26 years old and Sarma is 17 years old
b.	Mega is 25 years old and Sarma is 18 years old	d.	Mega is 27 years old and Sarma is 16 years old

Level of confidence	0	1	2	3	4	5

Reason

a. Let Mega be M and Sarma be S. Then the mathematical modeling of the statement 'Mega is 7 years older than Sarma' is $S = M + 7$

b. Let Mega be M and Sarma be S. Then the mathematical modeling of the statement 'Mega is 7 years older than Sarma' is $7M - S = 43$

c. Let Mega be M and Sarma be S. Then the mathematical modeling of the statement 'Mega is 7 years older than Sarma' is $M = S - 7$

d. Let Mega be M and Sarma be S. Then the mathematical modeling of the statement 'Mega is 7 years older than Sarma' is $M - S = 7$

Figure 5. Question item containing misconceptions

The question in Figure 5 could be answered correctly if the statement "Mega is 7 years younger than Sarma" is represented as S+7 = M or M – S = 7, nd the statement "their combined age is 43 years" is represented as M+S = 43. Therefore, the correct reasoning is indicated in answer option D. By selecting reason D and continuing with substitution or elimination, students should choose answer B.

Only 16% of students managed to choose the correct reasoning, indicating that the majority struggled to understand the proper algebraic relationships. This result reveals that misconceptions in the use of algebraic symbols are common. The distribution of students' reasoning choices is shown in Table 5.

Table 5. Distribution of students' reasoning choices

Reason Choice	Percentage of Students	CRI
A	28%	2.73
B	47%	2.45
C	8%	2.6
D*	16%	3.37

Based on Table 5, the highest percentage of incorrect reason chosen by students was for option B (47%), followed by option A (28%) and then option C (8%). In terms of CRI, incorrect reasons with high CRI values were reason A (CRI = 2.73) and reason C (CRI = 2.6). Reason B, which was chosen by the majority, had a CRI value of 2.45, close to the threshold for misconceptions.

Students who chose reason A modeled Mega’s age as being 7 years older than Sarma’s age with the equation S=M+7. This modeling does not align with the information in the problem, leading to errors in problem-solving. These students were unable to see the entire picture and did not understand how different parts interrelated in a system or process. This inaccurate mathematical modeling led to incorrect solutions.

Students who chose reason C modeled the statement "Mega is older by 7 years than Sarma" as M=S – 7, which is actually incorrect. This error demonstrates a misconception in understanding the concept of "older." Students thought "older" meant subtracting from Sarma's age rather than adding to it. n fact, if Mega is older, Mega's age should be greater than Sarma's age. Students have difficulty translating verbal sentences into algebraic form. '7 years older' might be intuitively understood correctly, but when they try to translate it into algebraic form, students make mistakes. They tend to focus more on manipulating operations (+ or -) without truly understanding the context or meaning of the problem

The distribution of students choosing reason B indicates that the majority of students, 47%, selected the incorrect reason with a CRI of 2.45, nearing the threshold for misconceptions. This suggests that the students who chose this reason are quite confident in their answer rationale, even though it is logically incorrect. This error is primarily related to the students' inability to organize information and connect it with the appropriate concepts. They tend to jump straight to what they believe is the correct answer without considering alternatives or analyzing further. Based on answer choices, 56% of students chose the correct answer, as seen in Table 6.

Table 6. Distribution of students' answer choices

Answer Choice	Percentage of Students	CRI
A	14%	2.88
B*	56%	3.55
C	16%	3.05
D	13%	1.9

Based on Table 6, the highest percentage of incorrect answers chosen by students was for option C (16%), followed by option A (14%) and then option D (13%). Examining the CRI values, incorrect answers with high CRI scores included choice C (CRI = 3.05) and choice A (CRI = 2.88). This data suggests that students experienced misconceptions related to the given question.

Question number 3 assessed algebraic thinking ability with an analytical thinking indicator. Students likely selected answer C if they believed that adding Sarma's and Mega's ages would yield the total stated in the problem, 43 years. The students based their reasoning on M = S + 7 because, given the provided modeling, they believed that Sarma's age is 17. This is due to the students' inability to logically organize information and understand the relationships between various pieces of information. Meanwhile, the students confidently chose answer A because the sum of Sarma's and Mega's ages matches the problem statement, which is 43 years. However, for the age difference between Sarma and Mega in option A, it is only 5 years. This occurs because the students experience a misconception of the concept, facing difficulties in understanding the concepts presented in the math problem and making errors in simplifying and organizing ideas. As a result, the information that Mega is seven years older than Sarma is not well understood by the students who chose answer A.

The students choosing answer A with a CRI of 2.88 indicates that they followed their first instinct or intuition without deeper reflection, leading them to fall into common logical traps. They accepted the answer that seemed correct without considering alternative possibilities or testing the underlying assumptions, and they remain vulnerable to errors due to a tendency to rely on intuition that may not always be accurate.

The students choosing answer D with a CRI of 1.9 indicates that this answer was selected with a lower level of confidence compared to answers with higher CRIs, such as 3.05. In this context, the students may not be very confident in their chosen answer, or they selected it because they felt they had no better options.

Discussion

Misconceptions arise when knowledge is not coherently connected or is inaccurate. Students often memorize facts or procedures without understanding the underlying principles, relying on short-term memory or accepted rules. This leads to incorrect conclusions or misalignment with the correct concept. Many students struggle to understand the relationship between algebraic variables and real-world contexts because they view algebra as isolated procedures, not linked to real-world applications or deeper understanding, making them more prone to misconceptions (Bye, 2016).

This study found that many students have misconceptions in understanding the relationships between variables. For example, students often mismodel the statements 'older' or 'younger' in algebraic equations. These misconceptions are evident in the incorrect answer choices, where students misinterpret the summation of variables as a simple operation without understanding the relationships between those variables. This is because students tend to make errors such as combining variables without understanding their relationships, leading to a failure in modelling the correct connections (Rittle-Johnson & Schneider, 2014).

These findings align with previous research which also indicates that students often struggle to translate verbal statements into algebraic symbols (Abidin, Darwis, & Sari, 2021; Mulungye, O'Connor, & Ndethiu, 2016). Additionally, students experience difficulties in identifying multiple variables, which leads to errors in understanding and solving specific problems, as well as challenges in applying the same principles to other problems with different structures (Yansa, Retnawati, & Janna, 2021; Yasseen, Yew, & Meng, 2020).

Misconceptions in the answers are caused by errors in understanding the fundamental concept of variables. Many students are confident in their answers, even though those answers are incorrect, indicating that they are unaware of the flaws in their reasoning (Pashler et al., 2007). Misconceptions in reasoning, on the other hand, often occur when students are unable to see the relationships between elements in a problem as a whole. They rely on their initial intuition without deeply considering the logic behind their answers (Budayasa & Budiarto, 2019)

Additionally, misconceptions also occur because these students often rely on their first intuition to answer questions, and incorrect reasoning indicates that these individuals may not reflect on the questions deeply. They tend to jump straight to what they believe is the correct answer without considering alternatives or analyzing further (Baiduri & Ulfah, 2022; El Khoiri & Widiati, 2017)

To address these misconceptions, it is important for teachers to use a more explicit teaching approach. Teachers need to introduce the concept of variables gradually by using real-life examples that connect everyday situations with algebraic concepts. For instance, when explaining the concept of 'older,' teachers can use analogies related to the ages of family members or friends, which can then be translated into algebraic symbols (Paulsen, 2006).

Additionally, ongoing assessment is crucial to ensure students' understanding. By using formative tests or short quizzes, teachers can identify misconceptions early on and adjust their teaching according to students' needs. Problem-based learning can also be implemented to encourage students to think more deeply about how to use algebra in real life (Lucariello, Tine, & Ganley, 2014; Tobey, 2017).

Conclusion

This study shows that misconceptions in algebraic thinking are a significant issue among students, particularly in understanding the concept of variables and the relationships between variables. The use of the Four-Tier Diagnostic Test combined with the Certainty of Response Index (CRI) has proven effective in identifying the types and levels of students' confidence in incorrect answers, which subsequently reveals the presence of misconceptions.

From the analysis, it was found that the majority of students struggle to understand verbal statements involving mathematical relationships such as 'older' or 'younger,' leading to errors in modeling algebraic equations. Students with misconceptions tend to be confident in their incorrect answers, indicating that their misunderstandings are deep-seated and not merely a result of ignorance.

The most common misconception is the students' inability to logically organize information and simplify ideas in algebraic form, along with a tendency to rely on intuition without further thinking. This study also shows that misunderstandings often occur among students who do not grasp the relationships between variables in a system of equations.

As an implication, there is a need for more systematic interventions in algebra teaching, particularly in building a deeper understanding of the concept of variables and algebraic operations. Ongoing assessment and the use of problem-based teaching strategies, along with real-world contexts, are highly recommended to address these misconceptions and ensure a more solid conceptual understanding among students.

The limitation of this study lies in not analyzing the causes of students' misconceptions. The causes of misconceptions include the teaching process carried out by the teacher, the instructional materials, and students' initial mathematical abilities. This study focuses more on analyzing misconceptions in students' answers and the reasoning they provide.

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Conflicts of Interest

The authors declare no conflict of interest regarding the publication of this manuscript. In addition, the authors have completed the ethical issues, including plagiarism, misconduct, data fabrication and/or falsification, double publication and/or submission, and redundancies.

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Author Contributions

Hestu Wilujeng: Developing instrument, conceptualization, writing - original draft; **Aristiawan:** Writing - review & editing, formal analysis, and methodology; **Joel I. Alvarez:** review & editing.

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