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Didactical design research on mathematical sequence material in vocational high schools for job preparation

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Abstract

The material of sequences and series has many benefits in daily life, so mastery of sequences and series is essential, especially for job selection. This study is based on Didactical Design Research (DDR), which focuses on five twelfth-grade vocational high school students as research subjects. This study uses a methodological framework, which includes three analysis stages: prospective, metapedagogical, and retrospective. The results of this study reveal that: (1) learning obstacles experienced by students are divided into three types, namely ontogenical obstacles, epistemological obstacles, and didactical obstacles; (2) the didactic design developed by the researcher includes hypothetical didactic design 1 (which includes material, examples, and practice questions regarding number arithmetic operations and number patterns), hypothetical didactic design 2 (which includes material, examples, and practice questions regarding arithmetic series and geometric series), and hypothetical didactic design 3 (which contains material, examples, and practice questions regarding types of number series in psychological test questions); (3) Learning obstacles can be overcome by utilizing didactic design; and (4) The empirical didactic design is a synthesis of the material and analysis obtained from hypothetical didactic designs 1, 2, and 3 which have been implemented effectively.

Keywords: didactical design; learning obstacles; learning trajectories; number sequence

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Introduction

Vocational High School is an educational tier where students acquire the skills necessary for their chosen professions, preparing them for the workforce. Vocational education contributes to developing a high-quality national workforce that is anticipated to maintain a symbiotic relationship with the demands of the labor market (Edi et al., 2017) & (Soleh et al., 2023). Globally, there is a growing interest in enhancing vocational education as a means to swiftly and efficiently integrate young individuals into the labor market by providing them with specialized skills (Hampf & Woessmann, 2017). The primary objective of vocational education is to prepare students for employment in a particular field (Santika et al., 2023). As time progresses, education must generate graduates proficient in their respective disciplines (Jazuli et al., 2022) & (Adiastuty, 2015). The fundamental prerequisites for effectively entering the labor market often include authorized grades and a high school education (Mårtensson et al., 2023).

The competencies acquired by students in vocational schools must be pertinent to the demands of the labor market. Mathematics is a fundamental discipline that significantly enhances vocational abilities in the industrial sector. Mathematical skills are essential in the industrial sector (Restiana, 2019). Although vocational schools aim to prepare graduates for work, this contrasts data from the Central Statistics Agency, which reports that the number of unemployed in Indonesia has reached 7.99 million people, with the unemployment rate for vocational school graduates reaching 9.60 based on news from CNN. Preliminary observations at SMK Kuningan Regency revealed that numerous students did not pass the psychological assessment, particularly in fundamental mathematics skills, which included substantial content on sequences. It is believed to be due to students' challenges completing the test questions. Hence, it is crucial to determine the learning impediments encountered by students.

Student learning obstacles can be anticipated by studying their challenges when addressing assigned questions and identifying the underlying causes and potential alternative answers. In this context, educators play a crucial role in providing customized instructional materials to meet students' requirements for industrial entrance examinations. Creating suitable instructional materials can enhance students' comprehension, reducing learning barriers. Consequently, it is essential to undertake research utilizing didactic design research (DDR), which encompasses three phases: the analysis of the didactic situation prior to the implementation of learning, meta-pedagogical analysis, and retrospective analysis, which connects the outcomes of the hypothetical didactic situation analysis with those of the meta pedagogical analysis. An empirical didactic design will be developed from these three phases, which may be further refined through a process encompassing the three stages of DDR. The instructional design seeks to assist students in comprehending industrial entrance examination questions.

Students frequently perceive mathematics education as insignificant, believing that future advancement solely relies on employment (Sumaji & Wahyudi, 2020). Consequently, a systematic comprehension of the significance of mathematical ability as a foundation for joining the industrial sector is essential. The primary objective of vocational mathematics education is to equip individuals for future employment or to enhance the skills of currently qualified workers (Frejd & Muhrman, 2022). Educators are paramount in achieving high-quality vocational high school education and are prepared to confront contemporary issues (Dwijanto et al., 2022). Students may engage in conversations if the educator identifies the underlying causes of challenges in learning mathematics (Kaufmann & Ryve, 2023). The inadequate student learning process results from insufficient didactic foresight in learning planning. The lack of exploration into students' responses to instructional situations results in teachers inadequately addressing emerging learning issues (Suryadi Didi, 2018). Prior research conducted by scholars at a vocational high school identified that pupils' learning challenges were marked by deficiencies in the interpretation of numerical symbols (Khairullah & Heriyana, 2023).

Educators must meticulously plan instruction to attain objectives and facilitate a more significant learning experience. Teaching materials serve as a resource to assist educators in preparing for instruction. Teaching materials are essential components of the learning process, specifically created, applied, and developed to address learning challenges—didactic conditions in students (Senjayawati & Kadarisma, 2020). The didactic design pertains to the creation of educational materials encompassing several elements, specifically learning obstacles and learning trajectories. The learning design plan aims to assess students' educational needs and assist teachers in addressing those needs (Haqq et al., 2019). Previous research has extensively linked didactic design with mathematical content, focusing on identifying learning hurdles and devising strategies to overcome these challenges (Haqq et al., 2019), (Refi Elfitra, 2017), (Sawitri et al., 2020) & (Hasibuan, 2018).

Among the several investigations, none have explicitly examined the learning impediments associated with the content of sequences in the psychological test items. The advancement of student recruiting into the workforce has been minimal thus far. One contributing cause is that kids do not pass the psychological assessment. Nevertheless, no pedagogical framework has been developed to facilitate students' comprehension of psychological exam questions. A didactic design must be developed to address didactic obstacles, which are significant elements affecting learning (Lestari & Umbara, 2022). This study aimed to create a more effective learning design and targeted strategies to assist students in overcoming challenges in solving psychological test questions related to sequences. The research employed Didactic Design Research, which comprises three stages: didactic situation analysis prior to learning, metapedagogical analysis, and retrospective analysis (Suryadi Didi, 2018).

Method

This study was conducted to formulate an alternative learning design in vocational high schools on sequence material related to basic mathematics for job selection psychological test questions based on learning obstacles experienced by students. This alternative learning design considers three aspects: the learning process, teaching materials, and learning obstacles (Haqq et al., 2019). These three aspects are interrelated phenomena that are studied qualitatively. Therefore, the method used in this study is qualitative research using a didactic design. In principle, this design develops teaching materials based on students' objectives, relevance, experience, and difficulties/obstacles (Rikayanti, 2018). According to Suryadi, this study includes three stages (Suryadi Didi, 2018). The stages of developing teaching materials are as follows:

In the Didactical Situation Analysis Stage, the researchers select teaching materials, look for references as literature, ask questions about the Ability to Solve Psychological Questions about Rows, conduct initial tests, analyze test results to identify students' difficulties in solving test questions, compile teaching materials (student worksheets) about linear programs. In addition, in the Metapedidactic Analysis Stage, the researchers deliver teaching materials (worksheets) that have been prepared and analyze student responses while working on worksheets. Lastly, in the Retrospective Analysis Stage, the researchers connect the results of student responses and solutions that have been predicted before, analyzing student responses in working on worksheets, conducting a final test to see the improvement of the results, analyzing the effectiveness of didactical design based on students' learning obstacles in working on mathematical reflective problems.

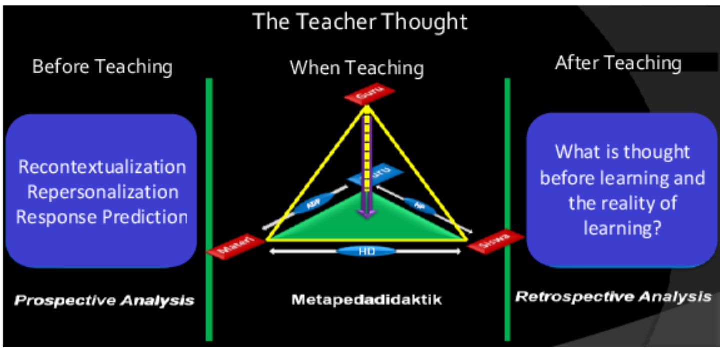


Figure 1. Scheme didactical design research (DDR)

This research was conducted in a twelfth-grade vocational high school with thirty-four students. The DDR-related research in this class used a deliberate sampling method to ensure a representative sample, which enabled it to gather information and understand the main topics of the research. As part of this method, six students were selected as the focus of the study and participated in interviews.

This research was conducted through continuous learning activities. In the initial stage, preliminary observations were made to identify the most relevant learning materials. Among the topics investigated, this sequence material emerged as a fascinating subject. The objectives of this research include finding out the description of learning obstacles in learning mathematics on the material of the sequence and getting a didactic design developed to overcome learning obstacles in learning mathematics on the material of the sequence.

Various research instruments were used to achieve these objectives. The written test used in this study was a respondent's initial ability test given to thirty-four students and lasted for ninety minutes. This test assesses students' understanding and ability to solve row problems, especially psychological test questions. This test also aims to identify potential learning obstacles by examining the completion process and analyzing student responses. In addition to the written test, in-depth interviews were also conducted with a selected group of six students. These interviews aimed to dig deeper into students' understanding of the row material, explore their problem-solving approaches during the test, and analyze their experiences in understanding the concept. All data collected from the respondent proficiency test, in-depth interviews, and supporting documents were carefully scrutinized to identify any learning barriers students may face.

This research focuses on individual actors or groups and aims to understand their perspectives on a particular phenomenon (Hitchcock, 1995). It seeks to capture how individuals define and interpret the phenomenon in a particular context and time (Aberdeen, 2013). The observed data from all prospective analysis processes were iteratively analyzed to refine the research questions. The observed data from all prospective analysis processes were analyzed repeatedly to refine the research questions so that the researcher could guarantee the validity and reliability of the research data based on the three analysis techniques.

Results

Didactic situation analysis before learning (prospective analysis)

The prospective analysis encompasses three categories of didactic scenario analysis: learning obstacle analysis, personalization and recontextualization, and didactic design development. Initially, learning impediments are recognized and delineated to comprehend the sequences within the psychological exam inquiries. Furthermore, re-personalization and recontextualization analyze the sequence context by considering students' learning obstacles. Based on the results of recontextualization and re-personalization, the last step is to create a didactic design. This stage produces a hypothetical didactic design that produces Student Worksheets and Hypothetical Learning Trajectories—learning Obstacle Analysis through Student Initial Ability Tests. At this stage, an initial ability test is conducted on five twelfth-grade students from the same class with varying academic abilities to capture students' learning obstacles from the sequence material, which contains three types of questions that have been adjusted to the needs of the industrial entrance test. Based on the results of the initial ability test, the researcher then analyzed the obstacles into three types of obstacles: (1) Ontogenic Obstacle (related to the development of student thinking), (2) Epistemological Obstacle (related to student understanding of the material), and (3) Didactic Obstacle (related to teaching, such as how teachers teach, what approaches, models or strategies are used). The analysis is as follows:

Table 1. Summary of ontogenic obstacles

Ontogenical Obstacle	Characteristics	Students Experiencing Obstacles
Psychological Ontogenic Obstacle	Students are less ready to learn because they lack motivation, whether from themselves or others, such as family or teachers.	Students A, B, D, and E
Instrumental Ontogenic Obstacle	Students do not know the formula to solve the problem.	All students
	Students do not know the steps to solve the problem.	Student A

Ontogenical Obstacle	Characteristics	Students Experiencing Obstacles
	Students make mistakes when representing problems, which results in errors in the calculation process.	Students A, B and C
Conceptual Ontogenic Obstacle	Students do not understand the concept of the question	Students A and B
	Students do not know the prerequisite material.	All students

Table 1 outlines the ontogenic obstacles students encounter, categorized into three main types: psychological, instrumental, and conceptual. Psychological ontogenic obstacles are characterized by a lack of intrinsic or extrinsic motivation to learn, which may stem from limited support from family or teachers; this challenge affects Students A, B, D, and E. Instrumental ontogenic obstacles include students' inability to identify the required formula, lack of understanding of the problem-solving steps, and errors in representing problems that lead to calculation mistakes, with all students experiencing these challenges, particularly Students A, B, and C. Conceptual ontogenic obstacles arise when students fail to grasp the concept of the question or lack the prerequisite knowledge necessary for problem-solving, which impacts Students A and B as well as all students generally.

Table 2. Summary of epistemological obstacles

Epistemological Obstacle	Characteristics	Students Experiencing Obstacles
Error of description	The absence of semantics and meaning from the sequences that are the main material in the psychological test questions.	Students A, B, D and E
	Students do not know the concept of arrays/subseries.	All Students
The error of ways of thinking	Students focus only on guessing the next number without thinking about the steps.	All Students
	Students cannot represent the information in the problem to form a pattern.	Students A, B and C
Error ways of understanding	Students assume that they can solve the sequence of psychological test questions only by adding and subtracting.	Students A and B
	Siswa tidak mengetahui langkah pengerjaan soal yang tepat.	All Students
Error of the calculation process	The student was correct in determining the solution concept, but an error occurred in the calculation process.	Students C and D
	Students do not complete the process of working on the questions (only guessing)	Students A and B

Table 2 highlights the epistemological obstacles that impede students' comprehension of sequence material, categorized into four types: errors in the description, errors in ways of thinking, errors in ways of understanding, and errors in the calculation process. The error of description involves students' lack of semantics and meaning in understanding sequences, particularly those in psychological test questions, as well as their unfamiliarity with the concept of arrays or subseries, which affects Students A, B, D, E, and all students. The error of ways of thinking is characterized by students' tendency to focus solely on guessing the following number without considering systematic steps and their difficulty in representing problem information to form patterns; this challenge impacts all students, particularly Students A, B, and C. The error of ways of understanding arises when students assume sequence problems can be solved merely through addition or subtraction and lack awareness of proper problem-solving steps, affecting Students A, B, and all students. Finally, the error of the calculation process occurs when students correctly identify the solution concept but make mistakes during calculations or fail to complete the problem-solving process, often resorting to guessing; this type of obstacle is observed in Students C and D for calculation errors and Students A and B for incomplete work.

Table 3. Summary of didactic obstacles

Didactical Obstacle	Characteristics	Students Experiencing Obstacles
The representation and approach provided by teachers are not diverse	Based on interviews with teachers, learning with students is carried out using various methods such as lectures, discussions and Q&A. Obstacles: Students are not open to expressing the learning obstacles they face. Sometimes, teachers only assign students to study the material and work on the questions given.	All Students
Lack of understanding of the concept of each type of sequence and the flow of material delivery is less systematic	The current learning system requires students to be more active in seeking information, so teaching materials are not detailed with explanations for each part of the material. The learning system requires students to study the material in more depth because the teaching materials are not designed to suit the student's initial abilities.	

Table 3 identifies the didactical obstacles associated with the teaching approaches employed by teachers, categorized into a lack of diverse representation and approach and a lack of conceptual understanding and systematic delivery. The lack of diverse representation and approach is characterized by teachers relying on lectures, discussions, and Q&A methods without tailoring these to students' specific needs, reluctance to communicate their learning difficulties, and instances where teachers assign independent study or problem-solving without sufficient guidance. This issue affects all students. The lack of conceptual understanding and systematic delivery arises from the inadequacy of learning materials, which lack detail and hinder students' comprehension of different sequence types, as well as the unsystematic flow of material delivery that fails to accommodate students' initial abilities. All students experience these challenges.

Before designing the didactic design, the researchers carried out a re-personalization and recontextualization process. During the re-personalization stage, the researcher analyzed the teaching materials used and reviewed the relevant context in the study to link the mathematical concepts taught. This context review involved analyzing the concept of sequences from various references and various example questions. A concept map of the analyzed mathematics teaching materials was used to begin concept exploration. The following is an illustration of a concept map of the teaching materials that the researcher analyzed:

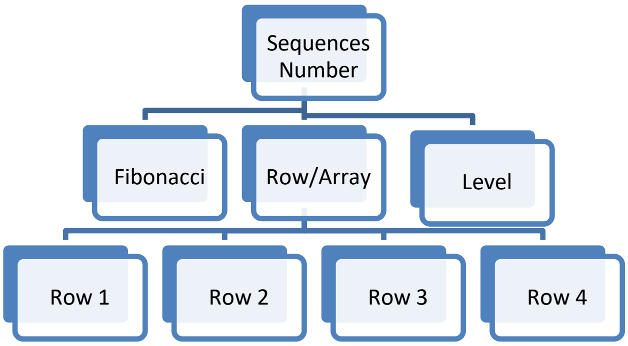


Figure 2. Concept map of number sequences

The concept map in Figure 2 explains that the concept of the material being analyzed is about the number sequence, where there are three types of number sequence: Fibonacci, array, and multilevel. There are four levels in the array type: array 1, array 2, array 3, and array 4.

Researchers explored the advantages and disadvantages of the teaching materials used, especially the number sequences material. The advantages of this teaching material are that it is simpler and easier to understand and has covered the concept of critical thinking in definitions, examples of questions, and practice questions. Meanwhile, the disadvantage of this teaching material is that it does not present initial mathematical abilities for sequences such as number operations. Therefore, the material will be used as an initial concept to understand the number sequence.

During the recontextualization phase, researchers developed an alternative learning trajectory informed by a comprehensive examination of the instructional material sources and the learning obstacle identification assessment outcomes. Identifying learning obstacles revealed that numerous students struggle with challenges about the notion of Number Sequences. Unstructured interviews between researchers and students indicated that pupils fundamentally lacked a comprehensive understanding of Number Sequences, comprehending only to the extent of answering questions. In addition, many students were weak in prerequisite materials such as number patterns and basic algebraic calculations, especially division.

The following presents an alternative learning flow resulting from the exploration of concepts and analysis of the learning obstacle identification test:

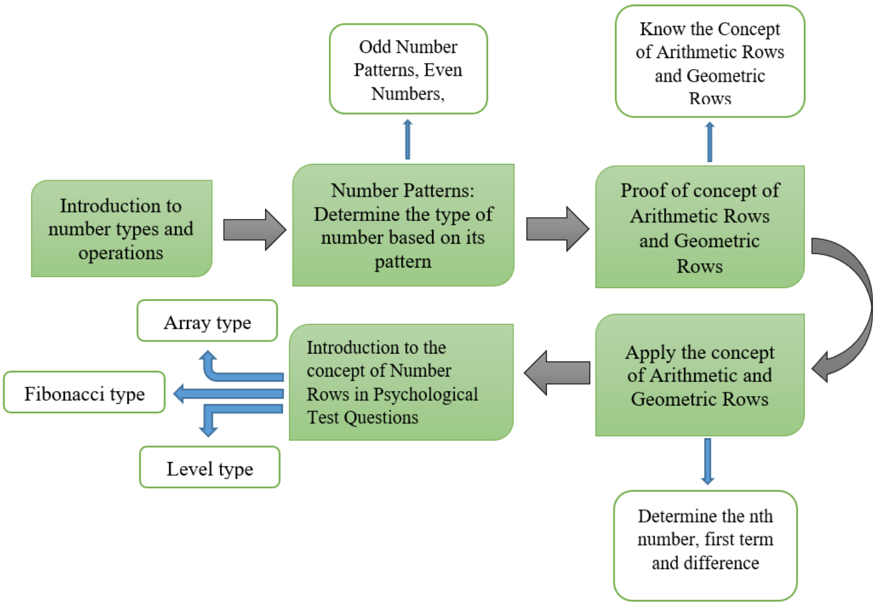


Figure 3. Alternative number sequence learning flow

Based on the learning sequence that has been arranged, the first material studied is about the introduction of types and operations of numbers. This material begins by explaining the types of numbers, including odd, even, prime numbers, and the like. Then, we continued with number operations, especially division, because, based on the results of interviews and initial tests, many students made mistakes in determining the results of division to exponentiation operations, roots, and mixed operations. Upon mastering numerical concepts and arithmetic procedures, students will proceed to the study of numerical patterns. In this topic, students are instructed to apply their intellect in identifying the pattern of a given numerical sequence to facilitate the prediction of the subsequent number in the sequence. Students will be introduced to Arithmetic and Geometric Sequences upon comprehending numerical patterns. In the introduction to this subject, students are tasked with identifying the nth term of both an arithmetic and geometric sequence and ascertaining the first term and its difference when the nth term is provided.

Additionally, students are presented with many problems/questions concerning number sequences with ambiguous patterns, such as those seen in arithmetic and geometric sequences. Students are given the concept of number sequence types, including Array Types, which are divided into arrays one to four, Fibonacci Types, and Level Types. In working on them when faced with the number sequence problem, students are asked to carry out the process stages on array one first in sequence to level. From this context, students will learn to find the following number from the given sequences.

Hypothetical learning trajectories analysis

Learning trajectories are examined to discern the process and evolution of students' comprehension of Number Sequences. The analysis at this level relies on the outcomes of the learning barrier identification exam and student interviews. Three main components need to be considered to achieve a complete HLT, namely (a) learning goals, (b) development of students' thinking and learning progress (developmental progression of thinking and learning), and (c) devices used in learning activities (sequence of instructional tasks). The following are details of the three components:

Table 4. Hypothetical learning trajectories analysis

LT Aspects	Description
Goal: Big ideas of Mathematics	1. Students have a comprehensive and focused understanding 2. Adapting actual learning according to current developments 3. Understanding concepts in understanding material about number sequences, whether presented through images, graphs or tables or not. 4. Students can describe, analyze, transform and compile material or problems.
Improving understanding: Learning Flow	1. There is an increase in student understanding when researchers apply learning according to the hypothetical didactic design created 2. Students are able to solve the problems given so that they can convince researchers that students have experienced an increase in understanding 3. Students are given the freedom to complete and conclude answers.
Learning tools: How to teach	1. The researcher designed hypothetical teaching modules and teaching materials for each meeting during the research. 2. The learning model used by the researcher is the contextual teaching learning method, which focuses on efforts to reduce students' learning obstacles.

Hypothetical didactic design stage

Hypothetical didactical design or hypothetical didactic design is a design of teaching materials that will be applied in the learning process. If, in its implementation, significant learning obstacles are found, or there is no increase in student understanding, then a retrospective analysis is needed. In this study, the researcher compiled three hypothetical didactic designs that have been adjusted to the sub-materials to be studied. Details of the three didactic designs have been outlined in the explanation below:

The hypothetical didactic designs provide a structured framework for addressing various mathematical concepts through sequential learning stages: Action Situation, Formulation Situation, Validation Situation, and Institutional Situation. Each design is tailored to specific topics, ensuring a comprehensive approach to teaching and learning.

Hypothetical Didactic Design 1 discusses Numbers and Arithmetic Operations. This design focuses on introducing students to fundamental numerical concepts and arithmetic operations. This stage includes several activities: (1) Action Situation includes introducing the types of numbers, observing their place and position, determining the results of basic arithmetic operations (addition, subtraction, multiplication, and division), and exploring exponents. These foundational tasks aim to build students' understanding of numbers as the building blocks of mathematics; (2) Formulation Situation includes the teacher explaining the various types of numbers, emphasizing patterns and positions in numerical operations. This stage underscores the importance of number placement in solving arithmetic problems; (3) The Validation Situation includes information about the definitions and types of numbers presented, supplemented by concept maps to help students visualize the scope of numerical material. Students are asked questions about their introduction to numbers and perform number operations to validate their understanding, and (4) Institutional Situation includes students analyzing and summarizing answers to reinforce their learning. Through self-discovery, they identify patterns and consolidate their understanding of numbers and arithmetic operations.

Hypothetical Didactic Design 2 discusses Arithmetic and Geometric Sequences. This design transitions into sequences, focusing on arithmetic and geometric sequences, their formulas, and applications. The first design includes the following activities: (1) Action Situation includes activities that guide students through understanding arithmetic and geometric sequences, calculating the nth term, and determining sequence elements based on the nth term. These activities build a robust conceptual foundation for sequence operations; (2) Formulation Situation includes teachers explaining the differences between arithmetic and geometric sequences and providing detailed descriptions of their formulas. Students are taught how to use these formulas to deduce other sequence elements; (3) Validation Situation includes practice questions employed to strengthen understanding. These exercises help students explore how the nth term and other elements are interrelated, reinforcing theoretical knowledge with practical application; and (4) Institutional Situation includes students analyzing their responses to identify and distinguish between arithmetic and geometric sequences. They summarize their learning, highlighting the unique characteristics of each sequence type.

Hypothetical Didactic Design 3 discusses Advanced Number Sequences. The final design addresses more complex number sequences commonly encountered in job selection tests, such as array types, Fibonacci sequences, and level-type sequences. The activities carried out are as follows: (1) Action Situation includes: Students are introduced to various sequence types, including array, Fibonacci, and level sequences, enhancing their ability to recognize and solve advanced sequence problems; (2) Formulation Situation includes Teachers explain the concepts underlying these sequences, their applications in psychological test questions, and how to predict subsequent terms based on sequence patterns; (3) Validation Situation includes: Students engage with concept maps that outline the material's scope, respond to questions related to sequence types, and practice solving problems involving these sequences. This stage strengthens their grasp of advanced sequence concepts, and (4) Institutional Situation includes Students analyzing their answers to develop strategies for identifying sequence types and predicting subsequent terms. They summarize their learning, articulating the distinctions and applications of various sequence types.

Metapedidactic analysis stage

Metapedadidactic analysis results from observation analysis conducted when the didactic design is implemented in the learning process. Before the design is implemented, anticipation and didactic predictions for various responses are prepared first. The researcher has developed metapedadidactic analysis during the learning process in this study.

In Hypothetical Didactic Design 1, an analysis has been carried out to overcome obstacles in student ontogeny. The researchers reflect, namely, motivating at the beginning of learning so that students feel motivated to continue learning. Then, the researchers dig up information related to statistical material that has been studied. Then, the researchers explained the things to be learned, the competencies to achieve, and the benefits of studying numbers. Furthermore, students were given Student Worksheets to facilitate discussion. During the discussion process, students work together to solve the problems given actively. Below is one example of the solution results given by students for didactical design 1.

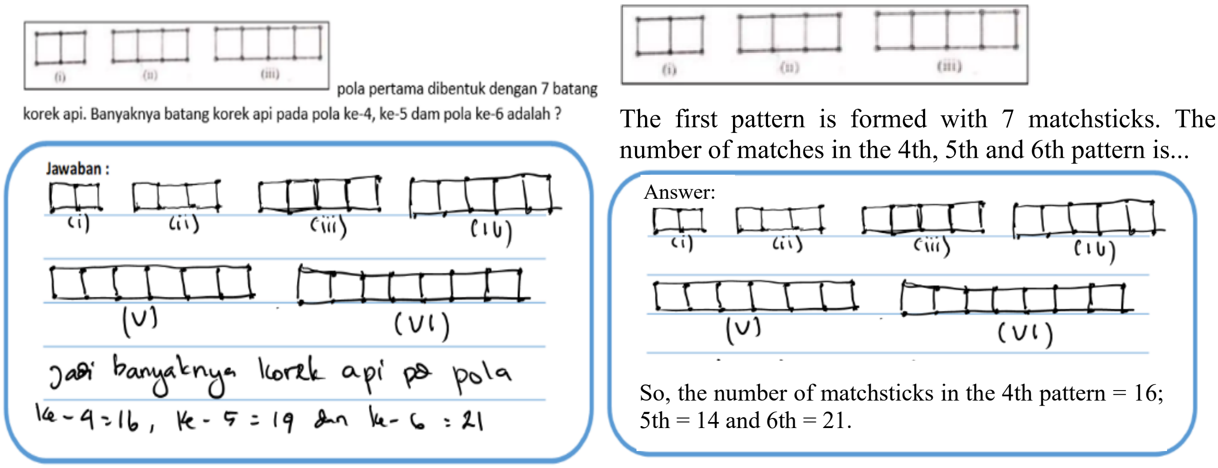
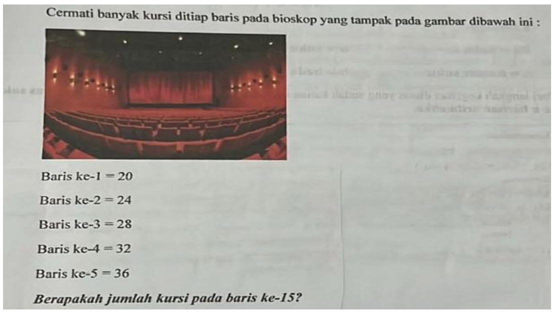


Figure 4. Appropriate answer to didactic design 1

The question in Figure 4 asks about number patterns illustrated in matchsticks. The questions were created to measure students' ability to understand number patterns. Almost all students gave similar answers and responded according to predictions. However, most still filled it in by continuing the image pattern, which took a long time despite the correct answer. The researcher anticipated this by directing students to find patterns in the number sequence. The conclusion on design 1 is that the design was implemented well, without revision, and students' knowledge increased.

Furthermore, in Hypothetical Didactic Design 2, researchers conducted an initial reflection before starting the learning process to minimize the obstacles experienced by students. Next, the researcher explained the definition and concept of Arithmetic and Geometric Sequences. At this stage of material delivery, students understand quickly. The researcher gave examples of problems related to Arithmetic Sequence and Geometric Sequence so that students know the difference between Arithmetic Sequence and Geometric Sequence. The following is an example of the solution results given by students for didactic design 2.



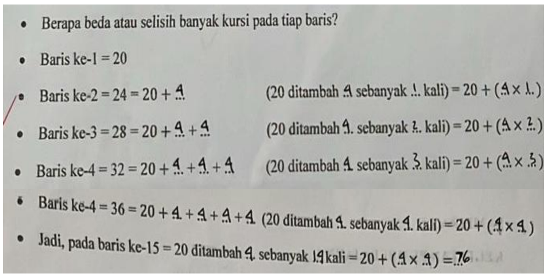
Cermati banyak kursi ditiap baris pada bioskop yang tampak pada gambar dibawah ini :

Baris ke-1 = 20
Baris ke-2 = 24
Baris ke-3 = 28
Baris ke-4 = 32
Baris ke-5 = 36

Berapakah jumlah kursi pada baris ke-15?

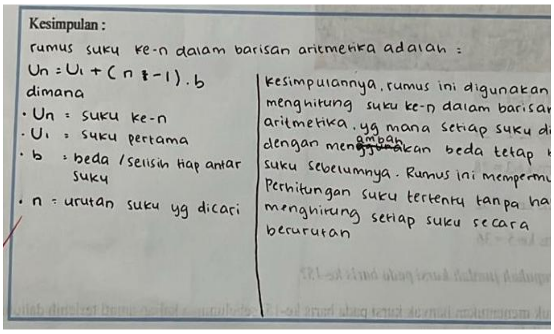
Look at the number of seats in the movie theater shown in the following figure:

1st row = 20
2nd row = 24
3rd row = 28
4th row = 32
5th row = 36
What is the number of seats in row 15?



- Berapa beda atau selisih banyak kursi pada tiap baris?
- Baris ke-1 = 20
- Baris ke-2 = 24 = 20 + 4 (20 ditambah 4 sebanyak 1 kali) = 20 + (4 x 1)
- Baris ke-3 = 28 = 20 + 4 + 4 (20 ditambah 4 sebanyak 2 kali) = 20 + (4 x 2)
- Baris ke-4 = 32 = 20 + 4 + 4 + 4 (20 ditambah 4 sebanyak 3 kali) = 20 + (4 x 3)
- Baris ke-4 = 36 = 20 + 4 + 4 + 4 + 4 (20 ditambah 4 sebanyak 4 kali) = 20 + (4 x 4)
- Jadi, pada baris ke-15 = 20 ditambah 4 sebanyak 14 kali = 20 + (4 x 4) = 76

- What is the difference in the number of seats in each row
- 1st row = 20
- 2nd row = 24 = 20 +4 (20 plus 4 for 1 time) = 20 + (4 x 1)
- 3rd row = 28 = 20 + 4 + 4 (20 plus 4 twice) = 20 + (4 x 2)
- 4th row = 32 = 20 + 4 + 4 + 4 (20 plus 4 for 3 times) = 20 + (4 x 3)
- 5th row = 36 = 20 + 4 + 4 + 4 + 4 (20 plus 4 for 4 times) = 20 + (4 x 4)
- So, 15th row plus 4 for 14 times = 20 + (4 x 14) = 76



Kesimpulan :
rumus suku ke-n dalam barisan aritmetika adalah :
$U_n = U_1 + (n-1).b$
dimana
- U_n : suku ke-n
- U_1 : suku pertama
- b : beda / selisih tiap antar suku
- n : urutan suku yg dicari

kesimpulannya, rumus ini digunakan menghitung suku ke-n dalam barisan aritmetika. yg mana setiap suku di... dengan menambahkan beda tetap ... suku sebelumnya. Rumus ini memperm... perhitungan suku tertentu tanpa ha... menghitung setiap suku secara berurutan

Conclusion:
The formula for the nth term in an arithmetic sequence is:
un = u1 + (n - 1).b
where
un: nth term
u1: first term
b: difference
n: the order of the terms being sought
In conclusion, this formula is used to calculate the e-n term in an arithmetic sequence where each term is different from the next. This formula takes into account a particular term without calculating each term sequentially.

Figure 5. Appropriate answers to didactic design 2

The question in Figure 5 describes the student's analysis in answering so that the pattern or formula of the arithmetic sequence is found. The answers given by the five students in the problem are the same; in other words, all students can understand the concept of Arithmetic and Geometric Sequences well. The conclusion on design 2 is that the design was implemented well, without revision, and students' knowledge increased.

The last design is Hypothetical Didactic Design 3, which researchers have done, namely designing the delivery of material sequentially and then providing practice questions. It is because researchers feel that this number line material is interrelated, so this is done so that the information received by students is not interrupted. Furthermore, to help students understand the material, researchers direct students to solve the problems in the Student Worksheet. The following is an example of an answer for didactical design 3.

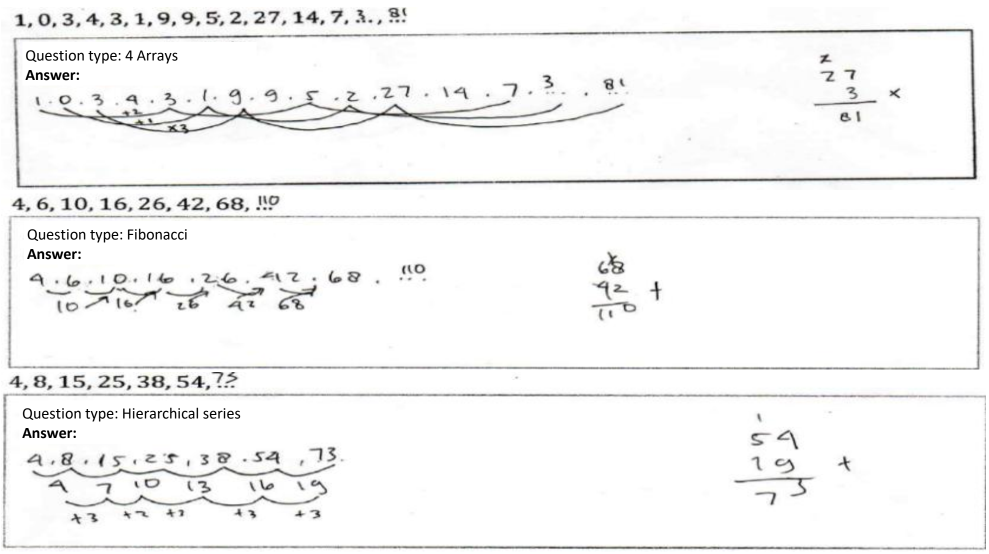


Figure 6. Correct didactic design 3 answers

The question in Figure 6 is about types of sequence patterns. The question is used to measure students in determining the type of sequence pattern and the following number. This question often appears in psychological tests when entering the industrial world. Three people gave the correct response according to the expected prediction; two others were wrong in determining the type of number sequence and the following number result. The researcher anticipated this difficulty by revising the type of questions given so that each type was completed first, starting from Array 1 - 4, Fibonacci, and level. The conclusion of Design 3 is that the design was implemented, the expansion of predictions and anticipations was revised, and the student understanding was increased.

Retrospective analysis

The researchers conducted a retrospective study to assess three executed hypothetical didactic designs. During the retrospective phase, the researcher correlates the findings from the study of the hypothetical didactic situation and metapedadidactic to formulate an empirical didactic design as an enhanced iteration of the initial version.

The meta-pedagogical study of hypothetical didactic design 1, centered on numerical operations and patterns, confirmed that the implementation proceeded as anticipated. Nearly all challenges were effectively foreseen as previously assessed. Hence, no enhancements were required for hypothetical didactic design 1. The meta-pedagogical analysis for hypothetical didactic design 2 concerning arithmetic and geometric sequences was successful. Nearly all challenges were effectively predicted based on prior assessments. Hence, adjustments for the hypothetical didactic design 2 were unnecessary.

Simultaneously, the findings from the meta-pedagogical analysis of hypothetical didactic design 3 concerning number sequences indicated that nearly all challenges were anticipated; however, there were instances where students erred in identifying the type of number sequence and in ascertaining the subsequent number in a sequence. This inaccuracy arises from pupils' challenges in comprehending the many sorts of number sequences offered. In light of these challenges, the hypothetical didactic design 3 was revised to address the emergence of additional issues encountered by students in problem-solving. In the preliminary design, students were instructed to ascertain the outcomes by identifying the subsequent number in each numerical sequence issue. This phase was revised to require students to identify the sort of number sequence initially, facilitating their ability to ascertain the subsequent number. Then, in the student worksheet, steps were added to solve the problem, where students had to check first, starting from array 1, array 2, and array 4; for the Fibonacci type, almost all students understood and could answer it. The initial didactic design framework generally only underwent slight modifications. This modification included a more precise sequence of work steps, more efficient learning time management, and increased prediction and anticipation of student responses to ensure that new difficulties did not arise and all difficulties could be adequately anticipated. This study also found that students needed to be reinforced again in their number arithmetic operations.

Discussion

Creating a didactic design to achieve successful mathematics learning is undoubtedly challenging, as numerous issues and inquiries may emerge. The research findings indicate that the proposed didactic design fulfills the specified criteria and positively influences students' mathematical problem-solving skills, particularly in number sequences. This study employs DDR to detect and address the learning difficulties encountered by students during educational activities (Fauzi & Suryadi, 2020). This learning hurdle is a crucial reference for researchers to ascertain students' conditions, which will inform their research. It aligns with Fauzia's (2017) assertion that the learning difficulties encountered by students must be considered in the development of didactic design. This didactic design research comprises three stages: the prospective analysis of the didactic situation prior to learning, represented by a hypothetical didactic design, and pedagogical didactic anticipation (ADP). ADP emphasizes overcoming learning obstacles that arise by the way teachers anticipate in the pedagogical and didactical relationship stage, which has been described in the metapedidactic analysis section, which results in a modification of the learning design in the third didactic design. The analysis of pedagogical didactic situations, or metapedadidactic analysis, and the retrospective analysis connect the hypothetical didactic situation analysis with the metapedadidactic situation analysis. In the prior number sequence study, it was determined that student learning faced impediments, including ontogenic obstacles (regarding the development of student cognition), epistemological obstacles (concerning student comprehension of material), and didactical obstacles (related to instructional methods).

The prospective analysis is an analysis of the didactical situation before learning in the form of hypothetical didactical design and pedagogical didactical anticipation (ADP). Improving the quality of classroom learning will align with teachers' moral and professional responsibility to provide fair and appropriate assessments through comprehensive information generated from prospective analysis (Scriven, 1990); this stage is critical in ensuring effective policy implementation (Yaakob et al., 2020). These learning impediments are responsible for students' difficulties in acquiring comprehensive knowledge. Hypothetical learning trajectories and didactic designs were developed to address student learning challenges and based on the researcher's findings regarding the presentation of material concepts aligned with the school curriculum, which was adapted for the job selection test through a process of didactic transposition. Didactic design emerges from research focused on learning obstacles, which are converted into a learning design (Pangestu Putri et al., 2020). Specific knowledge requirements in each ecosystem highlighted the need for knowledge transposition (Lombard & Weiss, 2018). Following the design and implementation of hypothetical learning trajectories and didactic frameworks, the findings indicated that the three didactic scenarios effectively facilitated the development of knowledge and skills related to number sequences.

In summary, there are no impediments to acquiring the notion of number sequences, including ontogenic obstacles (about students' cognitive capacity), epistemological hurdles (connected to student's comprehension of the content), and didactic obstacles (concerning teaching proficiency). To enhance the didactic design, the empirical didactic framework incorporates diverse questions, enabling students to expand their comprehension of solving numerical sequence issues.

The research conducted by Ardiansari et al. (2023) produced a didactic design that focused on student responses, sequences of activities, and didactic situations to encourage the learning process optimally. Therefore, it is essential to build alignment between conceptual images and students' concepts (Tsamir et al., 2015). Thus, when students are asked to explain the meaning of a particular concept, their success will depend on their previous learning experience (Jatisunda et al., 2024). Another thing was also obtained from the results of Fauzia's research (2017) on the didactic design of the arithmetic sequence concept that students must be given various exercises ranging from routine to non-routine questions so that students can increase their understanding and experience regarding the concept of arithmetic sequences and series. Teachers should also help students analyze number patterns so that students gain confidence. To ensure an efficient and effective learning process in number sequence, teachers should follow the research recommendations to align with the learning objectives (Jatisunda et al., 2024).

Conclusion

The didactic design mitigates learning obstacles in special number series within job selection psychological test questions effectively. Overall, student responses during the design implementation aligned with forecasts. However, there are situations where students have difficulty determining the type of number sequence and determining the following number in the number sequence. Anticipatory actions are taken based on student responses. The combination of the three hypothetical didactic designs that have gone through the revision stage and have been successfully implemented optimally is called empirical didactic design. Teachers can consider the teaching materials or materials used after knowing students' learning obstacles and learning trajectories in the number sequence material to minimize the emergence of student learning obstacles during the learning process.

Sequence material is not the only material tested in a psychological test to enter the job preparation. The implication of this research is a concept material for teachers in preparing learning designs on this sequence material according to learning obstacles and learning trajectories so that students can easily understand it.

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Conflict of Interest

The authors assert that there are no conflicts of interest pertaining to the publication of this manuscript. The writers have addressed ethical concerns, including plagiarism, infringement, falsification or fabrication of data, numerous publications or submissions, and redundancy.

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Author Contributions

Tio Heriyana: Conceptualization, writing - original draft, editing, and visualization; **Uba Umbara:** Writing - review & editing, formal analysis, and methodology; **Evan Farhan Wahyu Puadi:** Validation and supervision.

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