



How do junior high school students solve proportional reasoning problems?

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Abstract

Proportional reasoning is a crucial skill that students need to master to achieve success in learning mathematics at both the elementary and secondary levels. This study aimed to identify the strategies junior high school students used when solving proportional reasoning problems related to ratios and proportions. This qualitative descriptive research was conducted on six seventh-grade students at a junior high school in Belitang, selected through purposive sampling. Data was collected using tests and interviews and analyzed descriptively. Data validity was ensured through triangulation techniques. The findings revealed that students with low abilities used a repeated addition strategy to solve proportional reasoning problems. Meanwhile, students with medium abilities employed the multiplication strategy but occasionally made errors using the additive strategy—students with high abilities predominantly used multiplicative strategies. The study suggests that students with medium abilities do not consistently apply proper proportional reasoning strategies, highlighting a need for further research into their solution approaches. In addition, it is essential to design learning activities that specifically target the improvement of proportional reasoning skills for students with lower abilities, given the critical role this skill plays in mathematics education at both the primary and secondary levels.

Keywords: junior high school strategies; proportional reasoning problems; ratio and proportion

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Introduction

Proportional reasoning is the core point of many mathematics materials, especially in elementary and middle school mathematics curricula (Ayan et al., 2019; Izzatin, 2020; Tunç & Çakıroğlu, 2020). The development of students' mathematical abilities is greatly influenced by proportional reasoning, the standard for students' mathematical proficiency (Arican, 2018; Sari et al., 2023). Students must have proportional reasoning abilities to continue learning to a higher level (Cox & Root, 2020; Taufik, 2021; Utari et al., 2015; Weiland et al., 2021). Proportional reasoning is needed to understand and solve problems related to ratio and proportion (Khotimi et al., 2024; Sugiarni et al., 2024).

Proportional reasoning can be interpreted as a mental activity where students can find the relationship between two or more quantities (Prayitno et al., 2019). Meanwhile, Wahyuni (2022) and Muttaqin et al. (2017) define proportional reasoning as a mental activity that can understand changes in two quantities based on a multiplicative relationship. Lamon (2012) states that proportional reasoning uses multiplicative relationships to compare quantities and predict a quantity's value.

Students who can reason proportionally have specific characteristics. Lamon (2007) and Wahyuni (2022) agree that there are four characteristics of students with proportional characters, namely (1) understanding covariation, namely understanding the relationship between two quantities that vary with each other and being able to identify the variation of one quantity corresponds to another quantity, (2) recognizing proportional and non-proportional relationships, (3) developing many strategies to resolve proportions or ratios, (4) understanding ratios as separate entities. Meanwhile, according to Langrall and Swafford (2000), students who can reason proportionally have the following characteristics: thinking relatively, recognizing situations when using ratios correctly, understanding covariation, and forming units or using units to solve mathematical problems.

Proportion problems are generally divided into missing value problems and comparison problems. In missing value problems, students are given three out of four values in proportion and must determine the missing value. Meanwhile, students compare two ratios in comparison questions to determine whether they are equivalent or to assess which is larger or smaller (Arican, 2019; Oktaviani, 2019). Meanwhile, Johar et al. (2018) stated that the Rational Number Project developed three types of proportional reasoning problems: missing values, numerical comparisons, and qualitative predictions/comparisons.

There are various strategies that can be used to solve proportional problems. Proportional problems are problems related to the concepts of ratio and proportion. Chaim et al. (2012) stated that there are two strategies for solving proportional problems, namely formal strategies (using the proportional formula $\frac{a}{b} = \frac{c}{d}$, with $(a, b, c, d \neq 0)$) and pre-formal strategies such as additive strategy, intuitive, performing repeated additions, simplifying ratios, and finding unit values. Meanwhile, Johar divides proportional problem solving strategies into several groups, namely wrong strategies, including irregular calculations, determining the difference in solving problems (additive strategy), equality trial strategies; Correct strategies include repeated addition strategies, building up strategies by increasing the ratio or reducing the ratio, strategies

for simplifying the ratio to 1: m where m is an integer, factor of change strategies, unit value strategies, equality strategies, and calculation strategies using multiplication operations or division into known quantities (Lamon, 2020; Wahyuni, 2022).

Langrall and Swafford (2000) argue that there are four levels of students' proportional reasoning, namely Level 0 (Non-proportional Reasoning), where students use random strategies in solving problems and students are not able to identify the relationship between two quantities; Level 1 (Informal reasoning about proportional situations), where students can make representations in the form of pictures or models to understand the situation and make qualitative comparisons; Level 2 (Quantitative Reasoning), characterized by the use of scale factors or tables and using fractional similarities; Level 3 (Formal proportional reasoning), where students can use cross multiplication or fraction similarity solutions to solve proportional problems (Wahyuni, 2022).

Inhelder and Piaget classified the development of proportional reasoning into three stages: (1) the intuitive stage (ages 3-7 years). During this stage, children think intuitively and struggle to connect two quantities effectively; (2) concrete stage (Ages 8-12), children can relate two quantities based on real-life experiences. However, they cannot generalize rules or properties or understand formal mathematical principles. Students are also able to solve simple proportional problems; (3) the formal stage (ages 12-15) is characterized by understanding proportional relationships and the use of formal formulas as a tool for solving proportional problems efficiently (Chaim et al., 2012).

Previous studies related to proportional reasoning have been widely conducted, including the identification of solution strategies for proportional problems at the high school level (Khotimi et al., 2024), across various school levels (Cansız Aktaş, 2022), and misconceptions in proportional reasoning (I et al., 2018; Im & Jitendra, 2020). In contrast, this study focuses on the strategies junior high school students use to solve proportional reasoning problems.

The problem formulation in this research is: What is the strategy of junior high school students in solving proportional reasoning questions on ratio and proportion material? This research aims to determine what strategies junior high school students use to solve proportional reasoning questions on ratio and proportion material. The results of this research will provide new knowledge in mathematics learning regarding middle school students' strategies for solving proportional reasoning questions on ratio and proportion material. Teachers must understand students' different ways of solving a problem. Hence, they know what learning methods should be used if it is discovered that a student's problem-solving strategy is not appropriate.

Methods

This descriptive qualitative research aimed to uncover the strategies employed by junior high school students in solving proportional reasoning problems related to ratio and proportion topics. The study involved six 7th-grade students from one junior high school in Belitang with varied mathematical abilities (two high with code HS-01, HS-02; two medium with code MS-01, MS-02; and two low with code LS-01, LS-02). The limited subject selection of six students aims to obtain in-depth and varied data according to the characteristics of each student.

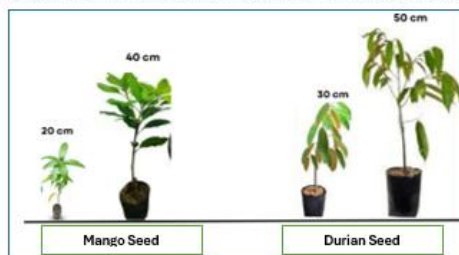
Researchers focus on exploring students' thinking processes and responses to the problems given, not on generalizing the results. The selection of research subjects used a purposive sampling method, where mathematics teachers were consulted to select students based on their differing levels of mathematical ability. From the two subjects for each ability, the researchers chose one subject with the correct answer and who could represent the answer from his ability category.

Data collection was carried out through tests and interviews. The test was designed to determine the strategies students used in solving proportional reasoning questions, while interviews were conducted to validate and strengthen the findings from the test results. The test included two descriptive questions that covered all indicators of proportional reasoning ability as outlined by (Langrall & Swafford, 2000). The questions were adapted from Wahyuni's (2022) book, *Proportional Reasoning*, which established credibility. The test was conducted at the beginning of data collection. Then, the researchers analyzed the test results based on the students' ability categories and the similarity of the answer patterns of each group of students with heterogeneous abilities. Then, the researcher conducted in-depth interviews with six research subjects. The collected data from the tests and interviews were analyzed descriptively and qualitatively to identify the strategies used by the students to solve proportional reasoning problems involving ratios and proportions. To ensure the validity of the data, a triangulation technique was applied by comparing the results of the tests with the responses from student interviews about the proportional reasoning problems they were given.

Results

The research was conducted by administering two proportional reasoning test questions related to ratio and proportion material. These test questions were presented descriptively to identify students' strategies to solve proportional reasoning questions. Students who are used as subjects and analyzed more deeply are students with different abilities who can answer questions correctly using their methods; in this case, low-ability students are represented by LS-01 students, medium-ability MS-02 represents students, and HS-01 represents high-ability students. The questions used in this research can be seen in Figure 1.

- 1 Mrs. Dewi's favorite fragrance oil is made by mixing 4 ml of rose oil and 3 ml of lavender oil. If 28 ml of rose oil is used, then calculate the amount of lavender oil that must be mixed to produce a fragrance oil according to Mrs. Dewi's preferences..
- 2 Mr. Beni planted mango and durian seeds together in his back garden. The following is a picture of the growth of the two seedlings after one month of planting.



Based on initial height, which seedling grows faster? Explain your reasons!

Figure 1. Proportional reasoning test problems

In question number 1 as shown in Figure 1, the question is a question of proportional reasoning in comparative form. The results of the proportional reasoning test show that there are three types of solving strategies that students use to answer question number 1. The first strategy can be seen in the steps for solving LS-01 students' proportional reasoning questions which are presented in Figure 2.

$$28\text{ml} - 21\text{ml}$$

$$9 + 9 + 9 + 9 + 9 + 9 + 9 = 28\text{ml}$$

$$3 + 3 + 3 + 3 + 3 + 3 + 3 = 21\text{ml}$$

Figure 2. Answer to question number 1 LS-01 students

Based on Figure 2, it can be seen that students solve proportional reasoning questions on proportion material using a repeated addition strategy. Students add 4 ml of rose oil until they get a result of 28 ml. Next, LS-01 students did the same thing, namely by adding 3 ml of lavender oil as many times as the addition of rose oil was repeated to obtain 21 ml of lavender oil which had to be mixed to produce a fragrance according to Mrs. Dewi's preferences. The results of the interview with subject LS-01 are described as follows.

- R : What is your strategy for answering question number 1? Try to explain.
 LS -01 : I answered by adding 4 ml until I found the result of 28 ml ma'am, then 3 ml was added as well as 4 ml. So found the answer is 21.
 R : Are you sure about your answer?
 LS -01 : Hmmm. Yes mom.

Based on the results of the interview above, information was obtained that strengthens the findings in the test results, so it can be seen that LS -01 subjects who have low mathematical abilities solve proportional reasoning questions using a repeated addition strategy.

Furthermore, the second strategy used by MS-02 students to solve proportional reasoning question number 1 presented in Figure 1 can be seen

$$= 21\text{ml}$$

$$= 7 \times 3 = 21\text{ ml minyak lavender}$$

$$= 7 \times 4 = 28\text{ ml minyak lavender maruar}$$

Translation:

$$= 21\text{ ml}$$

$$= 7 \times 3 = 21\text{ ml of Lavender oil}$$

$$= 7 \times 4 = 28\text{ ml of Rose oil}$$

Figure 3. Answer to question number 1 MS-02 students

Based on Figure 3, it can be seen that students solve proportional reasoning questions on proportion material using a multiplication strategy. Students use the information in the questions to solve the problem. In this question, it is known that 4 ml of rose oil was used and 3 ml of lavender oil. Furthermore, it was discovered that the rose oil used was 28 ml, so students used a multiplication strategy to find what number, if multiplied by 4 ml, the result was 28 ml. From this thinking, students remember that the product seven by four is 28. Because rose oil is

multiplied by 7, lavender oil is also multiplied by 7 so that the answer is 21 ml of lavender oil. The results of the interview with subject M2-01 are described as follows.

- R : *Explain how you would answer question number 1.*
 MS-02 : *The way I look for the multiplication of 4, the result is 28 ma'am, I find $4 \times 7 = 28$, then the 3 is also multiplied by 7 so that the fragrance doesn't change so the number is the same.*
 R : *Do you think your answer is correct or not?*
 MS-02 : *Yes ma'am, it's correct in my opinion.*

Based on the results of the interview above, it is known that MS-02 students with moderate mathematical abilities use a multiplication strategy to solve the proportional reasoning questions given. The interview results also strengthen the findings on the MS-02 student test results. So it can be concluded that subjects with moderate ability solve proportional reasoning problems using a multiplication strategy.

The strategy used by high ability students to solve proportional reasoning question number 1 can be seen in the answers to the HS-01 student test results which are presented in Figure 4.

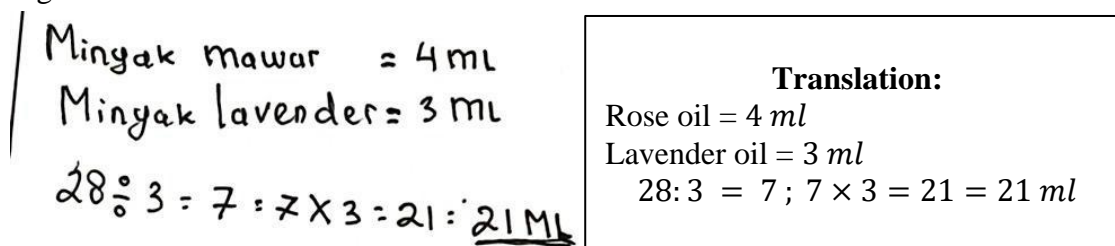


Figure 4. Answer to question number 1 HS-01 students

Based on Figure 4, it can be seen that students solve proportional reasoning questions on proportion material using a division strategy. Students divide the two pieces of information in question, namely dividing 28 ml by 3 ml and get the result 7. Based on the solution steps written by the students on the test results sheet, there is a calculation error made by the students, namely 28 divided by 3, the result is 7. This step is the steps were not quite right even though in the end the student got the correct result, namely 21 ml. To confirm the answers on the test result sheet, researchers confirm the test answers through interviews. The following are the results of interviews with subject HS-01.

- R : *Explain your strategy for solving problem number 1.*
 HS-01 : *In question, the mixture is 4 ml of rose oil, 3 ml of lavender oil. Then the rose oil makes 28 ml, ma'am, so I divide 28 ml by 4 ml, $28 : 4$, the result is 7. So for the lavender oil, I multiply it by 7 and get the result 21 ml, ma'am.*
 R : *Why do you multiply by 7?*
 HS-01 : *Because you got the quotient of $28 : 4$ which is 7, so the rose oil is multiplied 7 times, ma'am, so the 3 ml of lavender oil is also multiplied by 7 to match the fragrance of the perfume, ma'am.*
 R : *Let's look at your test results (showing the test answers), why is it $28 : 3$? Which one is correct?*

HS-01 : *Oh yes, ma'am, I was wrong on that one, ma'am, the correct one was divided into 4, the rose oil was 4 ml, ma'am. Sorry, ma'am, I wasn't careful.*

Based on the results of the interview above, it is known that students can solve proportional reasoning questions using their own strategy, namely the division strategy. Students can correct the answers they write during the test and give correct answers during the interview. So, based on the results of tests and interviews, it can be concluded that the strategy used by HS-01 students with high mathematical abilities is the division strategy.

Furthermore, for the answer to question number 2, the answers of students with low abilities can be seen from Figure 5 below.

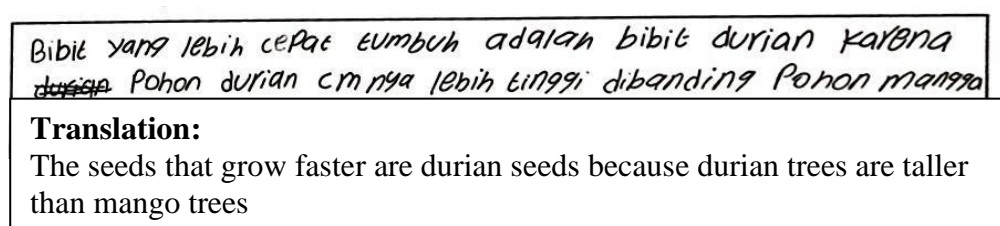


Figure 5. Answer to question number 2 LS-01 students

In Figure 5, it can be seen that students with low ability answered questions containing a comparison of the growth of two plants by looking at which plant was the tallest. Based on question number 2 in Figure 1, students answered that the plant seeds that grow faster are durian seeds because they are the tallest seeds. Excerpts from interviews with subject LS-01 regarding question number 2 are presented as follows.

R : *Look at the picture in question number 2. What do you think is the answer?*
 LS-01 : *Durian ma'am..*
 R : *Why durian? How do you answer question number 2?*
 LS-01 : *This is the picture of the tallest durian tree ma'am, if it's the tallest it means it grows faster ma'am compared to the others.*
 R : *Are you sure about your answer?*
 LS-01 : *Yes ma'am.*

Based on the results of the interview above, it can be seen that students solve questions by looking at which plant seeds are the tallest, those are the seeds that grow faster. The answers of students with medium abilities can be seen in Figure 6.

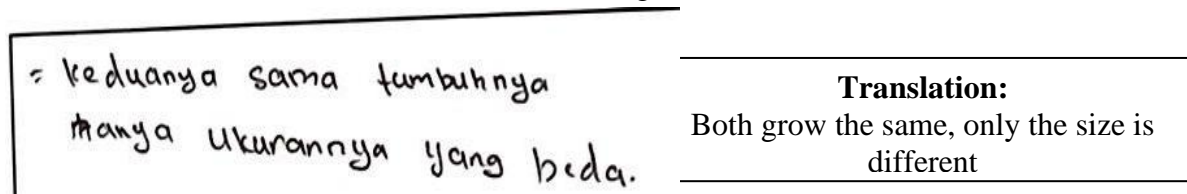


Figure 6. Answer to question number 2 MS-02 students

In Figure 6, it is known that students with medium ability answered that the growth of the two seedlings was the same, only the size was different. To obtain more comprehensive answers

and explanations, researchers conducted interviews, the results of interviews with MS-02 subjects are described as follows.

- R : *For question number 2, what do you think is the answer? Try explaining.*
 MS-02 : *The answer is, nothing grows faster ma'am, both have the same growth, if mango grows from 20 cm to 40 cm, if durian grows from 30 cm to 50 cm. Both of them have grown by 20 cm, ma'am. So both growths are the same.*
 R : *Are you sure about your answer?*
 MS-02 : *Yes ma'am, that's right...*

Based on the results of the interview above, it is known that students analyzed the questions and found out that the initial height of the mango seedlings was 20 cm and then grew to 40 cm, a growth of 20 cm occurred. In durian seedlings, the initial height is 30 cm and then grows to 50 cm, with growth of 20 cm also occurring. This is what underlies students' answers that the two seeds have the same growth.

The answers from students with high abilities for question number 2 can be seen in Figure 7 below.

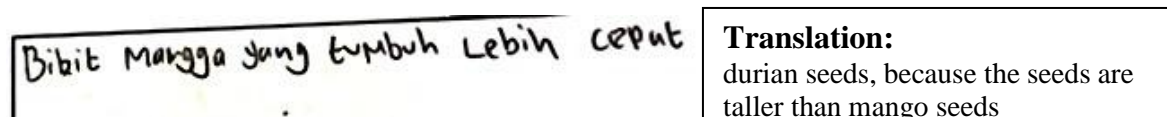


Figure 7. Answer to question number 2 HS-01 students

Based on Figure 7, it is known that students with high abilities answered that mango seeds grow faster than durian seeds. To clarify the answers to the test results, researchers conducted interviews with HS-01 subjects. The following is a summary of the results of the interview with the HS-01 subject.

- R : *What do you think is the correct answer to question number 2?*
 HS-01 : *Mango seeds ma'am.*
 R : *Why mango seeds? Try to explain how you got that answer.*
 HS-01 : *Because mangoes grow faster, from the initial 20 cm they continue to grow to 40 cm, that's double the growth ma'am, the durian doesn't grow twice as fast. So yes, mango is the answer.*

Based on the results of the interview above, it is known that from the analysis carried out by high ability students, the initial height of the mango seedlings was 20 cm and then grew to 40 cm. A growth of 20 cm occurred, meaning the mango seedlings grew 2 times the initial height. Meanwhile, for durian seedlings, the initial height was 30 cm and then grew to 50 cm. There was also growth of 20 cm, however, durian seedlings experienced slower growth because the growth of durian seedlings was less than double the initial height. So students get the answer that mango seeds grow faster. This shows that students use relative thinking strategies by using multiplication to answer question number 2.

Discussion

Based on the results of the proportional reasoning test conducted on six seventh-grade students at SMP Negeri 1 Belitang with varying abilities (2 high, two medium, and two low-ability students), it was discovered that the strategy used by students in the low-ability category to solve proportional reasoning problems was the additive strategy. It is evident from the students' answers to question number 1, which used repeated addition to solve proportional reasoning questions about proportions. In question number 2, students with low ability answered the question by looking at the differences in the height of plant seeds and thinking that the tallest seeds were the ones that grew faster. This answer is not appropriate for question number 2. It proves that using an additive strategy (by adding or subtracting) is not appropriate for solving problems regarding proportional reasoning. The results of this research strengthen the findings of Nugraha et al. (2016) that the mistakes of class VII junior high school students in working on proportional reasoning questions were due to the tendency to use additive methods. The results of this research also align with the findings of Nur and Sari (2022) that when working on proportional reasoning questions, students tend to use additive rather than multiplicative strategies.

The strategy used by students with moderate abilities is the multiplication strategy. It can be seen from the students' answers to question number 1 that students solve proportional reasoning questions on proportion material by finding multiplication numbers that match the information in the question and finding the correct answer from the multiplication results. Next, for question number 2, students with moderate ability answered the question using the additive method, namely by finding the difference in seedling height growth after one month of planting and the initial seedling height. From calculating the difference, it was found that the difference in the growth of the two plant seeds was the same, so students of medium ability concluded that they had the same height growth. The solution method used to find the difference includes an additive strategy. The results of medium ability students' answers to question number 1 and question number 2 were different, and students did not consistently use strategies for solving proportional reasoning questions. The answer to question number 1 uses a multiplication strategy and produces the correct answer.

In contrast, the answer to question number 2 uses an additive strategy (calculating the difference) and produces an incorrect answer. The results of this research are the opinion of Chaim et al. (2012) that students who choose to use an additive strategy to work on ratio and proportion questions, namely by calculating the difference between the two parts of the ratio, will get the wrong answer. The results of this research are also related to Wahyuni's (2022) opinion that the wrong strategy in solving proportional reasoning problems is to determine the difference in solving the problem (additive strategy). It is what makes students' abilities wrong when answering question number 2.

As for students with high abilities, based on the analysis of the results of the proportional reasoning test, in question number 1, the students answered using a division strategy. Students divide the information in the question to determine how many times to add rose oil. Meanwhile, to solve problem number 2, students with high abilities used a multiplication strategy to solve

the problem. They concluded that mango seedlings grew twice their initial height when planted, and durian seedlings grew slower, less than double their initial height when planted. The use of division and multiplication operations in solving proportional reasoning problems indicates that students use multiplicative strategies in solving problems. The results of this study align with the opinion of Anat et al. (2019) that proportional reasoning is related to the mathematical relationship of multiplication. So, to solve proportional reasoning problems, in order to get the correct answer, you have to use multiplication or division operations on known quantities (Wahyuni, 2022).

The findings of this study support previous research indicating that one of the main causes of students' errors in solving proportional problems is the use of additive reasoning or strategies. It aligns with the findings of (Dougherty et al., 2017; I et al., 2018; Im & Jitendra, 2020), who all emphasized that misconceptions in proportionality often arise when students mistakenly apply additive relationships instead of multiplicative ones in solving proportional reasoning tasks. Im and Jitendra's 2020 research showed that students' errors in proportional problems stemmed from using additive strategies. Similarly, I et al. (2018) revealed that students' difficulties in solving proportionality problems were due to reasoning based on additive concepts. This statement is further supported by Dougherty et al. (2017), who asserted that one common misconception in proportionality is the inappropriate application of additive rather than multiplicative reasoning in proportional contexts.

Conclusion

The strategy used by low-ability students to solve proportional reasoning questions is the repeated addition strategy; students with medium ability solve proportional reasoning questions using a multiplication strategy and still use additive strategies to solve proportional reasoning questions on ratio material. Moreover, proportion and students can solve proportional reasoning questions using a multiplicative strategy: division and multiplication operations. Based on the findings of this research, the problem-solving strategy for students with medium abilities uses two different strategies; for this reason, the researchers suggest that in future research, they can carry out more in-depth research regarding the strategies used by students with medium abilities.

This study was limited by the small number of participants, consisting of only six students with varying levels of mathematical ability. As a result, the findings may not be generalizable to the wider population of junior high school students. Additionally, the data were collected in a specific classroom context, which may influence how students approach and solve proportional reasoning problems. Despite these limitations, the study provides valuable insights into students' different strategies and highlights the importance of recognizing individual differences in reasoning. These findings inform classroom instruction by encouraging teachers to design tasks that cater to diverse student abilities and promote a deeper conceptual understanding of proportionality.

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Conflicts of Interest

The authors declare no conflict of interest regarding the publication of this manuscript.

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Author Contributions

FilianYunita Sari: Conceptualization, writing - original draft; **Zulkardi:** Editing, validation and supervision; **Ratu Ilma Indra Putri:** Validation and supervision; **Ely Susanti:** Validation and supervision.

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