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A local instructional theory for learning the concept of arithmetic sequence through *pocah piring* game

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Abstract

The nth-term formula is an arithmetic sequence subtopic that contributes significantly to explaining the fundamental concept of an arithmetic sequence. This study aimed to design a Local Instructional Theory (LIT) based on Realistic Mathematics Education (RME) to find the nth-term formula in an arithmetic sequence through the Pocah Piring game. This study was conducted on eighth-grade students in Palembang, Indonesia. This study used educational design research with a validation study, starting with a preliminary design, design experiments, and retrospective analysis. The learning trajectory in this study begins with rich circumstances that require mathematical reasoning rather than abstraction or exact definitions. Three structured learning activities integrating the Pocah Piring game were applied to help the students explore the concept of an arithmetic sequence. To effectively teach arithmetic sequences, instruction should progress from concrete representations, such as physical manipulatives and visual patterns that help students recognise sequential structures, to more abstract forms, including symbolic expressions and general formulas. The developed LIT accurately predicted students' thinking throughout the learning process, supporting their progression toward the intended learning objectives. Teachers are encouraged to use the LIT derived from this study to guide instruction and design alternative learning pathways when teaching arithmetic sequences.

Keywords: arithmetic sequence; educational design research; learning trajectory; local instructional theory; pocah piring game; realistic mathematics education

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Introduction

Education research has so far been viewed as a disappointment, as it only indirectly affects the improvement of instructional practice strategies (Gravemeijer et al., 2017), with limited application outside of the research's purview, making it seem distinct from direct instructional strategies (Howard-Jones et al., 2025; Prahmana, 2017). In general, educational research-supported experiments, practices, and programs have only marginally outperformed traditional methods (Akker et al., 2006; Haagen-Schützenhöfer & Hopf, 2020). Therefore, large-scale, quantifiable teaching methods or student learning advances are necessary in educational research. Specifically, in mathematics education, learning algebra has been a concern, ranging from a purely procedural mathematical perspective to complex mathematical abstractions (Hausberger, 2020; Zazkis & Liljedahl, 2002). The gap between students' knowledge and the abstract formal mathematical knowledge that must be acquired and the disparity between students' thinking perspectives are also reasons why students find it difficult to learn mathematics (Gravemeijer, 2008; Gravemeijer & Doorman, 1999; Schoenfeld, 2022). This gap often occurs during the transition from arithmetic to algebra, especially in arithmetic operations on algebraic expressions, number patterns, and number sequences, including arithmetic sequences (Jupri, 2022).

As one of the branches of mathematics that is considered as a difficult learning domain for students, the material of arithmetic sequences remains one of the topics of mathematics learning in both junior and senior high schools (Rachma & Rosjanuardi, 2021). Arithmetic Progression, or AP, is a series in which each new term after the first term is obtained by adding a constant b, called the common difference, to the previous term (Siemon, 2021; Zazkis & Liljedahl, 2002). If the first term of the series is a, then the arithmetic series is $a, a + b, a + 2b, a + 3b, ...$ where the nth term is $a + (n - 1)b$. More formally, we can state that for all a_i in an arithmetic sequence in any integers, $a \equiv c(mod\ b)$ where c is the common remainder and b is the difference (Siemon, 2021; Zazkis & Liljedahl, 2002). When selecting representations and materials for interactive arithmetic instruction, teachers must consider a variety of possible student interpretations (Drijvers et al., 2021). Therefore, mathematics instruction must emerge from contexts that are meaningful to students and build on their informal knowledge into formal knowledge (Mangelep et al., 2023; Fredriksen, 2021; Wijaya & Doorman, 2021; Quintero & Rosario, 2016). Teaching based on meaningful contexts allows students to connect their learning to their ideas and experiences (Fauzan et al., 2024). According to Domu and Mangelep (2020), a learning implementation plan will be effective if it includes predictions regarding the probability of student learning. It is related to developing students' thinking skills and understanding during teacher-designed learning activities.

In this context, conducting research based on the Design Research within the Realistic Mathematics Education (RME) approach is essential for addressing educational practice issues, notably in the teaching arithmetic sequences material (Putri et al., 2023; Jupri et al., 2022). Focusing on learning processes, distinctive design research aims to create theories and educational interventions (Gravemeijer, 2016). Since the mathematization of reality forms the foundation of mathematics, learning should begin with engaging students in this process (Van

Zanten & Van den Heuvel-Panhuizen, 2021). In this research, the RME theory is used because it is considered to be successful in guiding the design of learning materials for students in the Netherlands (Van den Heuvel-Panhuizen & Drijvers, 2020; Van Den Heuvel-Panhuizen, 2005) and has been adapted in Indonesia (Putri, 2024; Azizah et al., 2023; Zulkardi, 2020; Bustang et al., 2013). Additionally, the RME approach has been shown to significantly improve students' cognitive processes in understanding mathematical concepts, particularly arithmetic sequences (Jupri et al., 2022; Domu & Mangelep, 2020; Isik et al., 2020; Gee et al., 2018) because RME incorporates guided reinvention, didactical phenomenology, and the mediating model's principle (Ventistas et al., 2024; Doorman, 2019; Zulkardi, 2013; Sembiring et al., 2008).

Previous studies have successfully conducted local instructional theory (LIT) with RME in other areas of mathematics, such as probability (Fauzan et al., 2022), translation and reflection (Rawani et al., 2023), parallelogram (Zulfah et al., 2024), as well as different cultural contexts (Leton et al., 2025; Sahara et al., 2024; Risdiyanti & Prahmana, 2021). However, only limited literature or research uses local games as the context of the learning process (Qirom & Juandi, 2023), while Indonesia has many local games grounded in students' daily lives (Pramono et al., 2024), particularly for teaching abstract concepts such as arithmetic sequences. As none of the previous research has yet used the *Pocah Piring* game as the context in the arithmetic sequence learning process, this study proposes adopting the *Pocah Piring* game to create a local instructional theory for learning the concept of 'finding the formula for the nth term of an arithmetic sequence' using the Realistic Mathematics Education (RME) approach. The primary aim of this study is to design a local instructional theory (LIT) for teaching arithmetic sequences based on the RME approach, using the traditional *Pocah Piring* game as the context.

Pocah Piring is a traditional game that involves two competing groups. Etymologically, '*Pocah'* means 'breaking,' and '*Piring*' means 'plates.' As the name suggests, the game revolves around breaking a stack of objects called 'plates' arranged in a high pile. These 'plates' are typically objects that are easy to fall when stacked, such as stones, broken coconut shells, floor tiles, asbestos, bottle caps, or similar items (Kamaluddin et al., 2020). The number of "plates" used can vary based on the players' agreement, but the higher the stack, the more challenging and exciting the game becomes. One group acts as the guard, while the other serves as the leading group tasked with disrupting the arrangement of "plates" (Effendi et al., 2023). The leading group begins by throwing a ball to break the stack, after which they must attempt to reassemble the "plates" while the guard group throws the ball to disrupt their efforts. If a player is hit by the ball while reassembling, their turn ends, adding a layer of strategy and excitement to the game (Nasution & Siregar, 2017).

The selection of *Pocah Piring* over other contexts is based on several factors. First, the game is deeply embedded in local cultural practices, making it familiar and engaging for students. This familiarity can increase motivation and participation (Yu et al., 2021), as students can relate the game's activities to their own experiences (Inci et al., 2023; Sulistyanto et al., 2023; Vankus, 2021). Additionally, the game's physical nature, which involves stacking, breaking, and rearranging objects (Effendi et al., 2023), provides a tangible way for students to observe and analyze patterns, which is crucial for understanding arithmetic sequences.

Furthermore, the game's dynamic nature, as students incrementally add or remove objects, naturally introduces the concept of difference between consecutive elements, a core idea in arithmetic sequences. Through this engagement, students can better grasp the idea of constant differences in a sequence as students incrementally add or remove objects, creating a foundation for understanding the nth-term formula. The richness of this context lies in its ability to bridge students' informal, everyday experiences with formal mathematical concepts. It is hoped that *Pocah Piring*, as an educational tool, will bridge abstract mathematical concepts and students' daily activities, enabling them to establish meaningful connections and achieve improved academic performance in arithmetic sequences.

Methods

This study used educational design research with validation study type, which was an proper way to achieve research objectives, started with a preliminary design phase, design experiments phase, and retrospective analysis (Gravemeijer & Cobb, 2006). This research was conducted in Palembang with eight-grade students. The steps in this research can be seen in Figure 1 :

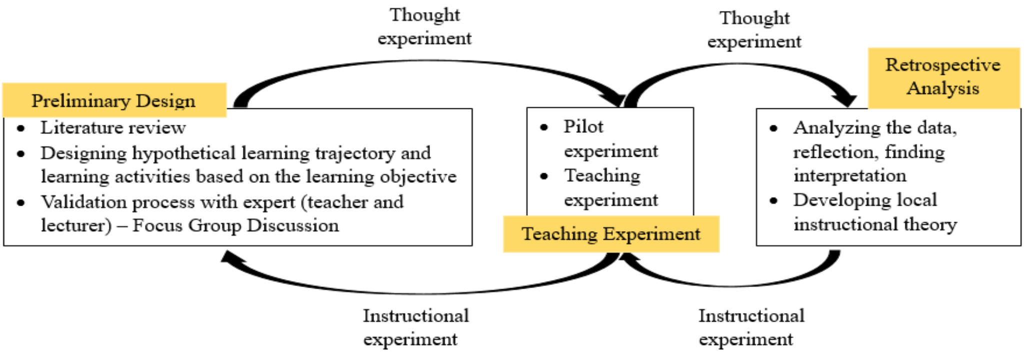


Figure 1. Design research method according to van den Akker et al. (2006)

The subject were eighth-grade students from a junior high school in Palembang. A total of 42 students were involved: 6 students participated in the pilot experiment, and 36 students participated in the teaching experiment. Data collection methods included student worksheets, teacher observations, and student interviews. Data was analyzed using qualitative descriptive. Observations and student work were analyzed to uncover the reasoning behind students' problem-solving processes and their conceptual understanding.

In the preliminary design phase, a literature review was conducted on arithmetic sequences and the development of RME-based learning activity sheets. The study's results consisted of an activity designed to achieve learning objectives related to arithmetic sequences for eighth-grade students. This design was developed for each stage of learning and included conjectures about students' activity trajectories in understanding the concept of the nth-term formula of an arithmetic sequence. Additionally, a focus group discussion with lecturers (the expert judgment), researchers, and mathematics teachers about the conditions of the students, their learning needs, and the activity sheets created was conducted as part of a validation procedure.

Next, in the design experiment stage, two trials of student activity sheets that had been designed were carried out in two stages. The first stage was a pilot experiment involving six students with high, medium, and low abilities. This stage aimed to revise the learning activity sheets that had been designed, obtain feedback for improving the Hypothetical Learning Trajectory (HLT), and evaluate the initial feasibility and implementation of activity sheets. After adjustments were made, the next stage was a teaching experiment involving 36 students, which was used to test the outcomes of the revision in order to investigate and hypothesize about the students' strategies and thinking processes during the actual learning process.

After completing the pilot and teaching experiments, the data collected from observations, student work, and feedback were analyzed retrospectively. The retrospective analysis involved identifying patterns in student learning, reflecting on the outcomes, and interpreting the data. This analysis aimed to develop a local instructional theory and evaluate the learning activities that had been implemented.

Results

Preliminary design

During this phase, researchers discussed with a model teacher to refine the research plan and reviewed a joint draft created and focused on incorporating the traditional game '*Pocah Piring*' into the study. The relationship between learning stages, learning activities, and basic concepts is presented in Figure 2.

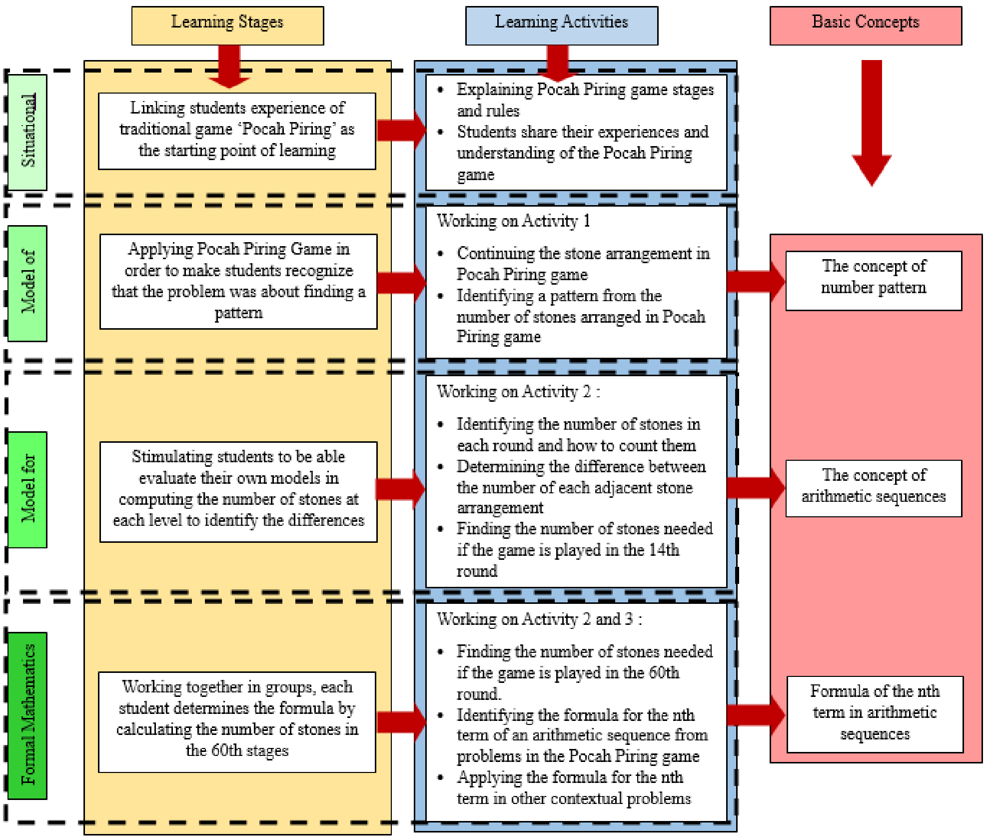


Figure 2. The relationship between learning stages, learning activities, and basic concepts

The teacher and researchers determined the material to be selected, specifically an arithmetic sequence, while considering adjustments based on the Merdeka curriculum. The researcher and teachers also discussed student activity sheets, research instruments, and the RME approach and conducted observations of students in the selected classroom who would serve as research subjects in the experimental design stage. The researchers discussed with the model teacher to further comprehend the students' abilities and conditions. It was discovered during the model teacher's interview that the arithmetic sequences content was taught in eighth-grade.

Based on Figure 2, the learning stages are arranged to align with students' experiences and cognitive development. At the 'situational stage,' students are introduced to the traditional game *Pocah Piring* to connect their experiences with the concept of number patterns. Furthermore, in the 'model of' stage, students begin to identify patterns in the arrangement of stones in the game, realizing that there is a regularity in the change in the number of stones in each round. At the 'model for stage,' students analyze the differences between the arrangements and create their mathematics models to determine the pattern of changes in the number of stones. Finally, at the 'formal mathematics stage,' students work collaboratively to compile a general formula for the nth term in an arithmetic sequence and apply it to predict the number of stones in the 60th round or other rounds. Furthermore, the HLT design that had been developed is depicted in Table 1.

Table 1. Design of HLT

Aims	Activity	Conjecture of Students' Thinking
To understand the concept of nth-term formula in arithmetic sequence	Activity 1 : A glimpse of Number Patterns (See Table 2) After you know one of the traditional games from Batak Toba, try to look at the picture of the stones used as playing tools for pocca piring below. Continue the arrangement of the stones below and count the number of the next stones 	• Students can continue the number of stone arrangements by drawing and writing the number. • Students can identify whether the number of stone piles in each round forms a number pattern and provide reasons for their statements. • Students can determine the next three numbers from the number sequence in question.
	Activity 2 (Question 1) : Find the Number of Stones in Each Round (See Table 3) **Let's Explore** **Traditional Pocca Piring Game from Batak Toba** A group of children play Pocca Piring on a field with different stone arrangements in each round. The following is the arrangement of stones used by the children during the game: 	Students can find out the number of stones in each round

Aims	Activity	Conjecture of Students' Thinking
	Activity 2 (Question 2): Find the Difference Between Two Piles of Stones From Adjacent Rounds (See Table 4)	• Students can determine the 'difference' between each round by calculating the difference in the number of stones used in the next round and the previous round. • Students can identify the concept of 'difference' in arithmetic sequences. • Students can provide a generalization of the 'difference' formula in arithmetic sequences that have been found.
	Activity 2 (Question 3): find the number of 14th stones (See Table 5)	• Students can find the number of stones used in the 14th round • Students can reason and model problems into mathematical models • Students can find the formula for the nth term of an arithmetic sequence
	Activity 2 (Question 4) : Find the number of the 60th stones (See Table 6)	• Students can manipulate simple algebraic calculations • Students can provide mathematical modeling of problems using their reasoning skills • Students can provide conclusions from their findings on the formula for the nth term of an arithmetic sequence from the given problem
	Activity 3 : Applying the Formula to a Real-World Problem (See Table 7)	• Students can understand the concept of the nth term formula in an arithmetic sequence • Students can use the nth term formula of an arithmetic sequence that they have understood to solve real-world problems.

Design experiment

There were two cycles in the design experiment stage: the teaching and pilot experiments. Six students with high, medium, and low ability levels took the HLT test during the pilot trial phase. In the experimental teaching phase, the outcomes of the pilot experiment phase were examined and tested. Changes in HLT were influenced by strategies that had not been implemented, events in the class that could not be anticipated, and activities that were too difficult to implement. Recognizing the significance of these influences, an HLT adjustment was made to provide the ideal conditions for the data that will be examined.

Based on the Hypothetical Learning Trajectory (HLT) created, the researcher tested the draft during the design experiment stage after consulting with the lecturer and teacher. This research developed a learning trajectory about 'finding a formula of the nth-term of an arithmetic sequence' through several learning activities for eighth-grade students. The learning activities consisted of four activities. Thirty-four students who were the focus of the study were given the redesigned HLT to test during the teaching experiment phase. The teacher started the lesson by asking students about the '*Pocah Piring*' game, a context used to clarify students' knowledge and provide a practical application for the learning process.

Activity 1: A glimpse of number patterns

The learning goal of Activity 1 was that students can identify and understand number sequences. The first activity was related to contextual activities, with the '*Pocah Piring*' as the given context.

Table 2. Students' answer in activity 1

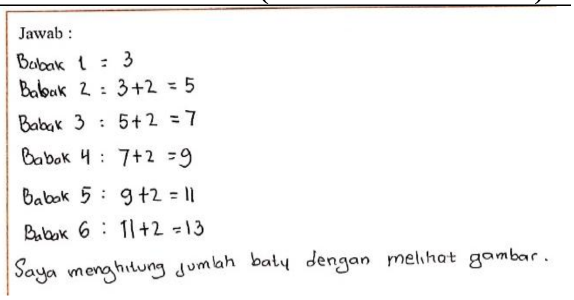
English Version	Students' Answer (Indonesian Version)
After you have done the activity, does the number of stones in activity 1 form a pattern? State your reasons. **Answer :** Yes, the number of stones in activity 1 forms a pattern. Because the stones are arranged according to a rule/pattern, namely a consistent addition rule between each image.	Setelah kamu mengerjakan aktivitas 1, apakah jumlah batu pada aktivitas 1 membentuk sebuah pola ? kemukakan alasanmu ! Jawab : Ya, jumlah batu pada aktivitas 1 membentuk sebuah pola. Karena batu-batu tersebut disusun mengikuti aturan /pola yaitu aturan penambahan yang konsisten di antara setiap gambar.
Continue the following numbers! find the next three numbers **Answer :** • 2 , 4 , 6 , 8 , ... , ... , ... Jawab : 2, 4, 6, 8, 10, 12, 14 (+2 +2 +2 +2 +2 +2) • 1 , 3 , 6 , 10 , ... , ... , ... Jawab : 1, 3, 6, 10, 15, 21, 28 (+2 +3 +4 +5 +6 +7)	Lanjutkan angka berikut ini ! Temukan tiga angka selanjutnya. • 2 , 4 , 6 , 8 , ... , ... , ... Jawab : 2, 4, 6, 8, 10, 12, 14 (+2 +2 +2 +2 +2 +2) • 1 , 3 , 6 , 10 , ... , ... , ... Jawab : 1, 3, 6, 10, 15, 21, 28 (+2 +3 +4 +5 +6 +7)

From the figure, the students answered the question given by stating that the number of stones in the game forms a pattern with the addition rule. In this activity, when students were asked whether the number of stones in activity 1 formed a pattern, students said that *'the number of stones in activity 1 formed a pattern, because the stones were arranged according to a rule or pattern, namely a consistent addition rule between each round'*. Students are also able to continue the numbers in the given number pattern and state the addition rule by writing +2 and +2, +3, +4, etc., in the answer.

Activity 2 (Question 1): Find the number of stones in each round

This activity aimed to help students understand the concept of arithmetic sequences and be able to find the formula for the nth term in an arithmetic sequence. The teacher reminded students of the context of the traditional game '*Pocah Piring*' based on the previous activity and asked them to observe the picture given.

Table 3. Students' answer in activity 2 question 1

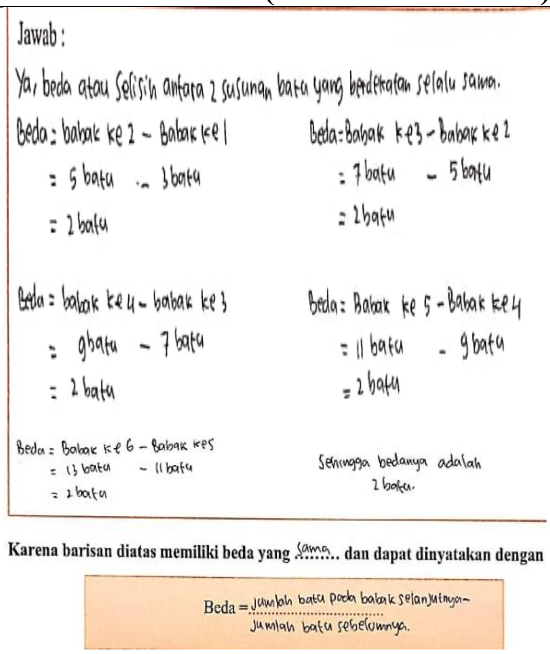
English Version	Students' Answer (Indonesian Version)
Round 1: 3 Round 2: 3+2 = 5 Round 3: 5+2 = 7 Round 4: 7+2 = 9 Round 5: 9+2 = 11 Round 6: 11+2 = 13 I count the stones by looking at the picture	 Jawab : Babak 1 : 3 Babak 2 : 3+2 = 5 Babak 3 : 5+2 = 7 Babak 4 : 7+2 = 9 Babak 5 : 9+2 = 11 Babak 6 : 11+2 = 13 Saya menghitung jumlah batu dengan melihat gambar.

Through this activity, students determined the pattern in the number of stones in each round by counting the number of stones in each round, as illustrated in the picture. Student 2 has demonstrated an understanding that the number pattern associated with the number of stones in each round follows an addition rule of 2, adding by +2 each time. This understanding leads them to comprehend the concept of 'difference,' which will be asked in the next question.

Activity 2 (Question 2): Find the difference between two piles of stones from adjacent rounds

In order for students to understand the concept of 'difference' in arithmetic sequences, Question 2 of this activity asked them to determine the difference in the number of stones used in each round of the *Pocah Piring* game. After that, students were also asked to write down the generalization results of the 'difference' model they had previously done in question 2 by writing the 'difference' formula in the answer on their activity sheet.

Table 4. Students' answer in activity 2 question 2

English Version	Students' Answer (Indonesian Version)
Yes, the difference between 2 adjacent stone structures is always the same. Difference = 2^{nd} round - 1^{st} round = 5 stones - 3 stones = 2 stones Difference = 3^{rd} round – 2^{nd} round = 7 stones - 5 stones = 2 stones Difference = 4^{th} round – 3^{rd} round = 7 stones - 5 stones = 2 stones Difference = 5^{th} round – 4^{th} round = 7 stones - 5 stones = 2 stones Difference = 6^{th} round – 5^{th} round = 7 stones - 5 stones = 2 stones so the difference is 2 stones Difference formula = number of stones in the next round – number of stones in the previous round	 Jawab: Ya, beda atau selisih antara 2 susunan batu yang berdekatan selalu sama. Beda : babak ke 2 - babak ke 1 : 5 batu - 3 batu : 2 batu Beda : babak ke 3 - babak ke 2 : 7 batu - 5 batu : 2 batu Beda : babak ke 4 - babak ke 3 : 9 batu - 7 batu : 2 batu Beda : babak ke 5 - babak ke 4 : 11 batu - 9 batu = 2 batu Beda : babak ke 6 - babak ke 5 = 13 batu - 11 batu = 2 batu Sehingga bedanya adalah 2 batu. Karena barisan diatas memiliki beda yang sama... dan dapat dinyatakan dengan Beda = jumlah batu pada babak selanjutnya - jumlah batu sebelumnya.

In this activity, students calculated the difference between two consecutive stone arrangements by subtracting the total number of stones in the subsequent round from the total in the preceding round. They also recognized that the difference in the sequence remained constant and were able to generalize this as a difference formula, conceptually expressed as 'difference = sum in the next round − sum in the previous round'. Although they did not represent it formally using algebraic notation (e.g., '$b = U_n - U_{n-1}$'), their understanding reflected an emerging grasp of the underlying mathematical structure.

Activity 2 (Question 3): find the number of 14th stones

In question 3, students were required to calculate how many stones were used in the fourteenth round of the *Pocah Piring* game. Students were also asked to write down their calculation method to solve the given problem. At this stage, students were expected to provide calculations that lead to the formula for the nth term in an arithmetic sequence.

Table 5. Students' answer in activity 2 question 3

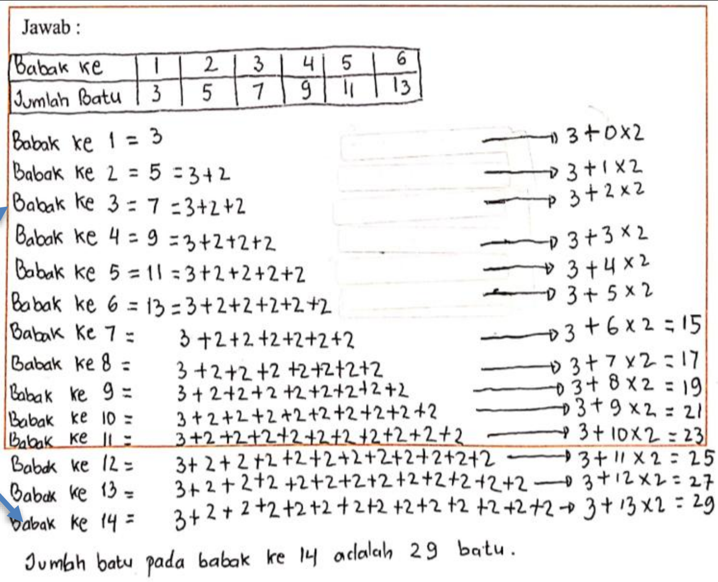
English Version

Round	1	2	3	4	5	6
Number of stones	3	5	7	9	11	13

Round

The number of stones in the 14th round is 29 stones

Students' Answer (Indonesian Version)


Jawab :

Babak ke	1	2	3	4	5	6
Jumlah Batu	3	5	7	9	11	13

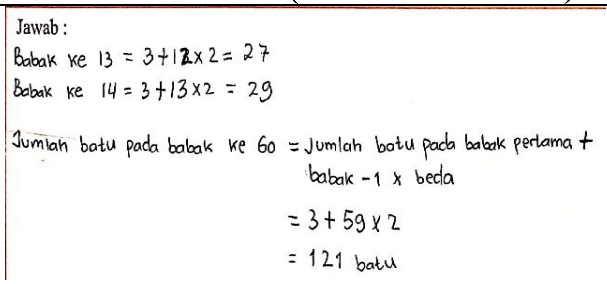
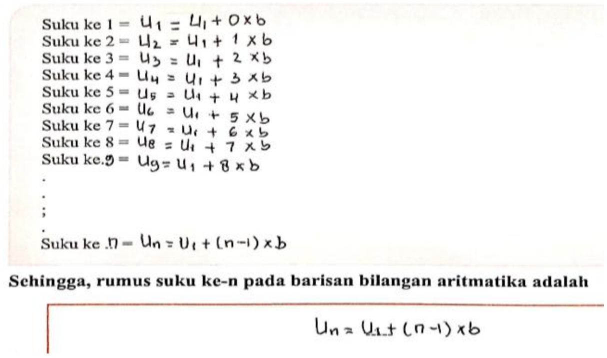
Babak ke 1 = 3 → 3+0×2
Babak ke 2 = 5 = 3+2 → 3+1×2
Babak ke 3 = 7 = 3+2+2 → 3+2×2
Babak ke 4 = 9 = 3+2+2+2 → 3+3×2
Babak ke 5 = 11 = 3+2+2+2+2 → 3+4×2
Babak ke 6 = 13 = 3+2+2+2+2+2 → 3+5×2
Babak ke 7 = 3+2+2+2+2+2+2 → 3+6×2 = 15
Babak ke 8 = 3+2+2+2+2+2+2+2 → 3+7×2 = 17
Babak ke 9 = 3+2+2+2+2+2+2+2+2 → 3+8×2 = 19
Babak ke 10 = 3+2+2+2+2+2+2+2+2+2 → 3+9×2 = 21
Babak ke 11 = 3+2+2+2+2+2+2+2+2+2+2 → 3+10×2 = 23
Babak ke 12 = 3+2+2+2+2+2+2+2+2+2+2+2 → 3+11×2 = 25
Babak ke 13 = 3+2+2+2+2+2+2+2+2+2+2+2+2 → 3+12×2 = 27
Babak ke 14 = 3+2+2+2+2+2+2+2+2+2+2+2+2+2 → 3+13×2 = 29
Jumlah batu pada babak ke 14 adalah 29 batu.

Throughout this activity, students attempted to construct a mathematical model of the given problem, and it was evident that their model led to the idea of the nth-term formula in an arithmetic sequence. Students were able to represent the number of stones using a table. Additionally, by writing 3+13×2 to answer the issue, students could construct a generalization of the addition of 2. By adding two n times, students would obtain a mathematical model that aligns with the solutions of the problem.

Activity 2 (Question 4): Find the number of the 60th stones

Students were asked to determine how many stones there were in the 60th stage in this question. By providing a relatively large number that would take a long time to calculate manually without a formula, this question aims to test students' mathematical modeling skills. It is hoped that they will be able to generalize their calculations, enabling them to determine the nth-term formula of arithmetic sequences.

Table 6. Students' answer in activity 2 question 4

English Version	Students' Answer (Indonesian Version)
Answer : Round 13 : $3 + 12 \times 2 = 27$ Round 14 : $3 + 13 \times 2 = 29$ **Number of stones in round 60** = number of stones in round 1 + round – 1 x difference $= 3 + 59 \times 2$ $= 121$ stones 1st term $= U_1 = U_1 + 0 \times b$ 2nd term $= U_2 = U_1 + 1 \times b$ 3rd term $= U_3 = U_1 + 2 \times b$ 4th term $= U_4 = U_1 + 3 \times b$ 5th term $= U_5 = U_1 + 4 \times b$ 6th term $= U_6 = U_1 + 5 \times b$ 7th term $= U_7 = U_1 + 6 \times b$ 8th term $= U_8 = U_1 + 7 \times b$ 9th term $= U_9 = U_1 + 8 \times b$ … nth term $= U_n = U_1 + (n-1) \times b$ So, the formula for the nth term in an arithmetic sequence is $U_n = U_1 + (n-1) \times b$	 

Based on the figure, student could use their own formula from the previous calculation in question number 3 and use that to answer question number 4. After that, they made a generalization by stating that the first term is $U_1 = U_1 + 0 \times b$ and continued to the nth term by stating the general formula

Activity 3: Applying the formula to a real-world problem

In the last activity, students were asked to apply the formula they had found previously to solve the real problems given. From this activity, it was expected that students would not only understand the concept of the nth-term formula in an arithmetic sequence but also solve real problems related to arithmetic sequences.

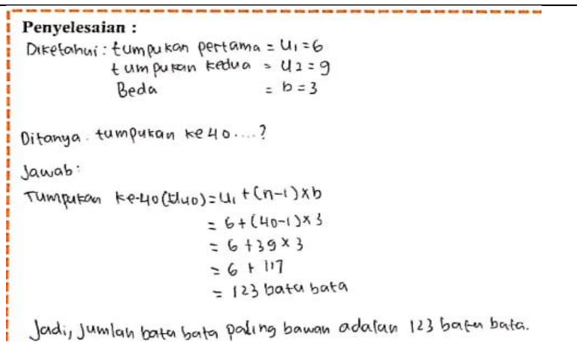
Table 7. Students' answer in activity 3

English Version	Students' Answer (Indonesian Version)
Question : Several bricks are stacked with different numbers in each pile. The number of bricks in the top pile is 6 bricks, right below it is 9 bricks, and so on. Each pile below it always has 3 more bricks than the pile above it. If there are a total of 40 piles from top to bottom, the number of bricks in the bottom pile is **Answer :** Known : First pile = U_1 = 6	**Pertanyaan :** Beberapa batu bata ditumpuk dengan jumlah berbeda-beda di setiap tumpukannya. Banyak batu bata di tumpukan paling atas ada 6 bata, tepat dibawahnya ada 9 bata, dan seterusnya. seriap tumpukan dibawahnya selalu lebih banyak 3 batu dari tumpukan diatasnya. jika terdapat total 40 tumpukan dari atas sampai bawah, jumlah batu bata pada tumpukan paling bawah adalah **Jawaban =**

Second pile = U_2 = 9
Difference = b =3
Asked : the number of stones in 40th pile
Answer :

$$\text{40th pile} = U_{40} = U_1 + (n-1) \times b$$
$$= 6 + (40-1) \times 3$$
$$= 6 + 39 \times 3$$
$$= 6 + 117$$
$$= 123 \text{ bricks}$$

So, the number of bottom stones is 123 bricks.


Penyelesaian :
Diketahui : tumpukan pertama = U_1 = 6
tumpukan kedua = U_2 = 9
Beda = b = 3
Ditanya : tumpukan ke 40 .... ?
Jawab :
Tumpukan ke-40 (U_{40}) = U_1 + (n−1) x b
= 6 + (40−1) x 3
= 6 + 39 x 3
= 6 + 117
= 123 batu bata
Jadi, jumlah batu bata paling bawah adalah 123 batu bata.

In the table above, it can be seen that students were able to state that the first pile is U_1, the second pile is U_2, and determine the 'difference' of the problem. Students could also apply the formulas that had been obtained and solve the given problem. By solving this problem, it could be shown that students understand the concept of the nth-term formula in an arithmetic sequence and use the nth-term formula of an arithmetic sequence they had understood to solve real-world problems.

Retrospective analysis

After conducting the second phase, it is shown that students' way of thinking in modeling the problem into mathematical formulas undergoes a horizontal mathematization process by providing the nth-term formula, which involves various symbols such as Un, U1, and b. The results of compiled data can be seen in the following iceberg illustration in Figure 3.

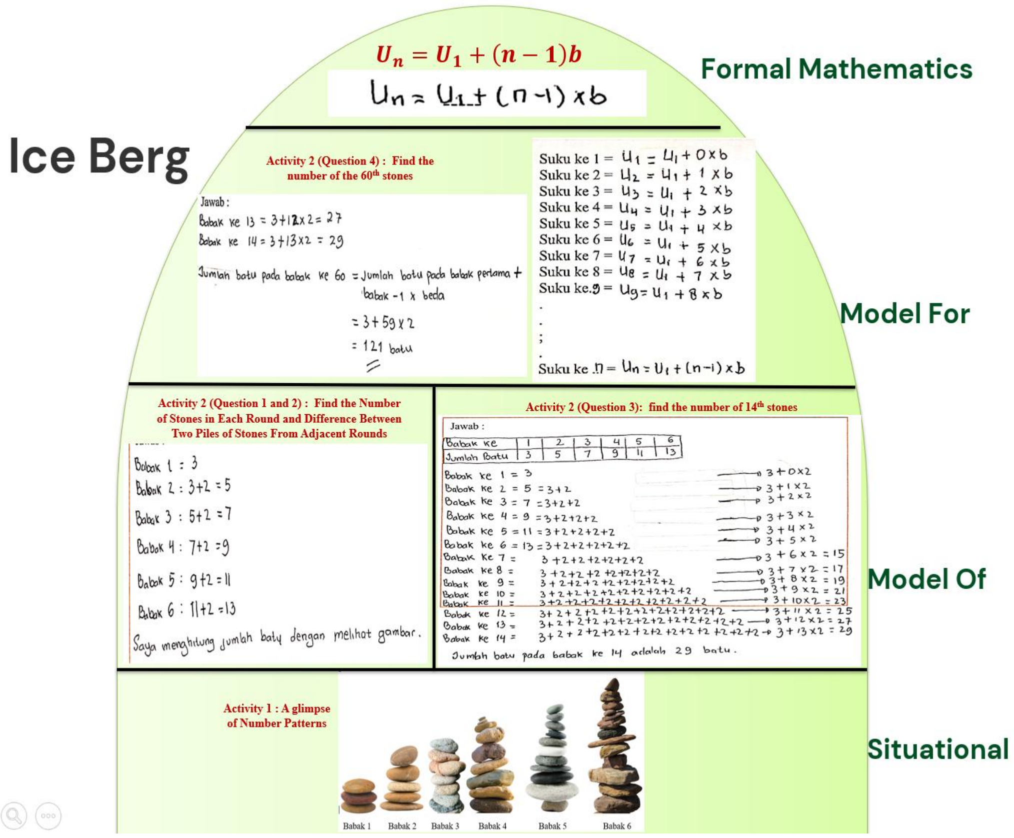


Figure 3. Iceberg design in nth term formula of arithmetic sequences

According to the results from the teaching experiments and retrospective analysis, the activities created in the Local Instructional Theory (LIT) may help students rethink the ideas of the nth term in the arithmetic sequence. It is evident that for gaining the learning objectives, as students can understand the concept of the nth-term formula in an arithmetic sequence. The teacher should start with context that is real to the student and make them understand the 'difference' concept first before jumping to the modeling stage.

The *Pocah Piring* game serves as an introductory activity to connect students' existing knowledge and experiences to the concept of arithmetic sequences by enganging in hands-on activities to recognize number patterns in the game ('situational' stage). Then, students begin by recognizing that the number of stones follows a pattern based on repeated addition (e.g., +2 each time). Students understand that the sequence follows a consistent rule of addition, identifying arithmetic sequences. After that, students determine the common difference between terms in the sequence by formalizing the concept of 'difference' as a fundamental property of arithmetic sequences. In this part, students begin to identify patterns in the game's progression, focusing on how the number of stones increases in a regular manner, thus laying the foundation for understanding arithmetic sequences ('model of' stage).

As the student already knows the 'difference' concept, challenge them with a smaller number first and then give them a bigger number so that they use their mathematical reasoning to find the nth-term using their own calculation. Students move from observation to analysis by identifying and formalizing the relationships between the numbers. This stage focuses on the formulation of patterns and the construction of a mathematical model for the arithmetic sequence ('model for' stage). In the last part, students use their previous understanding to derive the nth-term formula for the arithmetic sequence and apply it to real-world problems, such as predicting the number of stones in the 60th round ('formal mathematics' stage). Students are able to model real-world problems using the nth-term formula and gain confidence in its application. However, it took longer for the students to comprehend what the variables meant. It was found that students require more time to comprehend symbolic representations than verbal and pictorial ones, so instead of using the variable directly in activity 2, the teacher suggested using an abbreviation to represent the number of stones or allowing students to freely represent their own variables, such as round 1, round 2, etc. Then, as they worked on activity 3, which involved applying formulas to real-world problems, students learned about variables like Un, U1, b, etc.

Discussion

The *Pocah Piring* game effectively grounds abstract mathematical concepts in students' real-life experiences, making it easier for them to grasp the underlying patterns. Integrating the Pocah Piring game into mathematics instruction serves a dual purpose: it enhances students' enthusiasm and motivation (Ginting & Rakhmawati, 2024) while providing a structured yet flexible environment to validate the effectiveness of the designed learning trajectory. The game's mechanics (stacking, breaking, and reassembling objects) create a rich, real-world context that allows students to explore mathematical concepts in a tangible and engaging way.

By participating in the game, students are naturally motivated to investigate mathematical relationships, such as identifying patterns, predicting outcomes, and deriving formulas, all within a familiar and interactive activity. This aligns with the principles of Realistic Mathematics Education (RME), which emphasizes starting from meaningful (Inci et al., 2023), real-life contexts to guide students toward formal mathematical reasoning (Fauzan et al., 2024). The gradual development from concrete activities to abstract mathematical formulation supports cognitive development and ensures deep understanding (Fredriksen, 2021).

As most of the time, the conjecture about students' thinking were accurate, allowing teachers to employ well-prepared probing questions to help the students achieve the learning objectives. According to earlier research (Fauzan et al., 2020; Isik et al., 2020; Gee et al., 2018), the LIT was particularly beneficial in encouraging students to reinvent mathematical ideas, and these results support those findings. Learning trajectory shows that to teach the concept of arithmetic sequence effectively, instruction should begin with concrete examples, such as physical objects (e.g., stones, blocks) or visual patterns that help students see how sequences grow. Gradually, instructions can progress to a more abstract form using symbols and formulas. The iterative nature of the game fosters a deeper conceptual understanding of arithmetic sequences as they observe and analyze patterns over multiple rounds of play. The iterative refinement of instructional strategies based on student feedback ensures that the learning trajectory remains flexible and responsive to the needs of learners.

The game acts as a central tool for creating a learning trajectory that bridges students' informal experiences with formal mathematical concepts (Wijaya & Doorman, 2021). By embedding mathematical exploration within the game's mechanics, the study ensures that the learning process is practical and effective in real classroom settings. *Pocah Piring*, therefore, plays a critical role not only as an engaging educational tool but also as a learning media for validating instructional design. Its ability to connect abstract mathematical ideas, such as the nth-term formula in arithmetic sequences, to students' everyday experiences underscores its potential to significantly improve academic performance and foster meaningful learning. Teachers can use these findings to refine their instructional strategies, offering more interactive and context-rich experiences that align with students' everyday experiences, enhancing understanding and applying mathematical concepts..

Conclusion

A learning trajectory can be produced by using the traditional game "Pocah Piring" as the background since it can help students grasp the concept of "finding the nth term in arithmetic sequence". The research step, including preliminary design, design experiment, and retrospective analysis, can verify the validity of the local instructional theory (LIT). Three learning activities were applied to the arithmetic sequence content to accomplish the learning objectives. Students' way of thinking in understanding the concept of the nth-term formula in arithmetic sequence can be illustrated when they work on the activity. It will help teachers develop learning activities that are appropriate for their students. By conducting the local instructional theory in this research, teachers can utilize the Hypothetical Learning Trajectory

(HLT), which is based on Realistic Mathematic Education (RME) and uses valid arithmetic sequence material, as a guide when implementing learning and creating other learning flows.

This study's implications demonstrate how well culturally appropriate context, such as *Pocah Piring* game, may be included into abstract mathematical concepts, enhancing student engagement and understanding. The fact that this study was only tested in one school limits its applicability. The sample, consisting of students with similar academic levels and learning environments, may not represent the diverse range of students found in different schools. Further researchers can conduct similar research with trials in various schools with diverse conditions and backgrounds of students.

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Conflicts of Interest

The authors declare no conflict of interest regarding the publication of this manuscript. In addition, the authors have completed the ethical issues, including plagiarism, misconduct, data fabrication and/or falsification, double publication and/or submission, and redundancies.

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Author Contributions

Dhea Anisah Putri: Conceptualization, investigation, writing - original draft, editing, and visualization; **Zulkardi:** Supervision, formal analysis, and methodology; **Ely Susanti:** Writing - review & editing, validation. **Meryansumayeka:** Resources and project administration.

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