Jurnal Elemen, 11(3), 577-596, July 2025
https:/doi.org/10.29408/jel.v11i3.29859

e- ISSN: 2442 - 4226

Check for updates

Textbook analysis of integers content using the praxeology framework

Putri Fitriasari [1], Suhendra [1 *], Didi Suryadi [1, 2], Elah Nurlaelah [1]

[1] Department of Mathematics Education, Universitas Pendidikan Indonesia, West Java, Indonesia

[2] DDR Development Center Indonesia (PUSBANGDDRINDO), West Java, Indonesia

[*] Correspondence: suhendra@upi.edu

Abstract

Textbooks play a crucial role in the learning process, influencing instructional practices and students' cognitive development. However, some textbooks fail to support students in constructing new knowledge because of a lack of gradual concept progression. Consequently, textbook analysis has become a key focus of educational research. This study analysed a seventh-grade mathematics textbook on integers using the praxeology framework. It examines the material structure and identifies potential learning obstacles to learning. As part of Didactical Design Research (DDR) at the prospective stage, this study adopts an interpretive paradigm and qualitative approach. The praxeology framework, derived from the Anthropological Theory of the Didactic (ATD), serves as an analytical lens for this study. The researchers conducted an in-depth analysis of praxeological components, including task types, techniques, technologies, and the underlying theories. The findings revealed that the textbook systematically presented integer content but lacked support for epistemic knowledge construction. Additionally, potential didactic and ontogenic obstacles were identified. The findings contribute theoretically by demonstrating the framework's application to textbook analysis. Practically, it guides the alignment of mathematical tasks with learning goals and reduces the learning obstacles. The results inform policy by emphasising the development of contextually and pedagogically relevant textbooks, rather than direct adaptation.

Keywords: integer; praxeology; textbook analysis

How to cite: Fitriasari, P., Suhendra, Suryadi, D., & Nurlaelah, E. (2025). Textbook analysis of integers content using the praxeology framework. *Jurnal Elemen, 11*(3), 577-596. https://doi.org/10.29408/jel.v11i3.29859

Received: 18 March 2025 | Revised: 12 May 2025
Accepted: 10 June 2025 | Published: 30 July 2025

Introduction

In the formal education system, textbooks serve as the main pillar supporting the teaching-learning process. (Nomvuyo M et al., 2023) found that mathematics teachers often rely on textbooks not only as a source of content but also as a guide for making instructional decisions, particularly when planning lessons collaboratively. Meanwhile, Rezat et al. (2021) argue that textbooks should be viewed as instruments for promoting educational change, as they reflect curriculum intentions and influence how mathematics is taught and learned. Thus, textbooks can be understood as a form of "human action" deliberately constructed to facilitate the transfer of knowledge and structure teaching practices (Suryadi, 2023). Therefore, textbooks should be structured and clarify concepts to help students generate new knowledge (Fan et al., 2013; Remillard, 2000; Rezat, 2012; Zhang et al., 2022). Although textbooks play an important role in the learning process, research shows that many textbooks present unstructured material, resulting in students experiencing confusion in following the learning flow (Lodge et al., 2018; Pierard et al., 2020; Shepherd et al., 2009). Furthermore, numerous faults in embedding concepts can be fatal because they can lead to misconceptions and hinder students learning (Carson, 2007; X. Wang, 2024). Math textbooks, for example, should help students to gradually understand topics.

Mathematics textbooks should systematically introduce fundamental concepts progressively, ensuring their effective application in problem-solving. Studies indicate that well-structured textbooks organised with a logical sequence of ideas can significantly improve students' comprehension and facilitate the practical implementation of mathematical concepts (Dong et al., 2020; Kilpatrick et al., 2001; Klang et al., 2021; Sinaga et al., 2023). Furthermore, based on the theory of didactic situations (TDS), educational resources should foster learning environments that encourage students to actively explore and construct knowledge by engaging with relevant problem contexts (Brousseau, 1997). However, in practice, some learning materials do not adequately support this process, resulting in difficulties for students in understanding foundational concepts, which consequently impairs their problem-solving skills (Jäder et al., 2020; Özgeldi & Esen, 2010; Rzyankina et al., 2024). Although existing research highlights these challenges, it has not systematically employed a specific theoretical framework to analyse how these concepts are presented. Therefore, further studies should assess mathematics learning materials using established frameworks such as praxeology.

In terms of praxeology as a basic unit for understanding human action, mathematics textbooks as products of such action should present not only clear procedures for problem solving (techniques) but also the underlying reasons (technologies) and the theoretical foundations that support these approaches (Chevallard, 2006). This means that mathematics textbooks should explain in depth why a method is used and how it relates to fundamental concepts, allowing students to gain thorough and structured knowledge. Furthermore, praxeological examinations of textbooks seek to determine whether the approaches used are relevant to the ideas and technology underlying them, thereby facilitating meaningful learning for students. In addition, praxeology emphasises the importance of alignment between these components so that each task in the mathematics textbook is not only completed correctly but also builds new knowledge that can be applied in other contexts (Suryadi et al., 2023). Research on textbook analysis using the praxeology framework is important because of its significant role in supporting learning and reflecting on curriculum goals.

Several relevant studies on textbook analysis in mathematics education have highlighted the importance of this research in improving instructional quality. Fan et al. (2013) identified recent trends in textbook research, including cross-national comparisons and how teachers utilize textbooks to support curriculum implementation. Additionally, other studies have shown that well-designed mathematics textbooks enhance students engagement in learning. S. Wang et al. (2024) analyzed secondary school mathematics textbooks in China and Singapore, focusing on how students and teachers understand and apply mathematical concepts. Furthermore, Hendriyanto et al. (2023), Kusharyadi et al. (2024), Utami et al. (2024), and Yunianta et al. (2023) examined the presentation of mathematical concepts using the praxeology framework. However, these studies were limited to topics such as geometry, functions, sets, and sequences and series, meaning that their analyses addressed praxeology organisation only at the local level, focusing on a single sector.

This study adopts the praxeological framework from the Anthropological Theory of the Didactic (Chevallard, 2006), as applied by Wijayanti and Winslow (2017) in their analysis of praxeological epistemological models on the topic of proportion. More recently, Utami et al. (2025) conducted a praxeological analysis of function concepts in lower secondary textbooks in Japan and Indonesia, further demonstrating the framework's versatility across mathematical topics and educational contexts. Both studies utilised a referential praxeological model comprising four components: type of task (T), technique (τ), technology (θ), and theory (Θ). The similarity lies in the application of this model to examine the underlying structure of mathematical practices represented in textbooks and in the categorisation of tasks according to the techniques utilised. However, there are several notable differences between these two studies. Wijayanti and Winsløw's study focused on the topic of proportion in three Indonesian textbooks and developed a praxeological model as an analytical tool, classifying seven dominant task types (T1–T7) within the theme of proportion. In contrast, the present study examines the topic of integers and constructs its own praxeological model by identifying 11 task types from a single textbook, including an analysis of the learning obstacles that may arise from them. Thus, this study extends the approach of Wijayanti and Winsløw by applying it to a different topic and structure, demonstrating how variations in textbook design may shape or hinder students' mathematical knowledge construction within the Indonesian educational context.

In this context, textbooks play a crucial role as the primary resource for teachers, making systematic and structured presentations essential. Consequently, conducting this study is imperative to ensure that textbooks effectively support the learning process. Based on the problems described, the following problem formulations were proposed in this study:

RQ1: How is integer material presented in mathematics textbooks?

RQ2: What are the potential learning obstacles that may arise from the use of these textbooks?

Methods

This study was part of a Didactical Design Research (DDR) at the prospective stage, which focuses on identifying potential learning obstacles. It employs an interpretive paradigm with a qualitative approach and utilizes the praxeology framework from the Anthropological Theory of the Didactic (ATD) as an analytical tool. The interpretive paradigm was chosen to explore in depth the impact of mathematics textbooks on students' knowledge construction (Suryadi, 2023). The praxeology framework is used in this study to analyze the presentation of integer material in the textbook and to identify possible learning obstacles that may arise. Praxeology is a theory used to analyze human actions to determine whether they constitute knowledge (Chevallard, 2006). The researchers served as the primary instrument in data collection and analysis. The analysis focuses on praxeological components, namely types of tasks, techniques, technologies, and theories embedded in the material presentation.

The object analyzed in this study is a mathematics textbook entitled *"Matematika untuk SMP Kelas VII"* published in 2021 by the Center for Curriculum and Bookkeeping, Research and Development and Bookkeeping Agency, Ministry of Education and Culture, Republic of Indonesia. This textbook was selected because it is the official learning resource used by students at the school where the research was conducted, and therefore, it directly influences their learning process. Consequently, the scope of analysis in this study is limited to this textbook to maintain relevance with the objectives of the prospective stage of DDR.

This book is a translated, adapted, and reviewed version of the original work Mathematics for Junior High School 1st Level, authored by the Gakko Tosho team. The processes of translation, adaptation, and review were carried out by a team of experts under the auspices of the Ministry of Education and Culture of the Republic of Indonesia. After translation, the content was further adapted to align with the cultural and pedagogical context of mathematics education in Indonesia. The adapted version then underwent a review process to ensure its quality, content accuracy, and pedagogical relevance prior to publication. To facilitate the continuous improvement and refinement of this textbook, the development team has incorporated a note in the preface, encouraging educators and practitioners to provide constructive feedback for future editions.

This study specifically focused on analyzing the content related to integers in the selected textbook. The topic was chosen because it is introduced in seventh grade and represents a fundamental concept in secondary mathematics education. The analysis covered three main subtopics: (1) positive and negative integers, (2) addition and subtraction of integers, and (3) multiplication and division of integers.

In this study, the analysis begins by identifying the type of task (T) based on the given examples. Next, the technique (τ) is identified through the example provided for solving the problem in the textbook. The technique (τ) used to solve a given type of task (T) in the textbook is assumed to be the technique that students are most likely to use when working on the corresponding exercises. The rationale or justification for selecting a particular technique (τ) is referred to as the technology (θ). The technology (θ) serves to justify the use of the technique (τ). Furthermore, underlying the technology (θ) is a set of references or foundations that explain why that justification is valid, which is referred to as the theory (Θ). In short, praxeology is represented by four components [T/τ/θ/Θ]. The relationship among these four components and their development is illustrated in Table 1.

Table 1. Praxeology analysis (Adapted from (Chevallard, 2007) and (Yunianta et al., 2023))

Praxis block		**Logos block**	
Type of Tasks (T)	**Technique**	**Technology**	**Theory**
Type of task (a problem that has a specific objective to be solved)	The way a Person takes action to solve a given type of task.	A justification for performing a technique.	The foundation or reference underlying the existence of a technology.
In the context of mathematical textbook analysis			
Praxis block		**Logos block**	
Type of Tasks (T)	**Technique**	**Technology**	**Theory**
Types of student tasks presented in the textbook (problems that students are expected to solve)	Strategy that students are likely to use when solving a given task. This strategy can be identified through the teacher's guide, which provides solutions (answer keys) for each assigned task	Justification for the methods used by students to solve tasks in the textbook	A reliable and reasonable foundation or reference for justifying the methods used by students to solve tasks in the textbook

After the data were collected, the next step was analysis to identify each aspect of praxeology, including task type, techniques, technology, and theory. This identification aimed to understand how each material component is grouped and presented in the textbook. In addition, the analysis evaluated whether the presentation of integer materials met the criteria of unity, coherence, and flexibility. To ensure that the findings were accurate and acceptable, the author conducted a Focus Group Discussion (FGD) with three academic supervisors. In this forum, the results of the analysis were reviewed, discussed, and collectively confirmed.

Furthermore, the research examines potential learning obstacles that may occur as a result of using mathematics textbooks. These impediments may be didactic, epistemic, or ontogenic. Figure 1 epicts all stages of the research procedure for this study.

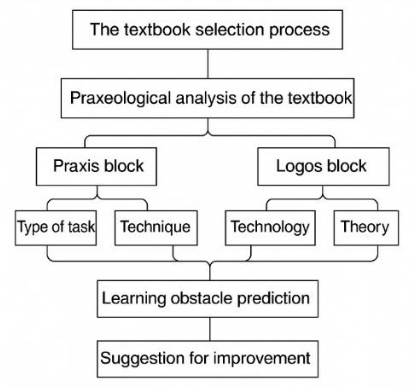


Figure 1. Overall research processes

Results

This study chose the topic of integers as the main focus, which is studied by seventh-grade students in Indonesia. Based on the teaching module in the curriculum document, the learning objectives to be achieved for integers are as follows: In this section, the research results are presented in a clear and detailed manner. The research results can be presented based on the research results at each stage of the research or research results that answer each problem formulation or others, as long as the research results are visible. The research results should be supported by empirical evidence.

This section may be divided into subheadings. This section should provide a concise and precise description of the experimental results, their interpretation, and the experimental conclusions that can be drawn.

Table 2. Learning materials and objectives

Topic	Learning Objectives
Integers	1) Explain the relationship between positive integers and negative integers by modeling them on a number line (direction and distance) 2) Use correct notation to express integer numbers 3) Compare and order integers and put them on a number line 4) Recognize and use the relationship between numbers and their inverse (inverse addition) to solve problems 5) Determine the result of the arithmetic operations of addition, subtraction, multiplication, and division of integers 6) Determine factors of integers 7) Recognize and use the fact that an integer can be written in exactly one way as the product of prime numbers 8) Connect prime factorization of two numbers Least Common Multiple (LCM) and Greatest Common Divisor (GCD) 9) Solve problems about integers related to daily life

Textbook authors should use the learning objectives for integers in curricular documents to create activities, materials, and tasks to fulfil these goals. However, the researchers' examination revealed contradictions between the learning objectives in the teaching module and the mathematics textbook, which could impact the achievement of the curriculum objectives. The teaching module emphasises deep conceptual understanding and the interconnection between concepts, such as the relationship between integers and factorisation and the use of formal notation. In contrast, textbooks favour a practical and operational approach with concrete activities and direct applications to facilitate student understanding.

Furthermore, to analyse the distribution of tasks on integers, the researchers reviewed that the material in the mathematics textbooks is divided into three subtopics, as shown in Table 3.

Table 3. Subtopic division of integers

No	Subtopic	Number of Task
1	Positive and negative numbers	12
2	Addition and subtraction	22
3	Multiplication and division	33

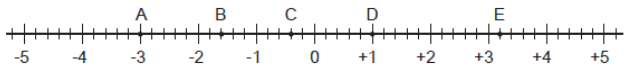
The integer material in mathematics textbooks is divided into three subtopics: positive and negative numbers, addition and subtraction, and multiplication and division. Each subtopic contained 12, 22, and 33 tasks, respectively. The multiplication and division subtopic contains more tasks than the other subtopics. This imbalance suggests that the textbook places greater emphasis on these sub-topics.

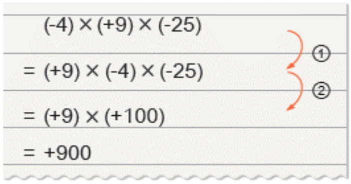
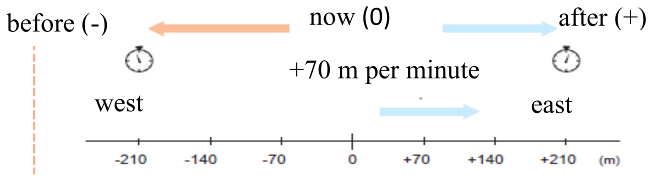
Presentation of integer materials in mathematics textbooks

Praxis: Type of task

The analysis begins with a review of the praxis block, which includes various types of tasks and techniques used in problem solving. Task types were identified based on the problems and activities presented throughout the textbook, from beginning to end, related to the topic of integers. These tasks were then grouped according to their similarities and labelled as T, with indices representing each task type. Based on the researchers' analysis, 11 types of tasks were identified, comprising 67 task instances, and their distribution is presented in Table 4. T_5 and T_{11} appeared most frequently in the textbooks and represented major themes within the topic of integers. These two dominant task types capture a significant portion of the " realised " curriculum in Indonesian schools and thus become the primary focus of the analysed material. Table 4 presents each task type along with sample problems, frequency of occurrence, and relative percentage of the total.

Table 4. Types and number of integer tasks

Type of task (T)	Number of Tasks (n = 67)
T_1: Identify positive and negative integers Example: The height of Mount Semeru is 3,676 meters above sea level, and the depth of the Java Trench is 7,140 meters below sea level. The base point of reference is the coastline. How do we express the quantities using positive and negative signs?	6 (8.96 %)
T_2: Presenting integers on a number line State the numbers corresponding to points A, B, C, D, E. Example: 	2 (2.99%)
T_3: Comparing integers Example: Compare the following pairs of numbers using inequality signs. 1) $+3\ ,\ +4$ 4) $-\frac{2}{3}\ ,\ -\frac{1}{3}$ 2) $-4\ ,\ -6$ 5) $+1\ ,\ -3\ ,0$ 3) $+0{,}1\ ,\ -0{,}2$ 6) $-2\ ,\ +5\ ,\ -5$	1 (1.49%)
T_4: Absolute numbers Example: Consecutively, determine the absolute values of -7 and +5.2	3 (4.48%)
T_5: Positive and negative integer operations Example: Calculate.	32 (47.76%)

Type of task (T)	Number of Tasks (n = 67)
1) $-7+(-3)\text{ x }2$ 3) $14-10\text{ x }(-3)$ 2) $8+(-20):(-4)$ 4) $(-6)\text{ x }(-5)\ -\ (-18):6$	
T_6: Decimal and fraction operations Example: Calculate. 1) $\left(-\frac{1}{3}\right):\frac{3}{4}$ 3) $6:\left(-\frac{4}{3}\right)$ 2) $\left(-\frac{3}{5}\right):(-\frac{9}{10})$ 4) $\left(-\frac{5}{6}\right):(-3)$	5 (7.46%)
T_7: Use commutative and associative properties of integer multiplication Example: Yuli calculated as follows. (-4) x (+9) x (-25) as shown in the count to the side. Explain the process behind calculations 1 and 2. 	1 (1.49%)
ative properties of multiplication of integers, decimals, and fractions Example: Calculate. 1) $(-50)\text{ x }(+17)\ x\ (-2)$ 2) $(+9)\text{ x }(-4.5)\ x\ (+2)$ 3) $\left(-\frac{1}{8}\right)\text{x }(+3.6)\ x\ (-8)$ 4) $\left(+\frac{1}{3}\right)\text{x }(-10)x\ (-\frac{3}{5})$	3 (4.48%)
T_9: Multiplication of integers, decimals, and fractions Example: Calculate. 1) $(-10)^2$ 4) $(0{,}3)^2$ 2) -10^2 5) $(-2)^3$ 3) $(-\frac{4}{7})^2$ 6) -2^3	3 (4.48%)
T_{10}: Distributive property of numbers Example: Answer the following problems by applying the distributive property 1) $28\ (-\frac{1}{4}+\frac{1}{7})$ 2) $\left(\frac{3}{4}-\frac{5}{6}\right)\text{x }36$ 3) $17\text{ x }9\ +\ 17\text{ x }(-8)$ 4) $69\text{ x }(-7.2)\ +\ 31\text{ x }(-7.2)$	1 (1.49%)
T_{11}: Use integers in real world and daily life Munir walked eastward at a speed of 70 m per minute. The starting point is set as 0 m. The direction to the east as the positive direction. Skipping one minute is counted as +1 minute. 1. At what point was Munir after walking one minute? After two minutes? Where was he a minute before? Two minutes before? Mark (with arrows) Munir's location using the diagram below. Example: 	10 (14.93%)

The researchers then identified the sequence of task types offered in the mathematics textbook. This step examines how the task layout affects the learning flow. Instructional resources help teachers design and deliver structured lessons, and the order and selection of tasks influence instructional techniques. The sequence of task types in the textbook, organized by subtopic, is presented in Table 5.

Table 5. Task sequence

No.	Subtopic	Page	Sequences
1	Positive and negative numbers	14 – 20	$T_1 \rightarrow T_2 \rightarrow T_1 \rightarrow T_2 \rightarrow T_3 \rightarrow T_4$
2	Addition and subtraction	21 – 35	$T_5 \rightarrow T_6 \rightarrow T_5 \rightarrow T_{11} \rightarrow T_6 \rightarrow T_5 \rightarrow T_6$
3	Multiplication and division	36 – 59	$T_{11} \rightarrow T_5 \rightarrow T_6 \rightarrow T_7 \rightarrow T_8 \rightarrow T_9 \rightarrow T_5 \rightarrow T_{10} \rightarrow T_{11}$

Praxis: Technique

Techniques in the praxis block are strategies or actions used to solve tasks. Each technique is denoted by the symbol τ with an index that represents the type of technique. In this study, the classification of techniques is based on the solution strategies that are likely to be used by students, which are identified through the analysis of solutions (answer keys) provided in the teacher's guide. These strategies are considered to represent the expected approach in solving tasks, although it does not rule out the possibility that students may use different strategies. Although the identification of techniques in this study focuses on the strategies presented in the teacher's guide, understanding the possible variations in student strategies remains important. Therefore, possible student answers are also analyzed to enrich the understanding of the approaches they might choose. The following are the techniques used in this study.

1. Visual (τ_1)
   Use of pictorial representations or diagrams to understand mathematical concepts. Visualizations include the use of number lines to show where positive and negative numbers are and the representation of absolute value as a distance from zero.
2. Logical (τ_2)
   Rules or principles of mathematical logic are used to draw conclusions.
3. Numeric (τ_3)
   Manipulate numbers to perform calculations. In the context of integers, this includes operations such as addition, subtraction, multiplication and division of positive and negative integers.
4. Symbolic (τ_4)
   Involves the use of mathematical symbols and algebraic expressions to represent mathematical operations and relationships.
5. Applicative (τ_5)
   Applying math concepts to real-world situations. The context of integers includes the use of integers to describe everyday phenomena such as temperature (below and above zero), altitude (above and below sea level), or gains and losses in a financial context.

6. Formal (τ_6)
   Standard rules or formal properties are used in mathematics to solve problems. This includes the application of theorems, commutative, associative, distributive and other rules.

The definition above provides a basic explanation of the techniques that students use to complete tasks; nevertheless, when applied to specific types of tasks, the approaches must be tailored to the context of the material that is being taught. Each sort of activity necessitates the application of more particular and targeted procedures, depending on the task's qualities and aims. The following are the strategies and explanations used to solve tasks involving integers.

Table 6. Techniques for each type of task integer

Type of Task	Technique	Description
T_1	τ_1, τ_3, and τ_5 τ_1: visual τ_3: numerical τ_5: applicative	τ_1: using a number line to understand the different positions of positive and negative numbers τ_3: using numbers to calculate and compare positive and negative numbers τ_5: relates the concept of positive and negative numbers to the real world such as temperature and altitude.
T_2	τ_1, τ_2 and τ_3 τ_1: visual τ_2: logical	τ_1: visualize the location of numbers on the number line. τ_2: use reasoning to determine the order and position of numbers on a number line.
T_3	τ_2 and τ_3 τ_2: logical τ_3: numerical	τ_2: understand the basic rules of integers comparison, such as that negative numbers are always smaller than positive numbers. τ_3: compare two numbers directly by looking at their values.
T_4	τ_1, τ_3 and τ_4 τ_1: visual τ_3: numerical τ_4: symbolic	τ_1: understand the concept of absolute value as the distance of a number from zero on a number line τ_3: perform calculations to find the absolute magnitude of a number τ_4: understand and express absolute value using correct mathematical notation.
T_5	τ_3, τ_4 and τ_6 τ_3: numeric τ_4: symbolic τ_6: formal	τ_3: perform calculations for positive and negative integer operations. τ_4: use mathematical symbols to express integer operations. τ_6: apply formal rules, such as commutative, associative, and distributive properties, in integer operations
T_6	τ_3, τ_4 and τ_6 τ_3: numeric τ_4: symbolic τ_6: formal	τ_3: perform direct calculations with decimal numbers and fractions. τ_4: use mathematical symbols correctly to express decimal and fractional number operations τ_6: apply formal mathematical rules, such as the distributive, commutative and associative properties in decimal and fraction operations.
T_7	τ_2 and τ_6 τ_2: logical τ_6: formal	τ_2: use logical reasoning to understand how the order or grouping of numbers does not affect the result in multiplication operations (commutative and associative properties). τ_6: apply standard mathematical properties, such as commutatively and associativity, in solving multiplication problems.
T_8	τ_2 and τ_6 τ_2: logical τ_6: formal	τ_2: logically understand that the order and grouping of numbers in multiplication, be it integers, decimals, or fractions, does not affect the result

Type of Task	Technique	Description
		τ_6: formally apply the commutative and associative properties to different types of numbers, including integers, decimals, and fractions.
T_9	τ_3, τ_4 and τ_6 τ_3: numerical τ_4: symbolic τ_6: formal	τ_3: calculate the result of multiplying integers, decimals and fractions directly τ_4: use exponent notation to express multiplication operations appropriately τ_6: apply formal rules of multiplication to simplify and solve more complex multiplication operations
T_{10}	τ_2 and τ_6 τ_2: logical τ_6: formal	τ_2: logically understand how multiplication is distributed into addition or subtraction in parentheses τ_6: formally apply distributive rules to simplify calculations and solve mathematical expressions.
T_{11}	τ_1, τ_3 and τ_5 τ_1: visual τ_3: numerical τ_5: applicative	τ_1: Help students visualize the use of integers in real-world situations τ_3: perform direct calculations using integers to solve problems in daily life. τ_5: applying integer concepts in real contexts

The logos: Technology and theory

The logos block consists of technology (θ) and theory (Θ). Technology is the justification for the technique chosen for each task type. While theory refers to the basis that explains the technology. In this context, the technology used consists of 3 steps: the application of number concepts ($\theta 1$), basic arithmetic operations (θ_2), and the application of integers in daily life (θ_3). Moreover, the theory used only focuses on one theory, that is integers.

Table 7. Analysis of praxeology in textbooks

Type of Task (T)	Technique (τ)	Technology (θ)	Theory (Θ)
T_1: Recognize positive and negative integers	τ_1: visual τ_3: numerical τ_5: applicative	θ_1 = Application of number concepts	Θ_1 = Integers
T_2: Present integers on a number line	τ_1: visual τ_2: logical		
T_3: Compare integers	τ_2: logical τ_3: numerical		
T_4: Absolute value	τ_1: visual τ_3: numerical τ_4: symbolic		
T_5: Positive and negative integer operations	τ_3: numerical τ_4: symbolic τ_6: formal	θ_2 = basic arithmetic operations	
T_6: Decimal and fraction operations	τ_3: numeric al τ_4: symbolic τ_6: formal		
T_7: Use commutative and associative properties of integer multiplication	τ_2: logical τ_6: formal		
T_8: Use commutative and associative properties of	τ_2: logical τ_6: formal		

Type of Task (T)	Technique (τ)	Technology (θ)	Theory (Θ)
multiplication of integers, decimals, and fractions			
T_9: Multiplication of integers, decimals, and fractions	τ_3: numerical τ_4: symbolic τ_6: formal		
T_{10}: Distributive property of numbers	τ_2: logical τ_6: formal		
T_{11}: Use integers in real world and daily life	τ_1: visual τ_3: numerical τ_5: applicative	θ_3 = applications of integers in everyday life	

As an example, on the basis of Table 7, the praxeological element (T_1/τ_1, τ_3, $\tau_5/\theta_1/\Theta_1$) involves tasks related to recognizing positive and negative number signs in the context of sea surface depth. This task type is represented by the following problem: "The height of Mount Semeru is 3,676 meters above sea level, and the depth of the Java Trench is 7,140 m below sea level. The base reference point is the coastline. How can these quantities be expressed via positive and negative signs?". Solving this problem requires employing visual (τ_1), numerical (τ_3), and applicative (τ_5) methods. The visual technique (τ_1) is necessary for representing the sea level position, the numerical technique (τ_3) is used to assign positive and negative signs to the numbers, and the applied technique (τ_5) is relevant since the problem is related to real-world scenarios. The technological component (θ_1) pertains to how students apply number concepts to solve the problem, whereas the theoretical component serves as the foundation for explaining the properties and rules of integers.

Potential learning obstacles in textbook use

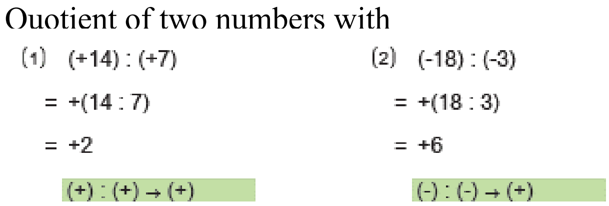
In this study, the potential for learning obstacles arises from the incomplete presentation of material, leading to gaps in understanding and difficulties in completing assigned tasks. These challenges occur when fundamental concepts are not systematically introduced before students are confronted with more complex problems. Addressing this issue requires designing an instructional approach that integrates the concepts of integers, fractions, and decimals, thereby facilitating a deeper conceptual understanding. The incorporation of instructional media as a supporting tool can enhance the visualization of abstract concepts, such as the use of a digital number line or visual simulations, to intuitively illustrate relationships within numerical systems. Furthermore, structuring the instructional design on the basis of established theoretical frameworks ensures a systematic and organized presentation of content. This not only aids in organizing material effectively but also guarantees that the acquired knowledge holds epistemic value. The following problems with fractions and decimals are found in the textbook:

Calculate

1) (0.3) + (1.2)	4) $\left(-\frac{3}{5}\right) + (-\frac{4}{5})$
2) (-0.7) + (0.5)	5) $\left(-\frac{1}{2}\right) + (-\frac{3}{4})$
3) (1.4) + (-0.9)	6) $\left(\frac{1}{4}\right) + (-\frac{5}{6})$

Figure 2. Fraction and decimal number problems in Indonesian math textbooks

Furthermore, the Indonesian textbook contains a conceptual error in the quotient of two numbers with different signs. The book provides examples of (+14): (+7) = +2 and (-18): (-3) = +6, stating that these two examples demonstrate the quotient of two numbers with different signs. However, this explanation is not consistent with the right mathematical concept. In the first example, the numbers (+14) and (+7) have the same sign, which is positive; hence, the result is valid. However, it does not account for the operation of numbers that have different signs. Similarly, in the second example, (-18) and (-3) are two negative numbers, so the quotient of these two numbers is positive, not an example of the quotient of numbers with different signs. Instead, to illustrate the quotient of numbers with different signs, the right example is that of (+18): (-3) = -6 or (-18): (+3) = -6, where one number is positive and the other is negative; hence, the result is negative. This error is likely caused by a typing error but needs to be corrected so that students do not misunderstand the sign rule in the division of positive and negative numbers. If this error is left unchecked, students may misinterpret the basic concept of number operations, which will affect their understanding of further material. In math learning, a correct understanding of the sign rule is essential for more complex operations.


Quotient of two numbers with

(1) (+14) : (+7)
= +(14 : 7)
= +2
(+) : (+) → (+)

(2) (-18) : (-3)
= +(18 : 3)
= +6
(-) : (-) → (+)

Figure 3. Typing errors in Indonesian math textbooks

Discussion

Presentation of integer materials in mathematics textbooks

The analysis revealed a significant discrepancy between the learning objectives stated in the curriculum and the content presented in the mathematics textbooks. The seventh-grade curriculum explicitly outlines several key objectives, including mastery of prime factorisation and conceptual connections among mathematical ideas such as LCM and GCD. However, these topics were entirely absent from the analysed textbooks. Instead, the textbook primarily focuses on procedural exercises involving operations such as addition and multiplication of integers, without incorporating activities designed to foster conceptual understanding, aligned with the curriculum. This indicates a misalignment between the intended and implemented curricula, as reflected in the textbook content.

This misalignment deserves critical attention, given the central role of textbooks in classroom instruction. Choppin et al. (2022) emphasize that alignment between curriculum and instructional materials is essential to ensure effective curriculum implementation. Teachers who rely heavily on textbooks may deliver instruction based solely on the available content, potentially neglecting essential learning objectives. Fan et al. (2021) similarly argue that when textbooks fail to comprehensively reflect curriculum goals, both teachers and students may be deprived of access to deeper mathematical insights. Zhao et al. (2023) further demonstrate that inconsistencies between curriculum, teaching practices, and instructional materials can negatively impact student outcomes by disrupting the coherence and progression of learning experiences.

Pedagogically, this lack of alignment may hinder the development of students' conceptual understanding. When instructional materials focus solely on procedural skills and exclude targeted conceptual content, students are less likely to engage in representational thinking, mathematical reasoning, and problem-solving. This finding also suggests that the textbook content is heavily centred on technical skills, a trend that becomes even more evident when examining the types of tasks and techniques embedded in the materials.

Further analysis revealed a significant imbalance in the distribution of tasks across subtopics. The subtopic "multiplication and division" contains 33 tasks, while "positive and negative numbers" included only 12. This disproportion indicates an excessive emphasis on procedural skills, particularly arithmetic operations, rather than foundational conceptual development, such as number placement on the number line or understanding the role of positive and negative signs. These foundational concepts are crucial for understanding subsequent topics.

Such an imbalance not only affects students' time allocation but also shapes their perception of mathematics as purely computational rather than conceptual in nature. Jukić Matić and Glasnović Gracin (2016) argue that the structure and distribution of textbook tasks directly influence students' thinking strategies and learning behaviors. Similarly, Huang et al. (2022) noted that unbalanced textbook content may hinder the development of higher-order thinking skills, as students are overexposed to repetitive procedural problems. In this context, disproportionate task allocation can limit students' learning experiences and disrupt the logical connections that should be fostered across mathematical concepts.

In addition to the unequal distribution of tasks, the analysis shows a dominance of procedural task types, particularly Task Type 5 (positive and negative integer operations), which accounted for nearly 48% of all tasks. This is followed by Task Type 11 (using integers in the real world and daily life), which, despite being contextual, remains grounded in basic arithmetic. The lack of task variation suggests that students are repeatedly exposed to routine and algorithmic activities rather than tasks that promote deeper conceptual engagement, such as comparing representations, generalising patterns, and constructing mathematical arguments. This procedural tendency is further reflected in the solution techniques emphasised in textbooks. Numerical ($\tau 3$) and formal ($\tau 6$) techniques are dominant, relying heavily on direct calculations and the application of standard arithmetic rules, such as commutativity, associativity, and distributivity. In contrast, visual ($\tau 1$), symbolic ($\tau 4$), and applicative ($\tau 5$) techniques appear less frequently and lack consistent integration into the learning sequence. These findings indicate that the textbook promotes rote procedural performance over relational understanding, which is essential for meaningful learning in mathematics.

The imbalance in task types and techniques has direct implications for the development of students' mathematical proficiency. Kilpatrick et al. (2001) identify five interrelated strands of mathematical proficiency: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. In this textbook, only procedural fluency is strongly emphasised, while other dimensions are notably under-represented. This observation is consistent with the findings of Fan et al. (2013), who noted that mathematics textbooks in various countries often focus on procedural presentation and provide limited opportunities for reasoning and conceptual engagement in the learning process. A similar conclusion was drawn by Kuncoro et al. (2024), who demonstrated that the absence of certain task types in Indonesian and Singaporean textbooks influenced students' conceptualisation of the material. Their study emphasised the importance of task diversity and structure in supporting both conceptual and procedural understanding of mathematics. Conversely, when this diversity is absent, as in the case of procedural task dominance found in the analysed textbook, students may be able to perform calculations correctly without understanding the rationale behind the procedures used.

From a didactical perspective, Brousseau (1997) highlights the importance of constructing didactical situations in which students actively engage with tasks that challenge their thinking and promote knowledge construction. However, if textbooks only provide closed-ended routine tasks, students' opportunities to participate in such meaningful learning experiences are significantly reduced. Thus, the dominance of procedural tasks and techniques in the textbook is not merely a matter of content variation; it fundamentally shapes how students experience and engage with mathematics as a discipline.

Potential learning obstacles in textbook use

The potential learning obstacles identified in the analysed textbook relate to the presentation of material that is not supported by adequate conceptual explanations in certain sections. Several topics, such as operations involving decimals and fractions, are presented directly as practice problems without introductory explanations that gradually build important conceptual understanding. Such presentations may lead to didactical learning obstacles, as the sequence of content delivery does not follow a logical conceptual progression or consider students' learning paths (Brousseau, 1997; Chevallard, 2006). When content is delivered without sufficient conceptual transitions, students experience confusion in understanding the interrelations among subtopics and tend to rely on procedural techniques to solve problems.

This finding is supported by Zhang et al. (2022), who revealed that a lack of conceptual understanding of a topic can be a major barrier to mathematics learning, particularly for students with learning difficulties. Lodge et al. (2018) also emphasized that unstructured textbook content may result in confusion and conceptual misunderstandings, especially when no explicit effort is made to build understanding progressively. In this context, the design of textbook material should incorporate the principles of learning progression and conceptual scaffolding (Huang et al., 2022), enabling students to develop a layered and coherent understanding.

In addition to didactical obstacles, the analysis also revealed the presence of ontogenic learning obstacles, which originate from students' cognitive unpreparedness due to a lack of relevant prior knowledge. This becomes evident when students are required to solve problems that integrate integers, fractions and decimals. Lutfi et al. (2021) found that students who have not fully mastered the concept of number systems struggle to transition from one form of numerical representation to another. This condition is further exacerbated by the lack of conceptual tasks that bridge students' understanding, as highlighted by Kilpatrick et al. (2001) in their framework of mathematical proficiency.

In the context of mathematics learning, it is important to design the sequencing of content and task types based on students' cognitive readiness to learn. One instructional approach that may be applied is the concrete–representational–abstract (CRA) sequence, as recommended by Al-Salahat (2022). This approach has been proven effective in helping students move gradually from concrete understanding to formal representation. Without such intervention, students are more likely to depend on procedural strategies without understanding the reasoning behind each step of the problem.

The implications of these obstacles are significant in several ways. Students are at risk of forming incorrect or superficial mental models when they are not adequately supported in developing conceptual understanding. In the long term, this may hinder their mastery of more complex and integrated mathematical content in the future. As emphasised by the National Council of Teachers of Mathematics (2000), understanding the relationships among different representations of numbers is foundational to fostering flexible mathematical reasoning and problem-solving. Resnick et al. (2023) also demonstrated that strong conceptual mastery of fractions and decimals is closely correlated with long-term success in mathematics.

Conclusion

This study employs a praxeology theoretical framework to analyse mathematics textbooks used in classroom instruction. The analysis was conducted by examining praxis and logos blocks, which encompass various types of tasks, techniques, technology, and theory. Based on these findings, the textbook materials are systematically structured but lack support for epistemic knowledge construction. However, the analysis also revealed the presence of potential didactic and ontogenic conceptual obstacles, as certain task types were introduced without prior explanation.

The researchers hope that these findings will influence teaching practices and curriculum design in Indonesia. In instructional practice, teachers should be more selective in choosing textbooks for classroom use, ensuring that the selected materials minimise potential learning obstacles. Additionally, instructional design should be grounded in established theoretical frameworks, making the learning process more systematic and effective in fostering deeper conceptual understanding among students. From a curriculum-design perspective, textbooks reflect the implemented curriculum. Therefore, textbook authors must align the instructional content with the applicable curriculum to ensure a structured learning process. Furthermore, the researchers recommend that the Indonesian government develop its own textbooks instead of adapting materials from other countries. This approach provides contextually relevant content that is better suited to students' capabilities and learning environments in Indonesia.

Limitations of the study This study had several limitations. First, the analysed textbooks were limited to those used in public schools in Indonesia, which may not fully represent the characteristics of the textbooks used in private schools or other educational institutions. Second, the scope of the analysis focused solely on integer-related topics, meaning that the findings may not capture potential didactic obstacles in other areas of mathematics.

Acknowledgment

The authors sincerely acknowledge the *Pusat Pembiayaan dan Asesmen Pendidikan Tinggi* (PPAPT) Ministry of Higher Education, Science, and technology of Republic Indonesia and the *Lembaga Pengelola Dana Pendidikan* (LPDP) for their generous financial support and invaluable contributions, which have been instrumental in facilitating this research. Their support has played a crucial role in the successful completion of this study.

Conflicts of Interest

The authors declare no conflict of interest regarding the publication of this manuscript.

Funding Statement

This research was conducted with the financial support of the *Pusat Pembiayaan dan Asesmen Pendidikan Tinggi* (PPAPT) Ministry of Higher Education, Science, and technology of Republic Indonesia and the *Lembaga Pengelola Dana Pendidikan* (LPDP), whose contributions were essential to the successful completion of this study (Grant No. 01837/BPPT/BPI.06/9/2024).

Author Contributions

Putri Fitriasari: Conceptualization, writing-original draft, editing, data curation, project administration, visualization, review, and validation; **Suhendra:** Conceptualization, data curation, supervision, review, validation, and methodology; **Didi Suryadi:** Conceptualization, review, validation, formal analysis, and methodology; **Elah Nurlaelah**: Review, validation, and formal analysis.

References

Al-Salahat, M. M. S. (2022). The effect of using concrete-representational-abstract sequence in teaching the perimeter of geometric shapes for students with learning disabilities. *International Journal of Education in Mathematics, Science and Technology*, *10*(2), 477–493. https://doi.org/10.46328/ijemst.2403

Brousseau, G. (1997). *Theory of didactical situations in mathematics*. Kluwer Academic Publishers.

Carson, J. (2007). A problem with problem solving: teaching thinking without teaching knowledge. *The Mathematics Educator*, *17*(2), 7–14.

Chevallard, Y. (2007). Readjusting didactics to a changing epistemology. *European Educational Research Journal*, *6*(2), 131–134. https://doi.org/10.2304/eerj.2007.6.2.131

Chevallard, Y. (2006). Steps towards a new epistemology in mathematics education.

Proceedings of the IV Congress of the European Society for Research in Mathematics Education, 21–30.

Choppin, J., Roth McDuffie, A., Drake, C., & Davis, J. (2022). The role of instructional materials in the relationship between the official curriculum and the enacted curriculum. *Mathematical Thinking and Learning*, *24*(2), 123–148. https://doi.org/10.1080/10986065.2020.1855376

Dong, A., Jong, M. S.-Y., & King, R. B. (2020). How does prior knowledge influence learning engagement? the mediating roles of cognitive load and help-seeking. *Frontiers in Psychology*, *11*, 1–10. https://doi.org/10.3389/fpsyg.2020.591203

Fan, L., Cheng, J., Xie, S., Luo, J., Wang, Y., & Sun, Y. (2021). Are textbooks facilitators or barriers for teachers' teaching and instructional change? An investigation of secondary mathematics teachers in Shanghai, China. *ZDM – Mathematics Education*, *53*(6), 1313–1330. https://doi.org/10.1007/s11858-021-01306-6

Fan, L., Zhu, Y., & Miao, Z. (2013). Textbook research in mathematics education: development status and directions. *ZDM-Mathematics Education*, *45*(5), 633–646. https://doi.org/10.1007/s11858-013-0539-x

Hendriyanto, A., Suryadi, D., Dahlan, J. A., & Juandi, D. (2023). Praxeology review: comparing Singaporean and Indonesian textbooks in introducing the concept of sets. *Eurasia Journal of Mathematics, Science and Technology Education*, *19*(2), 1–13. https://doi.org/10.29333/ejmste/12953

Huang, R., Tlili, A., Zhang, X., Sun, T., Wang, J., Sharma, R. C., Affouneh, S., Salha, S., Altinay, F., Altinay, Z., Olivier, J., Jemni, M., Wang, Y., Zhao, J., & Burgos, D. (2022). A comprehensive framework for comparing textbooks: insights from the literature and experts. *Sustainability*, *14*(6940), 1–19. https://doi.org/10.3390/su14116940

Jäder, J., Lithner, J., & Sidenvall, J. (2020). Mathematical problem solving in textbooks from twelve countries. *International Journal of Mathematical Education in Science and Technology*, *51*(7), 1120–1136. https://doi.org/10.1080/0020739X.2019.1656826

Jukić Matić, L., & Glasnović Gracin, D. (2016). The use of the textbook as an artefact in the classroom. *Journal Für Mathematik-Didaktik*, *37*(2), 349–374. https://doi.org/10.1007/s13138-016-0091-7

Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: helping children learn mathematics*. National Academy Press. https://doi.org/10.17226/10434

Klang, N., Karlsson, N., Kilborn, W., Eriksson, P., & Karlberg, M. (2021). Mathematical problem-solving through cooperative learning—the importance of peer acceptance and friendships. *Frontiers in Education*, *6*, 1–10. https://doi.org/10.3389/feduc.2021.710296

Kuncoro, K. S., Suryadi, D., Dahlan, J. A., & Jupri, A. (2024). Praxeological analysis in Indonesian and Singaporean mathematics textbooks: an understanding geometrical similarity by students. *Journal on Mathematics Education*, *15*(4), 1197–1218. https://doi.org/10.22342/jme.v15i4.pp1197-1218

Kusharyadi, R., Fatimah, S., & Kusnandi, K. (2024). A praxeological review of concept-sequence and series: comparing Malaysia and Indonesia textbooks. *Jurnal Elemen*, *10*(2), 239–259. https://doi.org/10.29408/jel.v10i2.24079

Lodge, J. M., Kennedy, G., Lockyer, L., Arguel, A., & Pachman, M. (2018). Understanding difficulties and resulting confusion in learning: an integrative review. *Frontiers in Education*, *3*, 1–10. https://doi.org/10.3389/feduc.2018.00049

Lutfi, M. K., Juandi, D., & Jupri, A. (2021). Students' ontogenic obstacle on the topic of triangle and quadrilateral. *Journal of Physics: Conference Series*, *1806*(1), 012108. https://doi.org/10.1088/1742-6596/1806/1/012108

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. NCTM. https://Www.Nctm.Org/Standards-and-Positions/Principles-and-

Standards/. https://www.nctm.org/Standards-and-Positions/Principles-and-Standards/

Nomvuyo M, T., David Sekao, R., & Ogbonnaya I, U. (2023). Mathematics teachers use of textbooks for instructional decision-making in lesson study. *Journal of Pedagogical Research*, *7*(4), 1–13. https://doi.org/10.33902/JPR.202320138

Özgeldi, M., & Esen, Y. (2010). Analysis of mathematical tasks in Turkish elementary school mathematics textbooks. *Procedia - Social and Behavioral Sciences*, *2*(2), 2277–2281. https://doi.org/10.1016/j.sbspro.2010.03.322

Pierard, C., Svihla, V., Clement, S. K., & Fazio, B. S. (2020). Undesirable difficulties: Investigating barriers students' learning with ebooks in a semester-length course. *College and Research Libraries*, *81*(2), 170–192. https://doi.org/10.5860/crl.81.2.170

Remillard, J. T. (2000). Can curriculum materials support teachers' learning? two fourth-grade teachers' use of a new mathematics text. *The Elementary School Journal*, *100*(4), 331–350. https://doi.org/10.1086/499645

Resnick, I., Newcombe, N., & Goldwater, M. (2023). Reasoning about fraction and decimal magnitudes, reasoning proportionally, and mathematics achievement in Australia and the United States. *Journal of Numerical Cognition*, *9*(1), 222–239. https://doi.org/10.5964/jnc.8249

Rezat, S. (2012). *Interactions of teachers' and students' use of mathematics textbooks* (G. Gueudet, B. Pepin, & L. Trouche (eds.); pp. 231–245). Springer Netherlands. https://doi.org/10.1007/978-94-007-1966-8_12

Rezat, S., Fan, L., & Pepin, B. (2021). Mathematics textbooks and curriculum resources as instruments for change. *ZDM – Mathematics Education*, *53*(6), 1189–1206. https://doi.org/10.1007/s11858-021-01309-3

Rzyankina, E., George, F., & Simpson, Z. (2024). Enhancing conceptual understanding in engineering mathematics through e-textbooks. *IEEE Transactions on Education*, *67*(4), 534–541. https://doi.org/10.1109/TE.2024.3387102

Shepherd, M. D., Selden, A., & Selden, J. (2009). Difficulties first-year university students have in reading their mathematics textbook. *Tennessee Technological University, Technical Report*, 1–45.

Sinaga, B., Sitorus, J., & Situmeang, T. (2023). The influence of students' problem-solving understanding and results of students' mathematics learning. *Frontiers in Education*, *8*. https://doi.org/10.3389/feduc.2023.1088556

Suryadi, D. (2023). *Jalan epistemik menghasilkan penegtahuan melalui didactical design research (DDR) [The epistemic trajectory of knowledge production through didactical design research (DDR)]*.

Suryadi, D., Itoh, T., & Isnarto. (2023). A prospective mathematics teacher's lesson planning: An in-depth analysis from the anthropological theory of the didactic. *Journal on Mathematics Education*, *14*(4), 723–739. https://doi.org/10.22342/jme.v14i4.pp723-740

Utami, N. S., Mizoguchi, T., Prabawanto, S., & Suryadi, D. (2025). A praxeological analysis of functions in lower secondary school: comparing the textbooks in Japan and Indonesia. *International Electronic Journal of Mathematics Education*, *20*(2), em0814. https://doi.org/10.29333/iejme/15818

Utami, N. S., Prabawanto, S., & Suryadi, D. (2024). How do Indonesian students learn function concepts? A praxeological analysis of textbook. *Journal on Mathematics Education*, *15*(2), 451–472. https://doi.org/10.22342/jme.v15i2.pp451-472

Wang, S., Winsløw, C., & Xu, B. (2024). Mathematical praxeologies in the Chinese and Singaporean secondary school textbooks : the case of probability. *Proceedings of the Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13)*, 4221–4228.

Wang, X. (2024). *Methodological approaches in exploring textbook structures*.

https://www.proquest.com/docview/3092180055

Wijayanti, D., & Winslow, C. (2017). Mathematical practice in textbooks analysis: praxeological reference models, the case of proportion. *Journal of Research in Mathematics Education*, *6*(3), 307–330. https://doi.org/10.17583/redimat.2017.2078

Yunianta, T. N. H., Suryadi, D., Dasari, D., & Herman, T. (2023). Textbook praxeological-didactical analysis: lessons learned from the Indonesian mathematics textbook. *Journal on Mathematics Education*, *14*(3 SE-Articles), 503–524. https://doi.org/10.22342/jme.v14i3.pp503-524

Zhang, S., Yu, S., Xiao, J., Liu, Y., & Jiang, T. (2022). The effects of concrete-representational-abstract sequence instruction on fractions for Chinese elementary students with mathematics learning disabilities. *International Journal of Science and Mathematics Education*, *20*(7), 1481–1498. https://doi.org/10.1007/s10763-021-10215-9

Zhao, L., Zhao, B., & Li, C. (2023). Alignment analysis of teaching–learning-assessment within the classroom: how teachers implement project-based learning under the curriculum standards. *Disciplinary and Interdisciplinary Science Education Research*, *5*(1), 13. https://doi.org/10.1186/s43031-023-00078-1