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Transformations of students' cognitive processes when solving PISA-like problems: A commognitive analysis

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Abstract

The mathematical literacy of Indonesian students in PISA 2022 was categorized as low, highlighting the importance of understanding students' cognitive processes in mathematical problem solving. This study examines transformations in the cognitive processes of a 15-year-old Indonesian student when solving PISA-like problems using Sfard's commognitive framework. A qualitative approach was employed to capture detailed learning dynamics. The participant was a 15-year-old student from a randomly selected junior high school in Rembang Regency, Central Java Province, Indonesia, purposefully selected based on the mathematics teacher's nomination for strong mathematical ability and clear evidence of cognitive shifts during problem solving. The tasks were developed by the researcher and adapted to relevant Indonesian contexts. Qualitative data—including written work, observations, and semi-structured interviews—were analyzed using mathematical literacy processes (formulating, employing, interpreting, and evaluating) and mapped onto four commognitive components: Word Use (WU), Visual Mediators (VM), Routines (R), and Narratives (N). Findings show that reflective self-evaluation supports cognitive restructuring, enabling movement from procedural errors toward coherent reasoning through shifts in WU, VM, and R. The study underscores the need for instructional designs that foster meta-level discourse, reflective thinking, and flexible visual re-representation to strengthen students' mathematical reasoning.

Keywords: cognitive process; commognitive; mathematical literacy; PISA-like mathematical problems

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Introduction

The findings from the Programme for International Student Assessment (PISA) highlight that Indonesian students continue to perform very low in mathematical literacy. Many Indonesian students encounter significant challenges in solving PISA-related problems (Bayirli et al., 2023; Fenanlampir et al., 2019; Stacey, 2011; Thien et al., 2015). These problems not only assess mathematical comprehension but also evaluate students' ability to apply mathematical concepts in practical, dynamic, and real-world contexts (Stacey & Turner, 2015). Solving these tasks requires students to move beyond routine procedures, engage in deeper conceptual understanding, and integrate various sources of information (Heyd-Metzuyanim & Sfard, 2012).

PISA is regarded as a key benchmark for assessing mathematical literacy on a global scale. Stacey (2011) defines mathematical literacy as the ability to use mathematical knowledge and skills effectively to solve real-life problems and communicate solutions. PISA specifically assesses the ability of 15-year-old students to apply mathematical understanding to solve complex real-world problems (OECD, 2023). The outcomes of these assessments provide valuable insights into students' mathematical literacy levels and their capacity for improvement across different countries (Boaler, 2016).

According to the OECD (2023), mathematical literacy is defined as the capacity to apply mathematical knowledge in real-world contexts. However, Indonesian students continue to struggle with PISA-like problems, particularly those requiring the practical application of mathematics in daily life contexts (Wijaya et al., 2014). Several critical challenges have been identified, including frequent conceptual errors, such as incorrect use of formulas and weak foundational understanding, which hinder their ability to solve more advanced problems (Bayirli et al., 2023; Fenanlampir et al., 2019). According to Andrews (2015), these challenges often cause shifts in students' cognitive processes when solving mathematical tasks.

Shifts in cognitive processes in mathematics education reveal how students adapt their understanding and thinking as they encounter increasingly complex problems. Barwell (2016) revealed that this shift involves a progression from intuitive thinking to structured and formal reasoning as problem complexity increases. This transition includes a movement from procedural thinking, which focuses on step-by-step processes, to deeper conceptual reasoning (DeCaro, 2016; Legesse et al., 2020; Stovner & Klette, 2022). These cognitive transformations are shaped by multiple factors, including learning experiences, social interactions, and exposure to unfamiliar, non-routine tasks, such as PISA-based problems. Additionally, errors in understanding or applying mathematical concepts often serve as key triggers for restructuring students' cognitive processes (Alvidrez et al., 2022).

To explore changes in students' cognitive processes when solving PISA-like problems, this study applies the commognitive framework introduced by Sfard (2008) . This framework underscores the central role of language in learning mathematics (Sfard, 2008). From this perspective, mathematical thinking emerges through interactions between individuals and various mediators, including words, visual representations, narratives, and routines (Cooper & Lavie, 2021; Heyd-Metzuyanim & Sfard, 2012). The Commognitive framework provides an effective lens for examining how students develop and adjust their problem-solving strategies when faced with PISA-style mathematical tasks.

Despite extensive research on mathematical literacy in PISA contexts, few studies have examined the transformations in students' cognitive processes when solving PISA-like problems through the commognitive framework. Most existing studies have focused on performance outcomes rather than the evolution of cognitive discourse. This study addresses this gap through an in-depth single-case analysis of cognitive shifts in mathematical-literacy. According to Yin (2018), a single-case design is appropriate for theory building when the case is information-rich and exhibits the phenomenon of interest in depth. This study aims to describe the cognitive shifts in mathematical literacy observed among 15-year-old Indonesian students when solving PISA-like problems through the lens of the commognitive framework. The research question guiding this study is as follows: How do students' cognitive processes transform when solving PISA-like mathematical tasks from a commognitive perspective?

Methods

This study employed a qualitative research approach with a descriptive design. The rationale for using a qualitative approach lies in the focus on exploring students cognitive processes when solving PISA-like problems. The qualitative data in question involve changes in the cognitive patterns of 15-year-old students' mathematical literacy when solving PISA-like mathematical problems, analyzed from a commognitive perspective. This data was described and analyzed in depth using mathematical literacy indicators from the PISA framework, which the researcher further developed into several sub-indicators. These sub-indicators were designed to meet the research's need for a detailed explanation of the mathematical literacy of 15-year-old students.

The first indicator of mathematical literacy, namely Formulating the Problems, consists of three sub-indicators that have been developed: (1) accurately identifying the information provided in the problem, (2) writing the information from the problem using correct mathematical statements, and (3) accurately analyzing what is being asked in the problem. The second indicator, Employing the Problems, includes three sub-indicators that have been developed: (1) developing an appropriate plan for solving the problem, (2) structuring patterns from the problem-solving plan accurately, and (3) writing strategies for the problem-solving plan precisely. The third indicator, Interpreting the Problem, comprises three sub-indicators that have been developed: (1) converting the problem into an appropriate mathematical form using variables, diagrams, or visual representations as necessary; (2) accurately implementing the problem-solving plan; and (3) interpreting the final results in accordance with the context of the problem. The fourth indicator, Evaluating Solutions, is composed of three sub-indicators that have been developed: (1) reviewing the strategies used in the problem-solving plan, (2) rechecking the obtained results for accuracy, and (3) writing a clear and accurate conclusion based on the problem's outcome. By operationalizing the indicators into sub-indicators, the researcher is better equipped to examine the stages and processes of students' mathematical literacy in greater detail. Furthermore, the analysis in this study is clarified through the use of commognitive components, which include Word Uses (WU), Visual Mediators (VM), routines (R), and narratives (N).

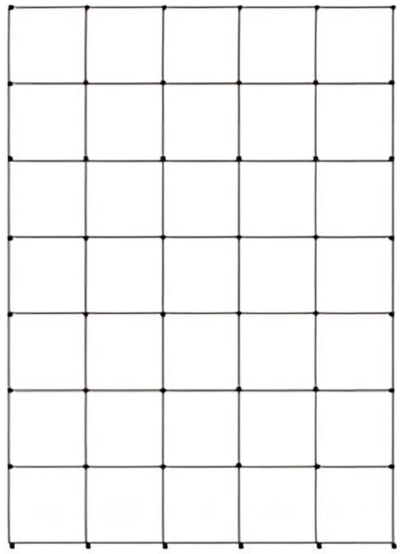
The subject of this study is a 15-year-old student from a randomly selected junior high school in Rembang Regency, Central Java Province, Indonesia. From this school, the participant was purposefully selected because the student was nominated by the mathematics teacher for demonstrating strong mathematical ability and exhibited rich indications of cognitive shifts when solving PISA-like mathematical problems. This aligns with the purposeful sampling strategies commonly used in theory-building case studies, where participants are chosen for their potential to provide in-depth and information-rich data (Patton, 2015; Yin, 2018).

The data collection in this study involved PISA-like mathematical problem tests, observations, and interviews with the research participants. The PISA-like mathematical problems were developed by the researcher and adapted to the Indonesian contextual framework, as shown in Table 1. The problems were validated by one professor and one associate professor of mathematics education at Universitas Negeri Semarang (UNNES). The validation process was conducted through a direct expert review, in which the professor and associate professor examined the tasks and provided qualitative judgments regarding their alignment with PISA characteristics—particularly realism, mathematical literacy demands, and required cognitive processes (OECD, 2023). Although no formal scoring rubric was used, both experts independently confirmed that the problems met the expected standards for relevance, realism, and cognitive demand, indicating a high level of content validity based on the expert consensus.

Observations were conducted throughout the problem-solving process, focusing on supporting data such as the participants' gestures, verbal expressions, and actions. These observational notes were used to complement the analysis of cognitive processes and strengthen the interpretation of students' discursive shifts. The interviews were conducted in a semi-structured format and lasted approximately 30 minutes. The session was recorded using a mobile phone camera to capture both verbal responses and relevant behavioral nuances of the participants. The recordings were transcribed verbatim in Bahasa Indonesia to preserve the original meaning and discourse. The transcript was then translated into English using AI-assisted translation, followed by manual refinement to ensure accuracy and conceptual fidelity. The data for this study included students' answers to the PISA-like problems, observational notes, and interview data. The specific PISA-like mathematical problems provided to the research subjects are presented in Table 1.

Table 1. PISA-like mathematical problems

PISA-Like Mathematical Problems
At the Festival Jaga Jarak: *Ajang Berkreasi Para Seniman Belia* (Social Distancing Festival: A Creative Platform for Young Artists) held in Parachiyangan Field, Majalengka Regency, West Java, the field allocated for festival participants is rectangular in shape, as illustrated below.



Picture sources : https://kumparan.com

To minimize the potential for rule violations, all participants must adhere to social distancing rules. The organizers have marked specific points on the field where participants are allowed to sit, ensuring compliance with the one-meter distancing rule. Based on the spacing, it is estimated that 48 participants can occupy the field.

To answer the questions, follow the steps in the activities below:

a) The organizers plan to set up advertising boards as part of sponsorship around the perimeter of the field, except for the front side, which is reserved for the event reception and judges' area. The organizers want to determine the perimeter of the participants' area first. One organizer attempted to calculate the perimeter by numbering each marked point around the area, arriving at a total perimeter of 24 meters. Do you agree with this result? If not, explain the steps to determine the correct perimeter and provide the result.

b) Determine the length of the advertising boards that the organizers will need to construct. Assume that there is a 1-meter distance between the participants' area and the advertising boards on all sides except the front side, which must have a 2-meter distance.

c) For the event's final report, the organizers need to include the dimensions of the field used for the festival. From the problem description, can the area of the festival field be determined? If so, calculate the area and also find the ratio of the participants' area to the total area of the festival field. Explain your steps in detail.

d) If the organizers are also asked to measure the perimeter of the entire festival field, describe the steps they can take to determine the perimeter. Provide a detailed explanation of the process.

The research data were analyzed based on a set of mathematical literacy indicators developed by the researcher. These indicators reflect the cognitive processes of 15-year-old students when solving PISA-like mathematical problems (OECD, 2023), and were further examined through the Commognitive perspective (Sfard, 2007). A detailed presentation of these indicators is provided in Table 2.

Table 2. Indicators of 15-year-old students' mathematical literacy in solving PISA-like mathematical problems from a commognitive perspective

Indicators of Mathematical Literacy	Sub-Indicators of Mathematical Literacy	Code	Commognitive
Formulating The Problems	Accurately identifying the information provided in the problem.	F1	WU, VM, R, N
	Writing the information from the problem using correct mathematical statements.	F2	WU, VM, R, N
	Analyzing what is being asked in the problem accurately.	F3	WU, VM, R, N
Employing The Problems	Developing an appropriate plan for solving the problem.	E1	WU, VM, R, N
	Structuring patterns from the problem-solving plan accurately.	E2	WU, VM, R, N
	Writing the strategies for the problem-solving plan precisely.	E3	WU, VM, R, N
Interpreting The Problem	Converting the problem into an appropriate mathematical form using variables, diagrams, or visual representations as necessary.	I1	WU, VM, R, N
	Accurately implementing the problem-solving plan.	I2	WU, VM, R, N
	Interpreting the final results in accordance with the context of the problem.	I3	WU, VM, R, N
Evaluating The Solutions	Reviewing the strategies used in the problem-solving plan.	Ev1	WU, VM, R, N
	Rechecking the obtained results for accuracy.	Ev2	WU, VM, R, N
	Writing a clear and accurate conclusion based on the problem's outcome.	Ev3	WU, VM, R, N

Results

The results of this study are used to answer the research question: how do changes in the cognitive patterns of students' mathematical literacy occur when solving PISA-like mathematical problems from a Commognitive perspective?. In this study, the subject is coded as S3.

1. Changes in the cognitive pattern of S3 in question (a)

At the stage of formulating the problem in question (a), S3 identified the length and width of the festival area as 7 meters and 5 meters by calculating using the diagram provided in the problem. In this activity, codes F1 and F2 were executed well by the subject. The subject then interpreted the length and width values into mathematical procedures. It was observed that initially, the subject had established a problem-solving plan using the concept of the area of a rectangle, as shown by the diagram marked with a cross in Figure 1 below.

Figure 1. Subject's response sheet for question (a)

However, after revisiting what was being asked in the problem, S3 realized that the question was about the perimeter of the festival area. As a result, S3 revised the answer using the concept of perimeter to solve question (a). In this activity, S3 was able to evaluate the cognitive process, reanalyze the question, and establish an appropriate problem-solving plan. Codes Ev1, Ev2, and Ev3 were demonstrated by S3 very effectively. S3's response is presented in Figure 2 below.

Figure 2. Subject's response sheet for question (a)

Below is a confirmation from the interview excerpt with S3:

R : Could you explain your answer to question (a)?

S3 : It asks to determine the perimeter... first, I find the length and width, which turned out to be 7 and 5, then using the formula 2(p+l), I get 2(7+5) = 24.

R : What concept did the committee use?

S3 : One of the committee members used a method of numbering each section around the perimeter. This is the same as the formula 2(p+l).

The explanation above showed that S3 was able to understand the basic concept of using the formula. The subject stated that the sections around the perimeter correspond to the sum of the lengths of each pair of opposite sides. S3 converted the problem into the appropriate mathematical form, established a correct problem-solving plan, and reviewed the strategy for solving the problem accurately. The codes F3, I1, I2, I3, E1, E2, and E3 were well executed by the subject. In terms of Commognitive components, the WU (Word Uses) applied were the terms "length," "width," and "perimeter." When determining the length and width, the VM (Visual Mediators) used by S3 were the diagrams in the problem. The subject's calculation process is shown in Figure 3 below.

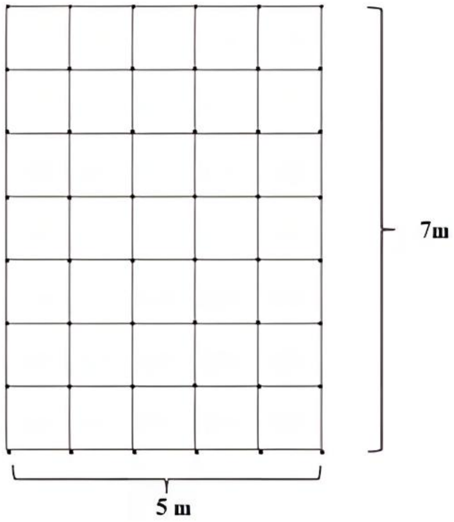


Figure 3. Subject's calculation process for question (a)

The subject determined the length and width of the festival area by counting one square to the next, which represented the length and width. The R (Routines) used by S3 were found in the process of counting one square to the next and applying the perimeter of a rectangle procedure. The N (Narratives) used by S3 were based on the concept of the perimeter of a rectangle. Below, the flow of changes in the subject's cognitive pattern for question (a) is presented in Figure 4.

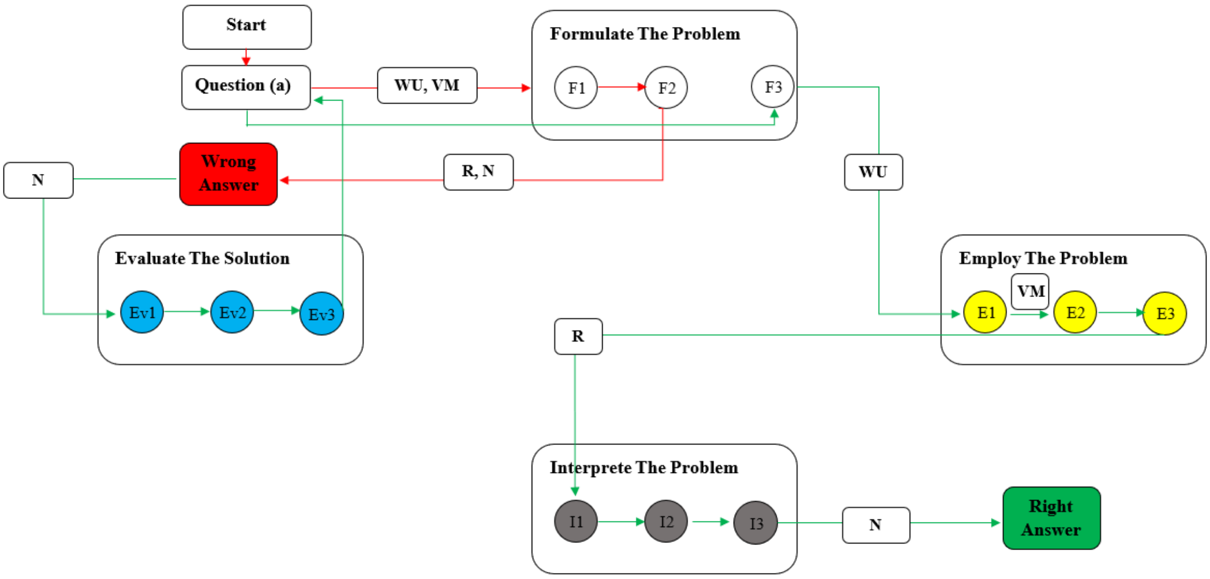


Figure 4. Flow of changes in the subject's cognitive pattern for question (a)

2. Changes in S3's cognitive pattern in question (b)

In question (b), S3 began by identifying the question and the given information in the problem. The question to be identified was the length of the brochure sign surrounding the participants. The information to be identified was that each side of the festival area was given a 1-meter distance from the brochure sign, except for the front side, which was given a 2-meter distance.

In this activity, the Word Uses (WU) included: "length of the participant area, length of the festival field, width of the participant area, and width of the festival field ". These WUs were used by the subject as a foundation to represent the problem. S3 illustrated the problem in Figure 5 below.

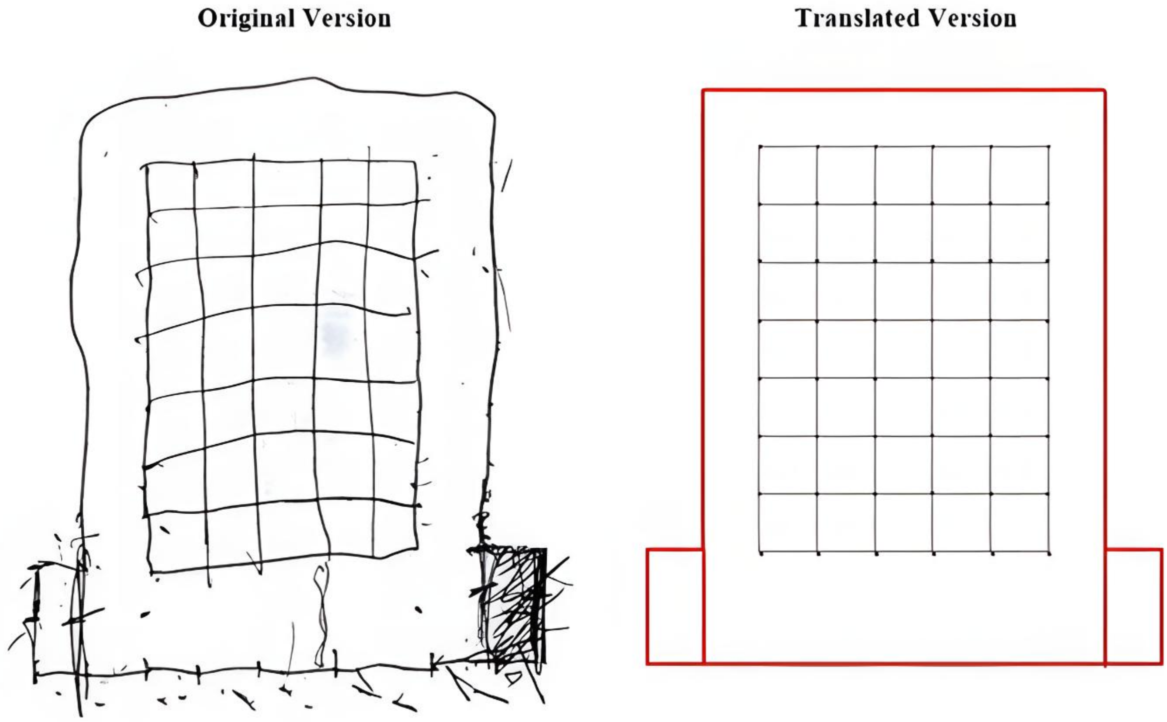


Figure 5. Subject's answer sheet for question (b)

In this case, the Visual Mediators (VM) used by the subject are the images they created. The subject solved the problem by determining the perimeter on the image they had drawn. Then, the subject calculated each segment one by one. The Routines (R) involved the procedure for calculating the perimeter. The components of R, WU, and VM were used by the subject as a foundation to construct their Narratives (N). The subject explained that to calculate the perimeter, they determined the length of each side that surrounded the area. This became the Narrative (N) in the Commognitive framework. The subject found that the result for the perimeter was 37 meters. In this activity, codes F1 and F2 were correctly followed by the subject. However, other codes were not properly implemented. In the in-depth interview session, it was revealed that the subject revised their cognitive process. Below is an excerpt from the interview with S3:

R : *How did you answer question (b)?*
S3 : *It asked to determine the perimeter... of the brochure sign. I calculated it by segments... hmm, I'm having trouble explaining it.*
R : *Can you illustrate the concept?*
S3 : *I started drawing according to the image.*
R : *How did you calculate the perimeter?*
S3 : *I calculated each segment, so the perimeter was 37 meters.*
R : *Is there anything wrong with your answer?*
S3 : *Hmm (pauses while re-checking the question and the answer)... I don't know yet.*

R : Does this also include the front side?
S3 : Oh... hmm... no. Aha... it should only be the perimeter of all sides except the front side.

The change in the cognitive process of S3 shows that the Visual Mediators (VM) they used were actually incorrect. The shift in S3's cognitive occurred when the researcher asked, "Does the front side also count?" At that point, S3 re-examined the problem-solving strategy they had previously created. In this activity, codes Ev1, Ev2, and Ev3 were correctly followed by the subject. After observing their answer again, S3 became aware of the error and began marking Word Uses (WU) to determine the size of the festival grounds. The WU used involved the length of 1 meter for each side, except for the front side, which was 2 meters long. This was because the front side, according to question (a), was to be used for the event's reception and the jury. In this activity, codes F1, F2, and F3 were followed correctly by the subject. The subject then organized a pattern and converted the problem into the appropriate mathematical form. They proceeded to analyze the lengths of the sides they needed to determine. S3 began recalculating the perimeter for the brochure sign. In this activity, codes E1, E2, and E3 were also properly followed by the subject. The problem-solving plan of the subject is shown in Figure 6 below.

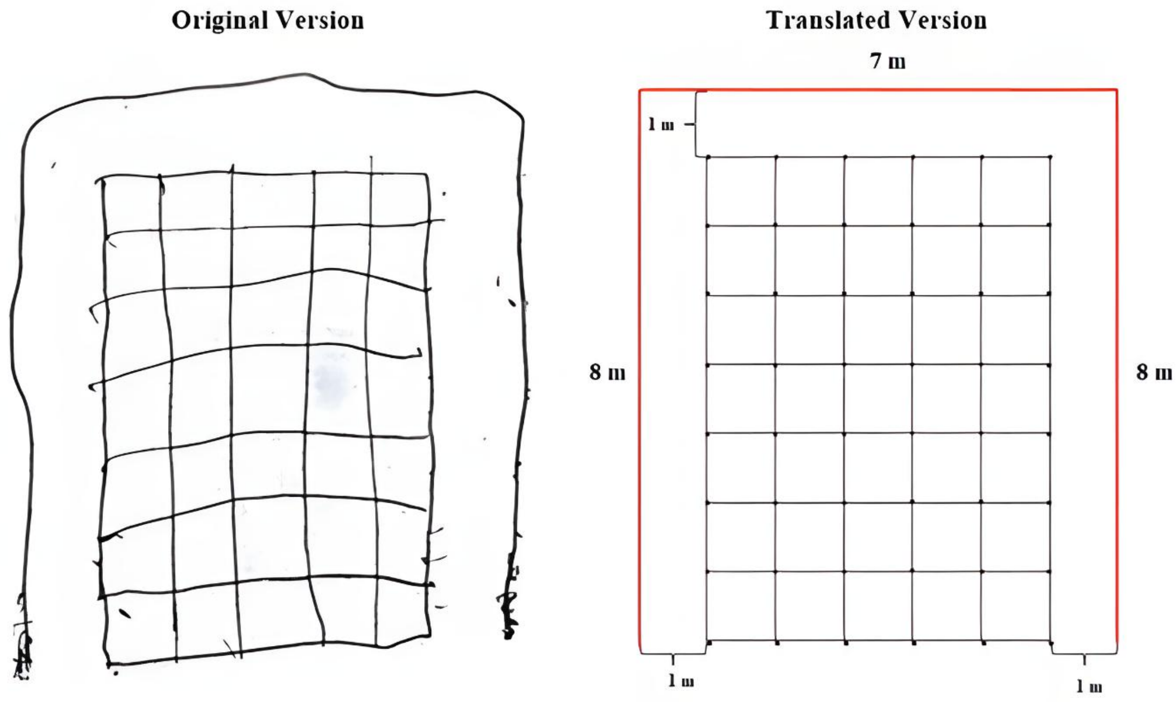


Figure 6. Revised answer sheet of the subject for question (b)

The subject only counted the sides surrounding the brochure sign. It was determined that the length was 8 meters, the width was 7 meters, and the total length of the brochure sign surrounding the area, according to the problem's rules, was 23 meters. In this activity, the subject converted the problem into the correct mathematical form.

For the Commognitive components, the Word Uses (WU) included the terms "length, width, and perimeter". The Visual Mediators (VM) used by S3 underwent a change in the schema from Figure 5 to Figure 6. The Routines (R) used by S3 involved the procedure for calculating the perimeter of the brochure sign surrounding the area. The Narratives (N) used by S3 were based on the understanding of the concept of the brochure sign surrounding the participant area.

In this activity, the codes I1, I2, and I3 were properly followed by the subject. Therefore, it can be concluded that the subject successfully followed all the codes for Ev, F, E, and I. The flow of the subject's changing cognitive process for question (b) is shown in Figure 7 below.

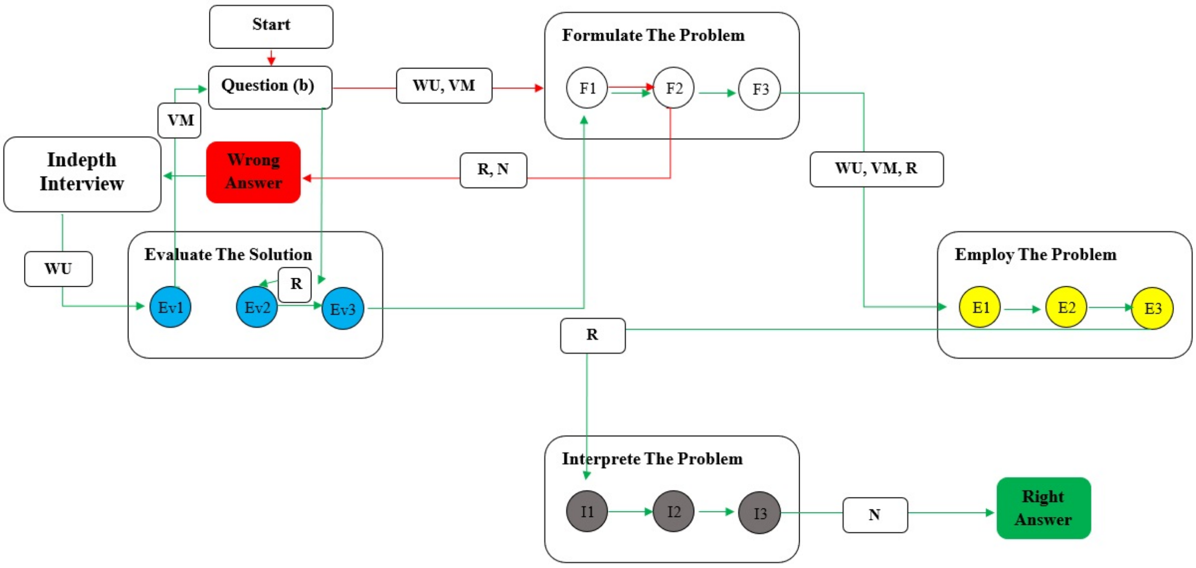


Figure 7. Flow of the subject's changing cognitive process for question (b)

3. Change in S3's cognitive process in question (c)

For question (c), initially, S3 answered that the area of the festival grounds could not be determined. According to the subject, only the area for the participants was known. However, during the interview session, the subject revised their response. The change in cognitive was that the area of the festival grounds and the comparison of areas could indeed be determined. The following is an excerpt from the interview with the subject:

R : *In question (c), you answered that the area of the festival grounds could not be determined. Could you explain?*

S3 : *Hmm... actually, after I revised my answer for question (b), I was able to determine the area of the festival grounds and the area comparison.*

R : *How did you do that?*

S3 : *For the participant area, I found the length to be 7 meters and the width to be 5 meters, so the area is 7 x 5 = 35. Meanwhile, for the festival grounds, the length is 10 meters and the width is 7 meters, so the area is 10 x 7 = 70. Therefore, the comparison of the areas is 35:70, which simplifies to 1:2.*

It turns out that the change in the cognitive process in question (b) led to an improvement in the solution for question (c). In this activity, S3 revisited the strategy and results they had obtained previously for question (c). The codes Ev1, Ev2, and Ev3 were revisited by the subject. They reformulated the problem by correctly identifying the information in the problem, writing it in mathematical terms, and accurately analyzing what was being asked in the problem. The Word Uses (WU) included: "length 7 m, width 5 m, area 7 x 5 = 35, length 10 m, width 7 m, area 10 x 7 = 70, the ratio 35:70, which simplifies to 1:2". In this activity, the codes F1, F2, and F3 were effectively followed by the subject. For the Visual Mediators (VM), the subject used the diagram they had created in question (b). The subject focused solely on the part highlighted in red in Figure 8 below.

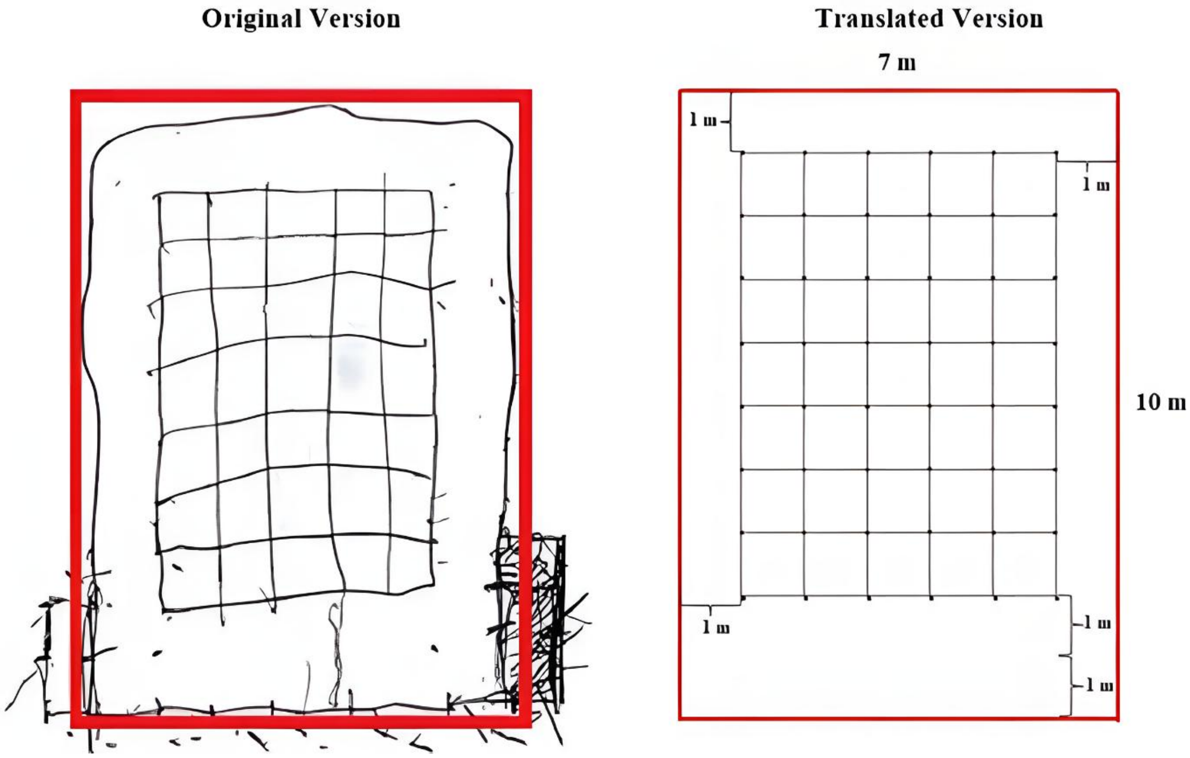


Figure 8. Revised answer sheet of the subject for question (c)

The change in the cognitive process enabled the subject to realize their mistake in question (b). In this case, the subject reinterpreted the information in the problem, focusing on the part highlighted in red in Figure 8. To determine the length and width, the subject used the diagram from question (b) and manually counted from one square to the next. For the length and width of the participant area, the subject began manually calculating and found that the length was 7 m and the width was 5 m, resulting in an area of 7 x 5 = 35. For the length and width of the festival area, the subject began manual calculations and determined that the length was 10 m and the width was 7 m, yielding an area of 10 x 7 = 70. In this case, the subject reassessed the information by formulating an appropriate problem-solving plan and accurately structuring the steps. For the Routines (R) used, S3 focused on finding the length and width from one square to another and applying the procedure for calculating the perimeter of a rectangle. The subject then concluded that the ratio was $\frac{\text{the area of the participant area}}{\text{the area of the festival area}} = \frac{35}{70} = \frac{1}{2}$. The codes E1, E2, E3, I1, I2, and I3 were successfully completed by the subject. For the Narratives (N) used, S3 focused on the concept of the area of a rectangle and ratio. Below is the subject's answer sheet after being asked to write down the changes in their cognitive process, as shown in Figure 9 below.

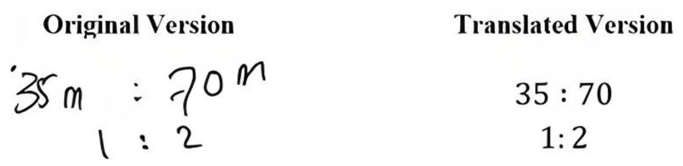


Figure 9. Revised answer sheet of the subject for question (c)

Commognitive plays a key role as a marker and explanation of the changes in the subject's cognitive process. The components of WU, VM, R, and N have been shown to effectively assist the subject in correctly solving the given non-routine problem. Below is the flow of the subject's change in cognitive process for question (c) as illustrated in Figure 10.

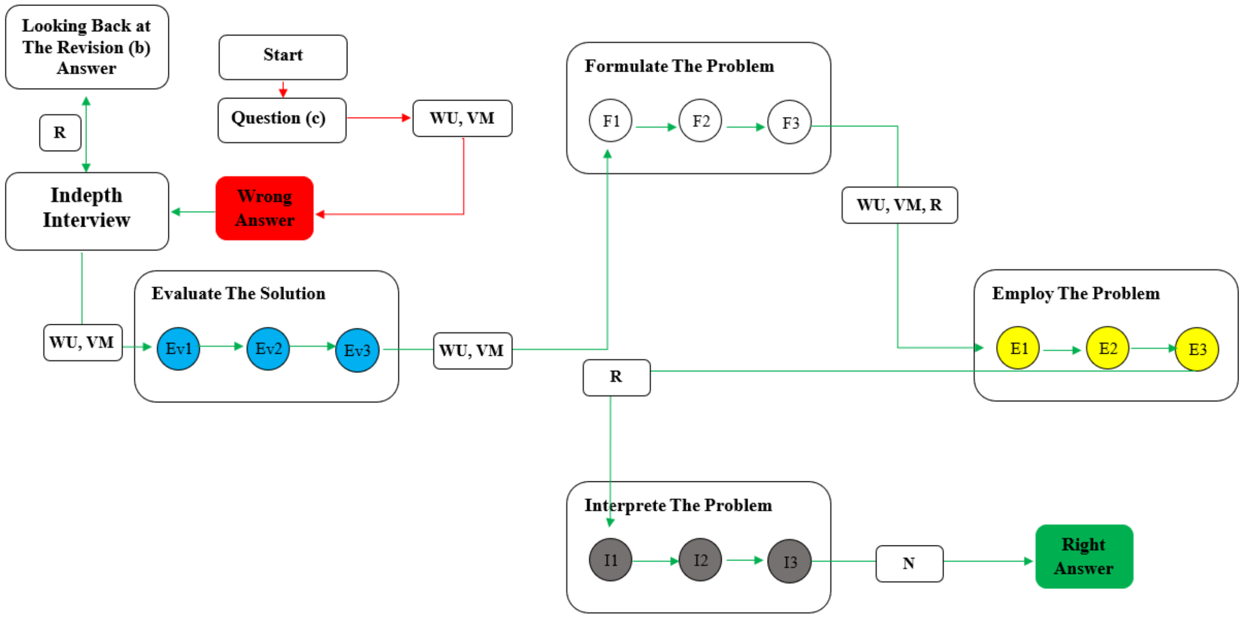


Figure 10. Flow of the subject's change in cognitive process for question (c)

4. Change in S3's cognitive process for question (d)

For question (d), it is evident that there was a revision made by S3. Initially, the subject answered that the perimeter in question (d) could not be determined because the length and width of the festival field were unknown. However, after revising questions (b) and (c), there was a change in the subject's cognitive process in re-answering question (d). The subject revisited the strategy and results obtained previously. In this activity, codes Ev1, Ev2, and Ev3 were effectively followed by the subject. The subject began to re-identify the information in the problem by distinguishing between the area for the participants and the festival field. The subject then determined the length and width of the festival field using the concepts from question (c). Below is the conceptual thinking used by the subject, taken from question (c), as shown in Figure 11.

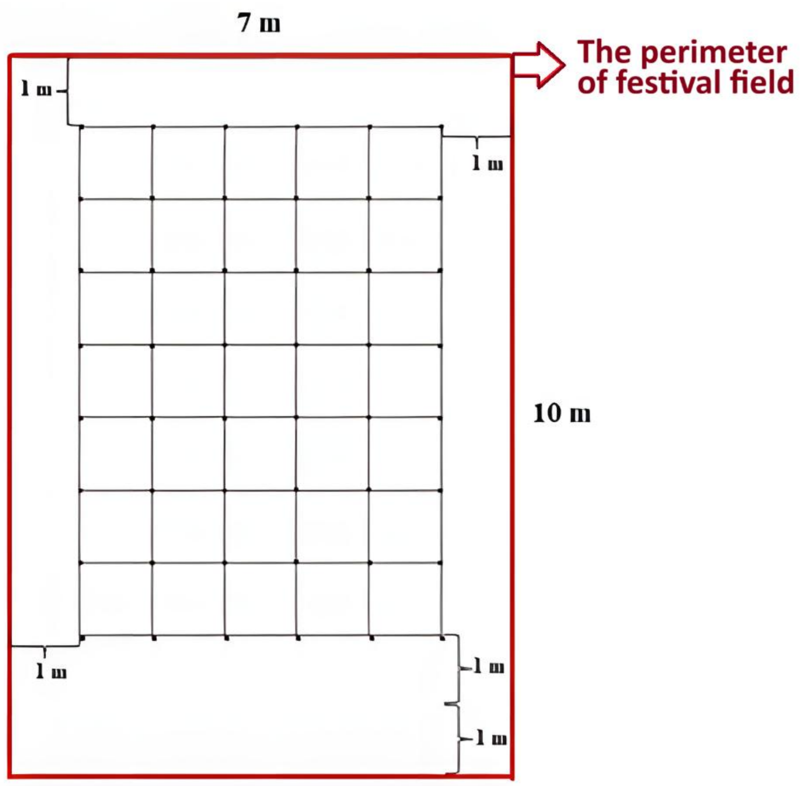


Figure 11. Conceptual cognitive of the subject for question (d)

For question (d), the subject was not directly illustrated on the answer sheet. Instead, the subject focused on the image from question (c). The subject began to formulate a solution plan by finding the perimeter of the festival field. From question (c), it had been determined that the length and width of the festival field were 10 m and 7 m, respectively. The subject then began to calculate the perimeter of the festival field. Below is the revised cognitive process shown on the subject's answer sheet in Figure 12.

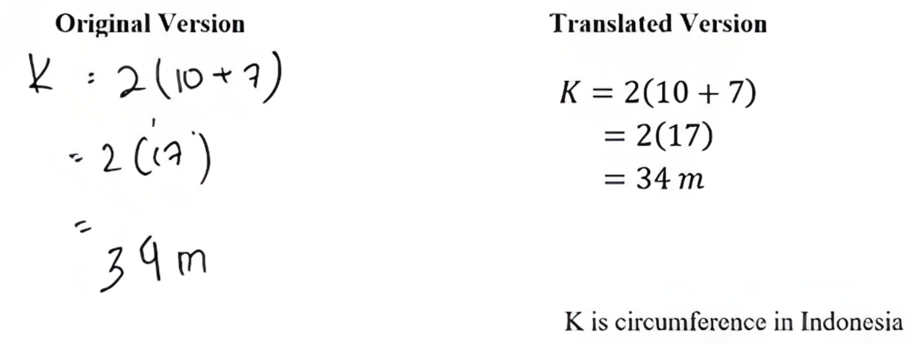


Figure 12. Revised answer sheet of the subject for question (d)

The subject applied the formula for the circumference: Circumference = 2 × (length + width). The result obtained was 34 m. The subject used the perimeter formula by multiplying 2 by the sum of the lengths of the opposite sides. This means that each side of the length and width has an opposite side of the same length. In this case, the subject understood the concept of the perimeter formula well. The process of solving question (d) again was done very smoothly by the subject.

For the Commognitive components, the Word Uses (WU) used are found in the words "length, width, and perimeter". The Visual Mediators (VM) used by the subject are found in the image created in question (c). For the Routines (R) used by the subject, they are found in the use of the perimeter formula for a rectangle. For the Narratives (N), the subject relied on the concept of the perimeter of a rectangle.

The changes in cognitive processes in question (c) resulted in changes in cognitive processes in question (d). Similarly, improvements in question (c) led to improvements in solving question (d). By revisiting the problem, the subject was able to reformulate the problem, reuse the problem, and reinterpret the problem very effectively. The codes F1, F2, F3, E1, E2, E3, I1, I2, and I3 were well followed by the subject. Below is the flow of changes in the subject's cognitive process for question (d) as shown in Figure 13.

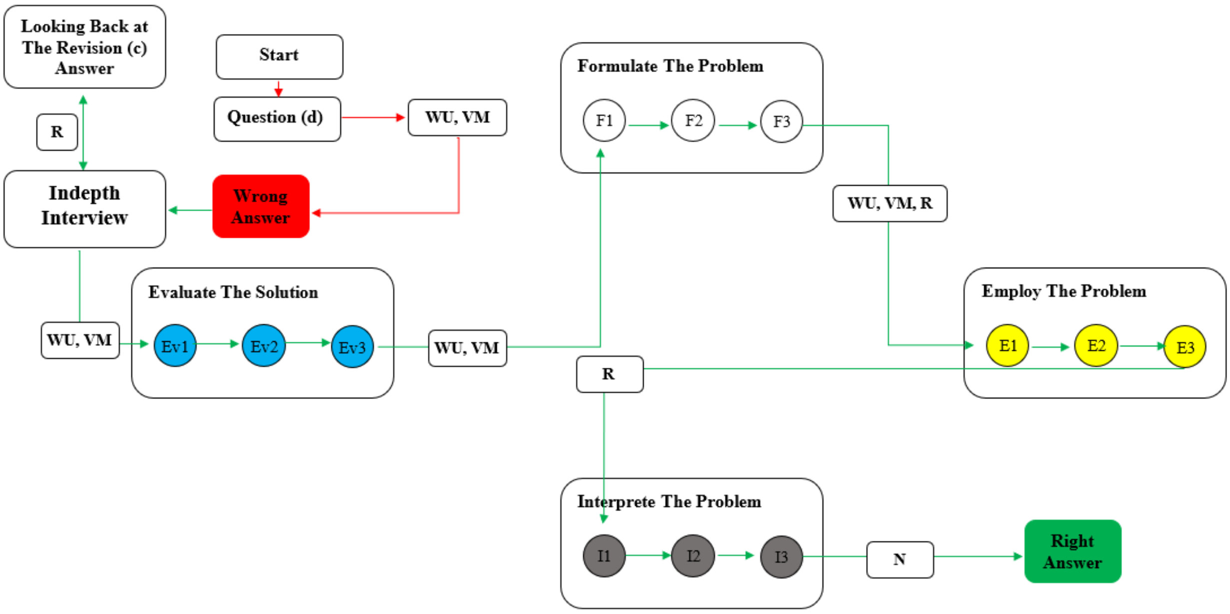


Figure 13. Flow of the subject's cognitive process changes for question (d)

Discussion

The cognitive transformation observed in student S3 while solving PISA-like mathematical problems highlights the essential role of self-evaluation and cognitive reflection in developing mathematical literacy. According to Nachlieli and Heyd-Metzuyanim (2022), this transformation reflects a shift from procedural engagement to metacognitive awareness. Initially, S3 demonstrated conceptual misunderstandings, such as using the area formula when the perimeter formula was needed. This aligns with the findings of Tabach and Nachlieli (2016), who reported that students often experience misunderstandings and errors when dealing with multiplicative strategies. Nevertheless, by revisiting and reanalyzing the problem-solving steps, the student was able to identify and correct these errors (Heyd-Metzuyanim & Graven, 2019). According to Manalo and Kapur (2018), this ability not only reflects the student's awareness of their errors but also demonstrates maturity in developing more effective problem-solving strategies. The process of self-evaluation (particularly evident in question (b)) was further supported by a dialog-based interview, which prompted the student to think critically and deeply about their learning (Koichu et al., 2022). In line with Kholid et al. (2022), reflection and self-evaluation are vital tools for identifying errors, refining problem-solving strategies, and generating improved solutions.

S3's use of Visual Mediators (VM), such as redrawing the problem to visualize the dimensions of the field, proved to be a key tool in understanding the relationships between various mathematical elements. According to Presmeg (2016), the student's drawings helped to organize complex information in a more structured manner, which was further enhanced by the application of routines (R), such as counting squares to calculate the perimeter. The integration of visual mediators and routines not only supports conceptual understanding but also fosters a systematic approach to thinking, which is essential for mathematical problem-solving (Lu et al., 2022; Zayyadi et al., 2019). This interpretation is consistent with the commognitive framework, as the student's shift in visual mediation was accompanied by a clear transformation of routines. S3's shift from counting segments to using $2(p + l)$ reflects early reification (Sfard, 2008)—treating the perimeter as an object rather than a process—facilitated by VM restructuring (Sfard, 2008).

The findings also highlight the significance of mastering fundamental mathematical concepts in shaping students' thinking. S3's ability to translate contextual information into appropriate mathematical forms, as demonstrated in (c), reflects a deepening understanding of concepts such as perimeter and area and their interrelationship. According to Annizar et al. (2020), this growth in understanding is triggered by the ongoing reassessment of contextual problems. Chua (2021) emphasizes the role of contextual learning in fostering mathematical literacy, as it connects concepts to real-world situations, improving students' comprehension and boosting their engagement in the learning process (Oktiningrum & Hartono, 2016; Putri, 2020; Wijayanto et al., 2024).

The correction process that S3 engaged in during problem-solving demonstrates that mathematical literacy involves more than just arriving at the correct answer; it also includes understanding the reasoning behind the solution and being able to identify and address errors (Asmara et al., 2024; Kusuma et al., 2021). This illustrates the importance of reflection in mathematics education, encouraging students to reassess their strategies and solutions, thereby strengthening the cognitive skills necessary to solve complex PISA-like problems (Putri & Alfani, 2020). These findings further emphasize the need for educators to create an environment that supports self-evaluation, allowing students to make mistakes, reflect on them, and correct them without fear of judgment (Alvidrez et al., 2022).

In terms of classroom implications, teachers should promote self-evaluation as an integral learning strategy, offering students the opportunity to reflect on their cognitive processes (Albab et al., 2014; Hough & Solomon, 2023). It is also essential to incorporate visual media and contextual learning approaches that make mathematics more relevant to students' lives (Kolar & Hodnik, 2021; Safura et al., 2018). Additionally, by utilizing problems similar to those found in PISA assessments, students can engage in deeper reflection, refine their problem-solving skills, and strengthen their conceptual understanding of mathematics (Borji et al., 2021; Retnowati et al., 2018). This approach lays a crucial foundation for preparing students to confront global challenges in mathematical literacy, as measured by the PISA program (Wijaya et al., 2024). While single-case analysis offers depth, the findings may not be generalizable. Replication with multiple students is required.

Conclusion

The transformation in students' cognitive processes when solving PISA-like mathematical problems is largely driven by their ability to engage in self-evaluation and cognitive reflection. This process allows students to identify errors, refine their strategies, and deepen their understanding of mathematical concepts. The reformulation of problems and the use of contextual problem-solving strategies have also proven effective in supporting conceptual understanding and building systematic cognitive patterns. However, the single-case design limits the generalizability of these findings. Therefore, future studies should involve multiple students across diverse ability levels to explore variations in cognitive development more comprehensively. We recommend commognitive scaffolding: prompting students to re-word (Explain using perimeter, not length), re-visualize (Redraw the field), and re-routinize (Try a faster method).

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Declarations

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