



Students' reasoning patterns in solving statistical data problems at the junior high school level: An APOS perspective

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Abstract

Many students face challenges in reasoning through statistical data problems; however, little is known about how their thinking processes develop through different cognitive stages. A qualitative descriptive design was employed to explore students' reasoning patterns in solving statistical data problems through the lens of Action, Process, Object, and Schema (APOS) theory. This study was conducted with seventh-grade students at a public junior high school in Kulon Progo, Indonesia. Data were collected through written tests and semi-structured interviews and were analyzed using coding techniques aligned with the APOS indicators. The results show that students with correct answers demonstrated conceptual reasoning and progressed through all APOS stages, whereas those with incorrect answers showed fragmented understanding and failed to complete the cognitive sequence. This study contributes to mathematics education by extending the APOS theory to statistical data representation at the secondary level.

Keywords: APOS theory; cognitive development; mathematical reasoning; problem-solving; student thinking patterns

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Introduction

The ability to solve problems is a core aspect of mathematics education that has received global attention (Pratiwi & Widjajanti, 2020). Mathematical processes such as problem solving, reasoning, communication, and modeling are very important (Sun et al., 2020). Students' conceptual understanding and procedural fluency are developed through their ability to solve problems, reason mathematically, communicate ideas, and apply mathematics in real-world contexts (Nilimaa, 2023). In particular, students' ability to construct new mathematical knowledge is regarded as a key outcome of problem solving, which is understood not only as an instructional objective but also as a fundamental means of learning mathematics (Olivares et al., 2021). At the international level, students continue to struggle with solving problems that require high-level reasoning, particularly in data representation and interpretation, which is evident in the low performance of students in mathematical literacy assessments such as PISA (OECD, 2019). Nationally, the same concern is reflected in Indonesian students' weak performance in interpreting and representing statistical data, which is crucial for understanding real-life information. This highlights the urgent need to analyze how students think and reason when faced with data-related problems in mathematics.

In response to this issue, researchers and educators have emphasized the importance of understanding students' cognitive structures to improve instruction. Expert opinion suggests that students' reasoning does not occur in a linear manner but rather through complex mental constructions (Dubinsky & McDonald, 2002). Researchers such as Chew and Cerbin (2021) have pointed out that students often memorize statistical procedures without fully understanding the underlying concepts. Consequently, students tend to make errors in interpreting data, choosing the right representation (e.g., graphs and tables), and linking data to real-world contexts. These challenges call for a framework that captures the depth and stages of students' thinking processes.

Several studies have applied various cognitive and constructivist frameworks to analyze mathematical thinking. For example, Mukuka et al. (2020) used the SOLO taxonomy to categorize students' understanding of statistical contexts. Meanwhile, studies such as those by Ugalde et al. (2021) have emphasized the need for learning environments that support cognitive development. However, only a few studies have examined the specific stages of students' mental constructions in solving statistical data problems using Action, Process, Object, Schema (APOS) theory. Although APOS theory has been widely applied to topics such as algebra, functions, and calculus (Dubinsky & McDonald, 2002; Trigueros et al., 2024), its application in the context of data representation remains underexplored.

This research seeks to fill this gap by applying APOS theory to understand students' reasoning patterns in solving statistical data problems. APOS theory, developed from Piagetian constructivism, offers a systematic lens for analyzing how students construct and internalize mathematical concepts. The theory posits that successful mathematical understanding involves moving through the stages of action, process, object, and schema (Dubinsky & McDonald, 2002). Applying this theory in the domain of data presentation is expected to uncover how

students mentally engage with statistical representations, such as bar charts, pie diagrams, and tables, and how these engagements reflect their reasoning.

Therefore, this study aims to describe students' reasoning patterns in solving statistical data problems through the lens of APOS theory. This study contributes to the literature on mathematical cognition and provides practical insights for educators in designing learning experiences that align with students' cognitive development. The findings are expected to support curriculum development and instructional design, particularly in teaching statistics at the secondary education level. Accordingly, this study addresses the following research question: How do students' reasoning patterns in solving statistical data problems manifest across APOS stages?

Methods

A qualitative descriptive design was employed to explore students' reasoning patterns in solving statistical data problems through the lens of the APOS theory. This study was conducted with seventh-grade students at a public junior high school in Kulon Progo, Indonesia. The participants were selected purposively, with the main criterion being that students produced either entirely correct responses or responses with two to three incorrect answers on a written test.

The data collection process involved two main instruments: written tests and semi-structured interviews. The written test was designed to evaluate how students solved problems related to data presentation and to determine the completeness and accuracy of their reasoning. The test consisted of three open-ended questions, each addressing different aspects of data representation, including pie charts, bar charts and frequency tables. The written test was developed based on the learning objectives of the data presentation. The indicators included the ability to read and interpret data, create bar/pie charts, and draw conclusions based on data representations. To ensure content validity, the test items were reviewed by two experts in mathematics education, and revisions were made based on their suggestions. The reliability of the test was examined through a pilot study with students of similar characteristics, and Cronbach's alpha ($\alpha = 0.82$) indicated that the instrument had acceptable internal consistency.

Of the 28 students who took the test, four were selected as research participants based on their written responses: two students with mostly correct answers and two with mostly incorrect answers. This small sample size is consistent with qualitative research norms, as the study prioritizes an in-depth exploration of students' reasoning processes rather than statistical generalization. The selected participants were considered information-rich cases that allowed for a detailed analysis of cognitive structures across the APOS stages. To facilitate analysis and maintain subject privacy, coding was performed for each subject, as presented in Table 1.

Table 1. Research subject coding list based on student answers

Category	Code
Correct Answer	SB1
	SB2
Wrong Answer	SS1
	SS2

The students were then interviewed to further explore their mental constructions in solving the problems. The interviews were semi-structured and flexible, allowing the researchers to probe deeper based on students' responses. The goal was to trace students' reasoning processes according to the four stages of APOS theory (Figure 1): (1) Action: how students identify and apply initial procedures; (2) Process: how they mentally coordinate and relate concepts; (3) Object: how they encapsulate their mental processes into structured understanding; and (4) Schema: how they integrate all elements into a coherent framework for problem-solving (Agustina et al., 2025; Irfan et al., 2020; Umam et al., 2025).

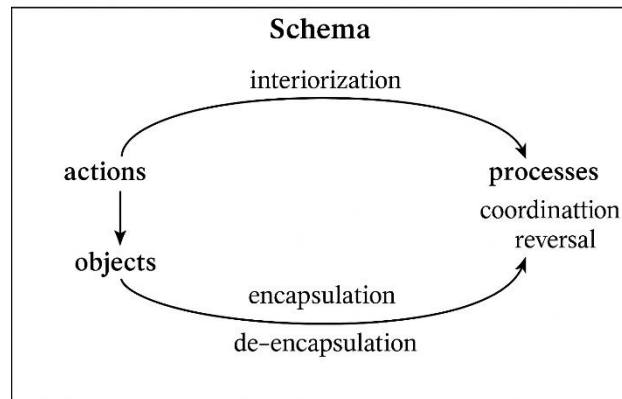


Figure 1. Four stages of APOS theory

The data were analyzed using qualitative techniques, including coding students' responses and guiding the interview process based on indicators derived from the APOS framework, all of which are presented in Table 2. These indicators were developed to represent each mental structure, as outlined by Dubinsky and McDonald (2002). All interview excerpts originally conducted in Bahasa Indonesia were translated into English to ensure clarity and consistency in reporting.

Table 2. Indicators of students' reasoning based on APOS theory

APOS Stage	Mental Mechanism	Indicator	Guiding Questions
Action	Interiorization	Student identifies known and unknown quantities in the problem	Can you explain what information is given and what is being asked in the problem?
		Student applies basic procedures or operations based on instructions or memory	What is the first step you took to solve the problem? How did you decide which data to use?
Process	Coordination & Reversal	Student connects current problem to previously learned concepts	Did you connect this problem to anything you have learned before?
		Student mentally manipulates data (e.g., calculates angles or frequencies without step-by-step aid)	What formula or method did you use, and why? Can you describe how you moved from one step to the next in your solution?

APOS Stage	Mental Mechanism	Indicator	Guiding Questions
			Were there any patterns or strategies that helped you solve the problem?
Object	Encapsulation & De-encapsulation	Student treats a process (e.g., percentage calculation) as a coherent object	How do you understand the formula or concept you used?
		Student selects appropriate formulas and adjusts them when needed	Why do you think this strategy or formula works in this situation?
		Student reflects on and corrects their own solution	If you made a mistake, how did you realize it and correct it?
			Could you explain this method to a friend?
		Student integrates multiple mathematical ideas to solve problems	What is your conclusion or final answer, and how did you arrive at it?
Schema	Generalization	Student explains the rationale behind each step of their solution	Can you summarize the steps you took and why you chose them?
		Student can transfer reasoning to new or slightly modified problems	How would you approach a similar problem in a different context?
			What did you learn from solving this problem?

Results

This section presents the findings of the study on students' reasoning patterns in solving statistical data problems through the lens of APOS theory. To further illustrate the students' reasoning patterns, this section presents two samples of students' written work (coded as SB1 and SS1), which were used during the interview for deeper analysis. All interview excerpts originally conducted in Bahasa Indonesia were translated into English by the researchers. The worksheet provides insight into how the student interpreted a pie chart problem involving proportional reasoning to determine the number of students participating in various extracurricular activities. The written solutions demonstrated the students' thought processes and were analyzed according to the APOS framework. Figure 2 shows a scanned copy of the student's SB1 worksheet.

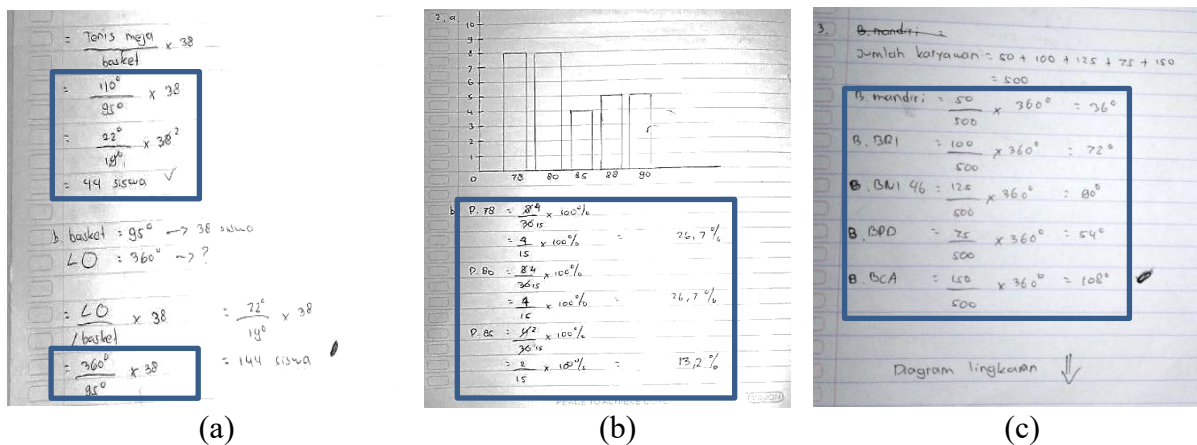


Figure 2. Worksheet SB1: (a) number 1; (b) number 2; (c) number 3

In solving the pie chart problem, SB1 applied a proportional reasoning strategy by comparing the central angle representing table tennis (110°) to that of basketball (95°), which was known to represent 38 students. The student used the proportion: $\frac{110^\circ}{95^\circ} \times 38 = 44$ students. This calculation was completed correctly, and the student proceeded to determine the total number of students represented in the pie chart by using the full circle (360°) as a reference: $\frac{360^\circ}{95^\circ} \times 38 = 144$ students. This solution shows that SB1 was able to identify relevant known and unknown quantities, select and apply proportional formulas appropriately, and structure their thinking in a logical sequence.

From the APOS perspective, SB1 exhibited a well-developed understanding of the task through all four stages. At the action level, SB1 accurately identified the angles given and correctly interpreted the number of students each sector represented. When moving to the process level, the student successfully coordinated proportional reasoning, applying it flexibly to determine values based on a new total. At the object stage, this reasoning was encapsulated into a generalizable strategy, as evidenced by SB1's consistent use of the same structure across different parts of the problem. Finally, at the schema level, SB1 demonstrated a comprehensive and integrated understanding of the task, indicating the development of a coherent mental model for interpreting data representation problems using angle-to-population relationships. This example provides strong evidence of cognitive development through the APOS stages and illustrates how student responses can be analyzed beyond mere correctness to reveal the underlying structure of mathematical thinking.

In contrast to SB1, SS1's worksheet illustrates several difficulties in solving the pie chart problem. The student attempted to determine the number of students based on the angle representing a specific activity but made multiple procedural and conceptual errors. For instance, in responding to the first problem, SS1 directly multiplied the angle by a number without establishing a proportional relationship, suggesting a surface-level application of mathematical operations without understanding the underlying structure. Figure 3 displays a portion of SS1's solution to the same pie chart problem given to SB1.

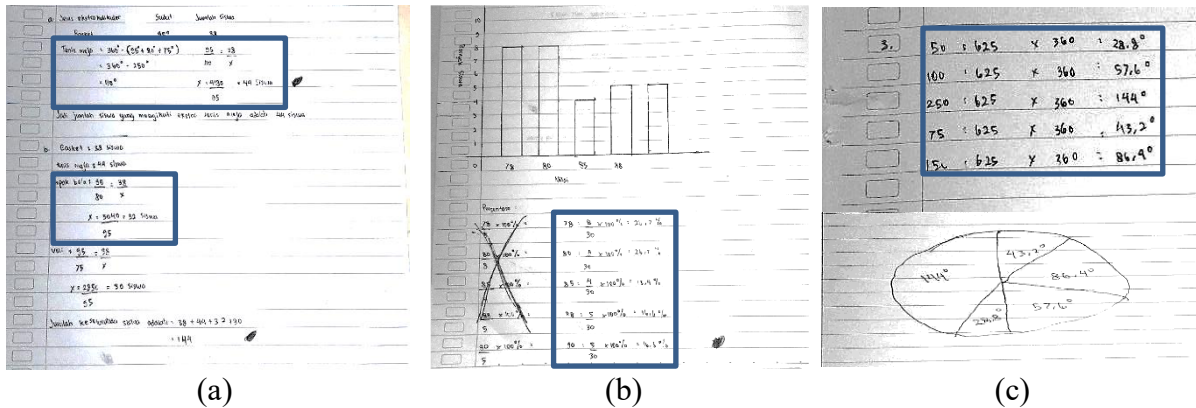


Figure 3. Worksheet SS1: (a) number 1; (b) number 2; (c) number 3

From the perspective of the APOS framework, SS1's responses revealed significant gaps across all four stages of understanding. At the action level, the student correctly identified the given angle and recognized that a calculation was needed, but proceeded with an unjustified operation, suggesting reliance on a memorized yet inappropriate procedure. At the process stage, SS1 was unable to coordinate relevant prior knowledge, showing no evidence of internalizing or mentally manipulating the relationship between angle measures and population data. This lack of conceptual processing marked a breakdown in understanding. At the object level, the student's approach did not evolve into a coherent or adaptable mathematical strategy; the formula used was applied mechanically without reflection, and the student struggled to articulate the reasoning behind it, even when prompted. Finally, at the schema stage, SS1's reasoning lacked integration of key mathematical concepts, resulting in unsupported answers and an inability to apply proportional reasoning to similar problems. These indicators collectively suggest that a schema for interpreting data using angle-to-population relationships had not yet been constructed.

Based on the analysis of student responses, their reasoning patterns align distinctly with the stages of the APOS framework. The findings across the four subjects: two with correct answers (SB1 and SB2) and two with incorrect answers (SS1 and SS2) reveal clear contrasts in how each stage of cognitive development manifests in problem solving.

At the Action stage, most students were able to recognize and articulate the known and unknown elements of the task. For example, SB1 and SB2 demonstrated clarity in identifying given data such as angles in pie charts or frequencies in tables and in understanding what was being asked. In contrast, SS1 and SS2 often misidentified or overlooked critical information, which suggests that their engagement was more procedural than conceptual. This reliance on

superficial cues or memorized steps, rather than meaningful interpretation, reflects a limited development at the action level.

In the Process stage, the differences became more pronounced. SB1 and SB2 were able to connect the problem to prior mathematical knowledge, such as recognizing proportional relationships and applying formulas accurately. Their ability to mentally coordinate various components of the data, and to structure intermediate steps logically, indicated internalized understanding. Meanwhile, SS1 and SS2 struggled to establish these connections. Their application of formulas was inconsistent, and they were often unable to explain their methods, indicating a lack of internalization and difficulty in coordinating learned procedures with the demands of the task.

At the Object stage, SB1 and SB2 showed evidence of having encapsulated their procedures into conceptual objects. They not only used formulas like $\frac{\text{part}}{\text{whole}} \times 360^\circ$ or $\frac{\text{frequency}}{\text{total}} \times 100\%$ appropriately, but also demonstrated flexibility and justification in their use during interviews. This suggests that they had transformed these procedures into stable mental constructs. On the other hand, SS1 and SS2 either applied formulas incorrectly or were unable to articulate why a certain approach was chosen, indicating that their procedural knowledge had not yet developed into well-formed mathematical objects.

Finally, in the Schema stage, SB1 and SB2 exhibited a well-integrated understanding of the overall problem. They were able to explain their reasoning step by step, connect various ideas coherently, and justify their conclusions with reference to the data. Their solutions reflected a comprehensive mental model of how the different mathematical elements interact. In contrast, SS1 and SS2 lacked such integration. Their reasoning was often fragmented, with gaps in logic and unclear justifications. This suggests that they had not yet formed a robust schema for reasoning about statistical data through proportional relationships. Overall, this analysis highlights the importance of supporting students in moving beyond procedural engagement toward deeper conceptual understanding, enabling them to build coherent mental models that align with the APOS framework.

A comparative analysis across the four subjects (Table 4) shows that students with correct answers progressed through all APOS stages, reflecting strong conceptual understanding and mental coordination. Conversely, those with incorrect responses demonstrated fragmented reasoning, often limited to the Action or early Process stages.

Table 4. Students' reasoning patterns based on APOS theory

Subject	Action	Process	Object	Schema
SB1	Able to identify and explain known and unknown information clearly.	Able to connect prior knowledge to the problem and recall relevant concepts.	Able to select and apply correct formulas and justify the steps.	Able to integrate all steps logically and explain reasoning clearly.
SB2	Able to identify and explain	Able to connect prior	Able to apply correct formulas	Able to explain the entire problem-

Subject	Action	Process	Object	Schema
	known and unknown information clearly.	knowledge to the problem and recall relevant concepts.	and revise errors when necessary.	solving process and final conclusion.
SS1	Able to write down known and unknown information but had difficulty explaining it.	Unable to relate previous knowledge to the current problem consistently.	Able to use formulas but could not justify their usage or correct errors.	Unable to fully explain reasoning or summarize steps coherently.
SS2	Struggled to identify and explain the information given in the problem.	Could not connect the problem with prior knowledge and applied steps inconsistently.	Unable to apply correct formulas and justify the mathematical procedures.	Failed to integrate steps or explain the reasoning behind answers.

Discussion

This study demonstrates that students' success in solving statistical data problems is closely related to their ability to construct coherent mental structures across the APOS stages. Rather than merely applying procedures, students who developed integrated action, process, object, and schema structures were able to reason conceptually and justify their solutions. From a cognitive perspective, this finding supports the central assumption of APOS theory that meaningful mathematical understanding emerges through the progressive coordination and encapsulation of mental processes. This interpretation is consistent with the research of (Trigueros et al., 2024), which emphasizes the role of schema construction in supporting conceptual coherence across mathematical representations. In the context of statistical reasoning, this progression appears to be particularly critical for tasks involving proportional relationships and data representation, where conceptual integration is essential for accurate interpretation.

In contrast, when such cognitive integration across the APOS stages does not occur, students' reasoning tends to remain fragmented and procedural. From an APOS perspective, this reflects difficulties in interiorizing actions into coherent processes and encapsulating these processes into stable mathematical objects. Consequently, students may rely on memorized formulas without understanding the proportional relationships underlying statistical representations. Similar patterns of fragmented reasoning have been documented by Jäder and Johansson (2025) and Lenz et al. (2024), who reported that procedural fluency without conceptual coordination often leads to superficial problem solving and limited transfer. This incomplete cognitive construction prevents the formation of robust schemas, limiting students' ability to justify their solutions or transfer their reasoning to similar data-related problems. These findings suggest that errors in statistical problem solving are not isolated procedural

mistakes but rather manifestations of deeper conceptual gaps in students' cognitive development.

An important contribution of this study is the extension of APOS theory beyond its traditional domains of algebra, functions, and calculus into the area of statistical data representation. While previous APOS-based research has primarily focused on symbolic and analytic mathematical objects (Borji et al., 2025), the present findings demonstrate that APOS theory is also effective in capturing students' reasoning processes when engaging with graphical and contextual statistical representations, such as pie charts, bar charts, and frequency tables. This extension highlights that the mental mechanisms proposed in APOS theory—interiorization, coordination, encapsulation, and generalization—are equally relevant in domains where interpretation and proportional reasoning play a central role. Consequently, this study broadens the theoretical scope of APOS and provides empirical support for its applicability in statistics education at the secondary level.

The findings also have important implications for instructional design in statistics education. Students who successfully progressed through the APOS stages benefited from learning experiences that implicitly supported conceptual coordination and reflection. In contrast, students whose reasoning remained procedural require instructional interventions that explicitly target the Action and Process stages, such as activities that emphasize identifying relevant information, interpreting representations, and articulating relationships between data elements. Teachers should therefore design learning tasks that scaffold students' transitions from concrete actions to abstract schemas, for example, through guided questioning, comparison of multiple representations, and opportunities for explanation and justification.

Furthermore, APOS theory offers a valuable diagnostic framework for instructional decision-making. By identifying the APOS stage at which students experience difficulty, teachers can provide targeted support that aligns with students' cognitive needs rather than relying on uniform procedural instruction. In statistics learning, this may involve shifting the emphasis from formula application toward reasoning about proportions, relationships, and meanings embedded in data representations. Such cognitively aligned instructional designs have the potential to foster deeper statistical understanding and support students' development of transferable reasoning skills.

In conclusion, this discussion underscores that applying APOS theory to statistical problem-solving not only enriches theoretical understanding of students' cognitive development but also provides concrete guidance for designing instruction that supports meaningful learning in statistics. By extending APOS into this domain, the study contributes to both mathematics education research and classroom practice, particularly in addressing persistent challenges in students' statistical reasoning at the secondary school level.

Conclusion

Students demonstrated different reasoning patterns in solving statistical data problems, which can be described through the stages of the APOS theory. Students who produced correct solutions were generally able to identify known and unknown information, apply procedures

accurately, and connect prior knowledge to the problem at hand. They showed evidence of encapsulating mathematical processes into coherent objects and were able to integrate these elements into a complete mental schema. In contrast, students who produced incorrect responses often struggled to internalize the procedures conceptually and demonstrated fragmented reasoning that did not fully progress through the APOS stages.

These findings highlight the importance of understanding students' cognitive development in mathematics and suggest that teachers should pay closer attention to the stages students go through when constructing mathematical knowledge. Instructional strategies that guide students through mental constructions from concrete actions to abstract schemas may enhance their problem-solving skills, particularly in the context of statistical data. The implication of this study is that the APOS framework can serve as a valuable diagnostic and pedagogical tool for identifying where students' thinking may be hindered and for designing interventions that are cognitively aligned with their developmental stage. However, this study is limited by its small sample size and context-specific findings. Future research involving a more diverse participant group and a broader range of mathematical topics is needed to strengthen the generalizability of these results.

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