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Secondary students' informal strategies in solving enumeration problems prior to formal combinatorics instruction

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Abstract

Although enumeration problems are fundamental in combinatorics, little is known about how students intuitively approach such enumeration problems before receiving formal instruction. This exploratory qualitative study investigated the initial strategies employed by twelve-grade students in solving enumeration problems prior to formal instruction on enumeration rules. Fifteen students from a public senior high school in Kerinci, Indonesia, who had not yet learned combinatorics in the curriculum, participated in this study. Data were collected through students' written responses to three combinatorial problems presented in different real-life contexts and further explored through semi-structured interviews. Only responses demonstrating coherent and interpretable strategy were analyzed. The findings reveal three dominant strategies: listing all possible arrangements, generalizing patterns, and applying the multiplication principle. These findings indicate that students possess intuitive approaches that can serve as a foundation for formal combinatorial reasoning. The study aligns with the Realistic Mathematics Education (RME) perspective, emphasizing the importance of guided reinvention and contextual mathematization, and proposes implications for designing learning trajectories that build on students' informal reasoning in secondary mathematics education.

Keywords: combinatorial reasoning; combinatorics; counting principles; enumeration problems; student initial strategies

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Introduction

The ability to solve problems involving selection and arrangement is increasingly vital in today's complex and data-driven world. Whether consciously or not, individuals are often required to make choices from a set of possibilities—such as planning travel routes, organizing schedules, or selecting items within constraints—which inherently involve combinatorial reasoning and numeracy. Globally, such skills are recognized as core components of mathematical literacy (OECD, 2019, 2023), yet reports indicate that many students struggle with tasks requiring these forms of thinking (Mullis et al., 2020). In Indonesia, the 2022 PISA results revealed that the majority of 15-year-old students perform below minimum proficiency in mathematics, particularly in tasks that require creative problem solving and application of combinatorial reasoning in everyday contexts (Kusmaryono & Kusumaningsih, 2023). These findings raise serious concerns about the preparedness of students to role effectively in quantitative environments, both in academic and real-world settings.

Researchers and educators worldwide have responded to this issue by emphasizing the need to develop both combinatorial thinking and numeracy as integral parts of mathematics education (Geiger et al., 2015; Lockwood, 2013). Combinatorial thinking refers to the ability to systematically explore and quantify possibilities in a given problem context, such as through enumeration, arrangement, and selection (English, 1991; Lockwood, 2013; Salavatinejad et al., 2021). Numeracy, by contrast, entails the capacity to interpret, evaluate, and act on quantitative information to make reasoned decisions (Geiger et al., 2015; OECD, 2023). Both are essential not only for academic success but also for informed participation in everyday life, such as budgeting, planning, and risk assessment.

However, previous research has largely focused students' post-instructional performance, often evaluating how well students apply permutation or combination formulas after formal teaching. As Matitaputty et al. (2022) that focus on identifying student mistakes conceptually and procedurally in permutations and combinations. While such studies offer insights into procedural knowledge and conceptual errors, they overlook how students initially make sense of counting problems using their informal mathematical understanding. The study has compared the strategies of students with and without instruction combinatorics instruction. For example, Lamanna et al. (2022), but this study did not identify in detail the strategies of students who had not received combinatorics instruction. These studies leaving a research gap in how students approach enumeration problems intuitively prior to instruction.

Addressing this gap is crucial for two reasons. First, identifying students' initial strategies provides a window into their intuitive and informal thinking, revealing the cognitive resources they bring to bear when confronting unfamiliar problems. Understanding students' initial strategies in solving enumeration problems is essential for supporting the development of relational rather than merely instrumental understanding of combinatorial reasoning (Skemp, 1978). Second, it offers a powerful basis for designing instructional trajectories that begin with students' existing knowledge, consistent with the principles of Realistic Mathematics Education (RME). RME, developed in the Netherlands and adapted in Indonesia (van den Heuvel-Panhuizen & Drijvers, 2014; Zulkardi et al., 2020), emphasizes learning mathematics is seen as a process of guided reinvention, where students reconstruct formal mathematical concepts from their informal experiences through mathematization—the progressive organization and formalization of problem situations (Solomon et al., 2021).

By understanding students' spontaneous approaches to enumeration problems, educators can design tasks that bridge informal reasoning with formal counting principles in a cognitively authentic procedure. Exploring such intuitive yet sometimes incomplete strategies aligns with the notion of productive failure (Kapur, 2016), where students' initial struggles to make sense of problems can serve as a valuable foundation for subsequent conceptual learning. In combinatorics, this means that students can move from intuitive enumeration (e.g., listing all possibilities) toward systematic organization (e.g., tree diagrams) and eventually toward abstract symbolic reasoning, such as the multiplication principle (English, 1991; Lockwood, 2013).

This study investigates how Indonesian twelfth-grade students solve enumeration problems before receiving any formal instruction in counting principles. The problems were embedded in everyday-related contexts that allowed for multiple solution strategies. The aim is to identify and categorize the initial strategies students use and to discuss their implications for designing RME-based instructional sequences. By doing so, the study contributes to a deeper understanding of students' informal reasoning in enumeration rules, and supports the development of mathematics learning that is both contextually relevant and cognitively meaningful.

Methods

This study employed a qualitative exploratory design to investigate students' initial strategies in solving enumeration problems before receiving any formal instruction. Qualitative research is particularly suited for uncovering the meaning behind students' responses and exploring their thought processes (Creswell & Poth, 2018). The study focused on identifying and classifying of naturally occurring strategies as a foundation for future instructional design based on Realistic Mathematics Education (RME) principles.

The research was conducted before participants—twelfth-grade students—received any formal instruction in counting principles (permutations, combinations, or the multiplication principles) in their mathematics curriculum. This timing was crucial to ensure that students relied solely on their prior mathematical knowledge and informal reasoning. Participants consisted of 15 students (aged 18–19 years, 7 males and 8 females) from a public senior high school Jambi, Indonesia, who voluntarily agreed to take part in the study. The selection was based on purposive sampling, a common practice in qualitative research when the goal is to explore specific cognitive phenomena in depth (Patton, 2015). Data saturation was achieved when no new strategies emerged after analyzing all participant's response, indicating that the diversity of strategies had stabilized across students.

To elicit students' initial strategies, the researcher administered a pretest consisting of three enumeration problems presented in different real-life contexts but requiring equivalent mathematical reasoning. Each student was asked to choose and solve one problem to reduce cognitive load and allow deeper reasoning within a single context. However, this approach also limited cross-problem comparisons, which is acknowledged as a methodological limitation of the study. The problems are as follows:

> ***Problem 1.*** *In an exam, the teacher gives 4 questions. To ensure that students do not cheat on each other, students are not allowed to answer the questions in the same order as other students. How many different orders are there for answering the 4 questions?*
>
> ***Problem 2.*** *During Eid al-Fitr, you plan to visit the homes of Mr. Beni, Mrs. Dalius, Mrs. Emi, and Mrs. Fitri. How many different routes are there to visit the 4 teachers' homes?*
>
> ***Problem 3.*** *Suppose you want to create a PIN for your cellphone screen lock consisting of 4 digits consisting of the numbers 0, 1, 2, and 3. How many different PINs can you create?*

Problems 1 and 2 represent permutation tasks without repetition ($4! = 24$), whereas Problem 3 is intentionally open to dual interpretation—students may treat it as either with or without repetition. This ambiguity was deliberately retained as a methodological feature to reveal how students' reason about combinatorial conditions when the context does not explicitly specify constraints.

Data analysis was conducted using content analysis, students' written responses were collected and reviewed, focusing on identifying distinct and meaningful solution strategies. Only responses that reflected distinct strategies—whether correct or partially correct—were analyzed in depth, while responses exhibited misconceptions or relied on external aids were excluded from the classification. Following the test, semi-structured interviews were conducted with selected students to clarify their reasoning, confirm their thinking process, and avoid misinterpretation of their written work. Interviews are a well-established method for capturing students' cognitive strategies in mathematics education (Goldin, 2000).

Triangulation between written responses and interview data ensured credibility and internal validity of the findings so that research results were more valid, accurate, and reduced potential bias. Ethical procedures were followed throughout the study: informed consent was obtained from all participants and initials were used to ensure anonymity.

Results

The written test was administered under a specific rule: students seated next to each other were not allowed to solve the same problem. This rule aimed to minimize academic dishonesty and encourage individual reasoning. Figure 1 presents an illustration of the students' seating arrangement and their respective problem choices (e.g., RD (3) indicates that student RD attempted problem number 3).

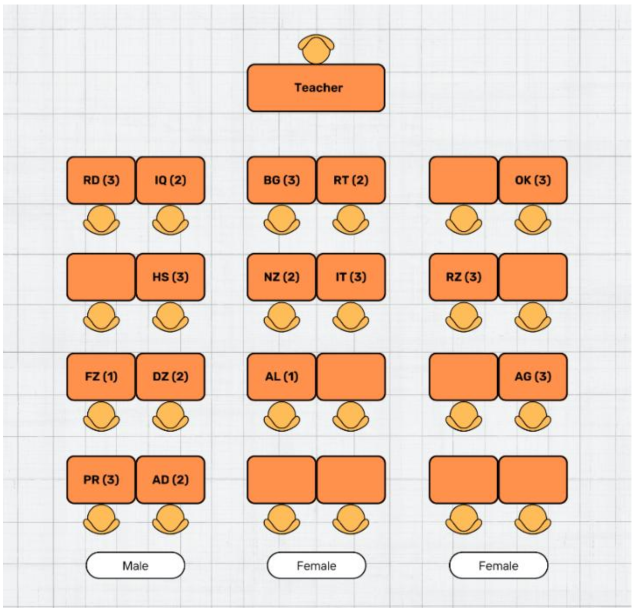


Figure 1. Ilustration of students' seating arrangement and their problem choices

Problem 1: Question order arrangement

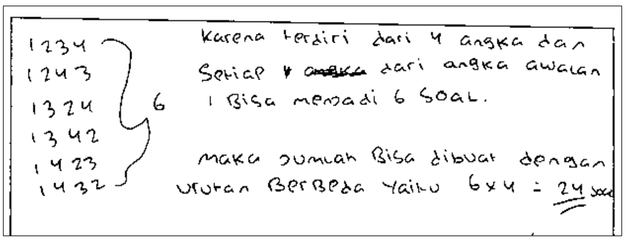
Only two students, AL and FZ, attempted Problem 1. AL manually attempted to list all possible orders of answering four questions using trial-and-error. AL managed to produce only six sequences, such as 1234, 4312, 2413, 1324, 2134, and 3142. However, the responses lacked structure and did not suggest a systematic method of enumeration.

"My answer is still lacking but I hesitate to make all the arrangements because it is too much." – AL

FZ, in other hand, proposed a distinctive strategy by starting with six unique arrangements starting with Question 1. Then generalizing the number of possible arrangements that would result if the sequence began with other questions.

"I suspect that if the sequence (of working on the questions) starts with another number (question) it will produce 6 sequences as well." – FZ

Although FZ could not fully justify why each starting point would lead to exactly six arrangements (see Figure 2), their strategy demonstrated an emerging awareness of recursive or multiplicative reasoning, which could serve as an intuitive basis for introducing tree diagrams or the multiplication principle in future instruction.



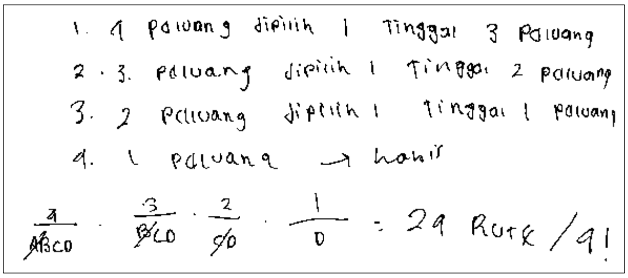
Translation:
(Arrangements)
Because it consists of 4 digits and every first digit like 1 can make 6 arrangements.
So, the number of different arrangement that can be made are 6 x 4 = 24.

Figure 2. FZ's attempt to generalize the total number of orders from different starting questions

Problem 2: Visiting teacher's houses

Five students—RT, NZ, IQ, DZ, and AD—attempted this problem. RT and IQ independently applied the multiplication principle by noting the decreasing number of choices at each stage of the trip. IQ, for instance, reasoned that the first house could be chosen in 4 ways, the second in 3, and so on (see Figure 3). However, during interviews, neither RT nor IQ could justify why multiplication was the appropriate operation to combine these choices, indicating partial understanding of the principle.

"If you add 4, 3, 2, and 1, the result is only 10. So I tried using multiplication." – IQ



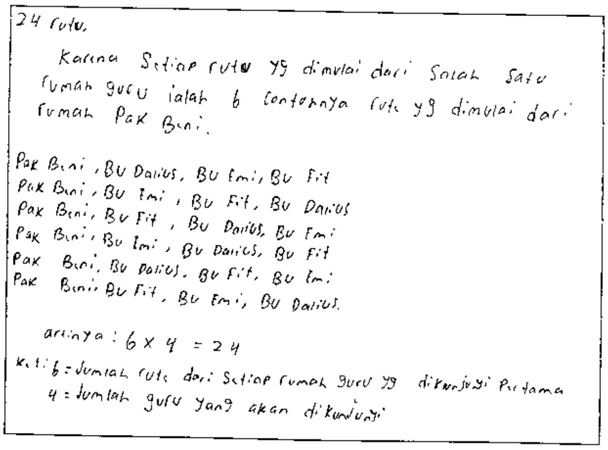
Translation:
1. (Given) 4 options, chosen 1, left 3 options
2. (Left) 3 options, chosen 1, left 2 options
3. (Left) 2 options, chosen 1, left 1 options
4. (Left) 1

4 x 3 x 2 x 1 = 24 routes or 4!
~~A~~BCD ~~B~~CD ~~C~~D D

Figure 3. IQ's use of logical step-by-step reasoning before multiplication

AD used a listing strategy similar to that of FZ in Problem 1. AD started by assuming the trip began at Mr. Beni's house, enumerated six different routes from that point, and then inferred a total of 24 routes by multiplying by 4 potential starting points (see Figure 4). Interviews revealed that AD had already listed all the possibilities on a separate sheet before making the generalization.

"After I tried starting from Mrs. Dalius, the result was also 6. I guess this applies to others too." – AD

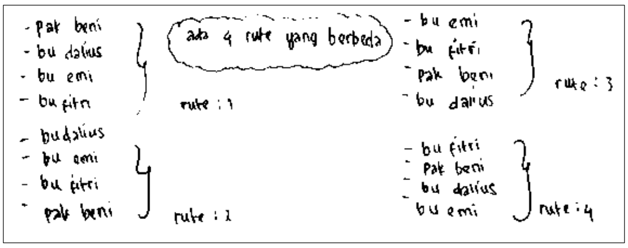


Translation:
24 routes
Because each route that begin from one teacher house have 6 (routes). For example, here the routes that begin from Mr. Beni's house.
(Arrangements)
It means 6 x 4 = 24 (routes)
NB:
6 = the number of routes from each teacher house that visited first
4 = the number of teachers (house) that will be visited

Figure 4. AD's strategy: listing from one starting point, then multiplying by the number of starting points

NZ presented only four routes, mistakenly assuming that different starting points alone constituted different routes. This reflects a misconception, failing to recognize that multiple permutations can originate from the same starting point (see Figure 5).

"I think there are only 4 routes because I started from Mr. Beni, Mrs. Dalius, Mrs. Emi, and Mrs. Fitri." – NZ



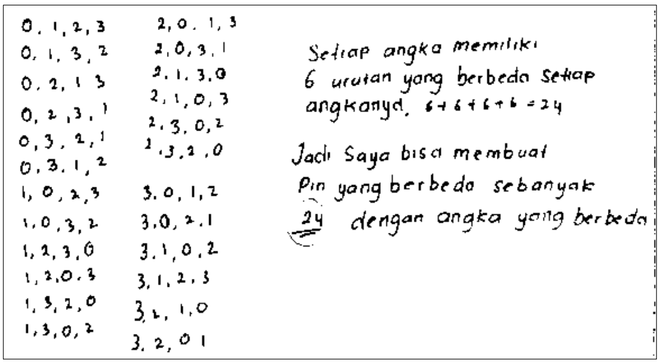
Translation:
There are 4 differnet routes.
- Mr. Beni - Mrs. Dalius - Mrs. Emi - Mrs. Fitri
- Mrs. Dalius - Mrs. Emi - Mrs. Fitri - Mr. Beni
- Mrs. Emi - Mrs. Fitri - Mr. Beni - Mrs. Dalius
- Mrs. Fitri - Mr. Beni - Mrs. Dalius - Mrs. Emi

Figure 5. NZ's misinterpretation of route variation based solely on starting points

DZ showed a more fundamental misunderstanding by interpreting "route" as a single segment of travel between two houses rather than a full sequence. For example, DZ considered traveling from Mr. Beni's to Mrs. Dalius's house as a complete route. This error indicates a failure to grasp the problem's structure.

Problem 3: Creating a 4-digit PIN

This problem was chosen by most students: OK, RZ, AG, BG, IT, RD, HS, and PR. OK, RZ, AG, IT, and PR used enumeration by listing possible PINs. Among them, only RZ successfully listed 24 unique combinations by fixing the starting digit and varying subsequent digits systematically (see Figure 6).



Translation:

Each digit have 6 different arrangements, so 6 + 6 + 6 + 6 = 24.

(Arrangements)

So, I can make 24 different PIN, by different digits.

Figure 6. RZ's systematic enumeration of unique pins based on starting digit

Interestingly, RZ used repeated addition rather than multiplication, suggesting a completely manual enumeration process. On the other hand, OK and AG produced repeated combinations due to a lack of systematic listing, as shown in Figure 7.

"I didn't check the same order. What matters is the total is 6 × 4. – OK

"I believe it is true because sequences starting from different numbers have the same total." – AG

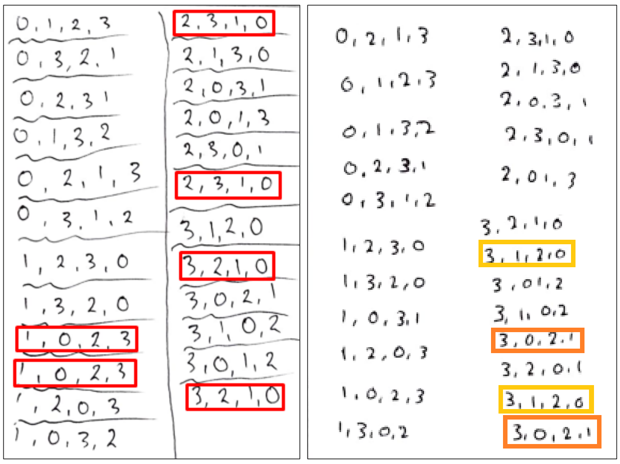
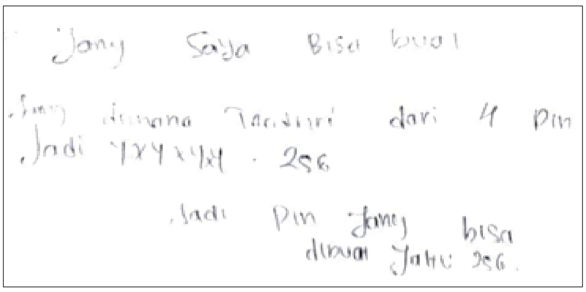


Figure 7. Overlapping PINs in OK's (left) and AG's enumerations (right)

In interviews, OK revealed that the listing assumed no repeated digits, indicating partial awareness of repetition rules. However, both IT and PR failed to list all possible combinations, producing only 22 and 6 combinations, respectively, some of which involved repeated digits (e.g., 1122, 0000).

RD applied the multiplication principle by multiplying 4 × 4 × 4 × 4 = 256, assuming each digit could be filled with any of the four available numbers (0–3). While RD was confident in this approach, interviews revealed that RD could not justify the use of multiplication (see Figure 8).

"These numbers can be repeated, so the number of choices remains the same (at each step). ... Just multiply them to get the largest number." – RD



Translation:
That is I can make.
That consist of 4 PIN.
So, 4 x 4 x 4 x 4 = 256
So, the number of PINs that can be made are 256.

Figure 8. RD's use of the multiplication rule without supporting explanation

HS used the strategy of listing combinations starting with one digit (0), similar to FZ and AD, then multiplied by 4 to arrive at 24 PINs. Lastly, BG gave two answers: 256 (with repetition allowed) and 24 (no repetition), but admitted during interviews that the answers were generated using ChatGPT, and not through personal reasoning. Consequently, BG's response was excluded from strategy classification for reasons of methodological integrity.

Variation of students' strategies

Apart from DZ and BG, students' solutions were classified into three general strategies: (1) listing all unique arrangements, (2) generalizing patterns of arrangement, and (3) using the multiplication principle. The distribution of these strategies is summarized in Table 1.

Table 1. Summary of students' strategies in solving enumeration problems

Strategy	Students	Frequency (n = 15)	Percentage	Description
Listing Unique Arrangements	OK, RZ, AG, IT, NZ, AL, PR	7	47%	Manually listing all possible outcomes
Generalizing Patterns	FZ, AD, HS	3	20%	Identifying structure or pattern before generalizing result
Using Multiplication Principle	RT, IQ, RD	3	20%	Directly applying multiplication without full justification
No Clear Strategy or Misguided	DZ, BG	2	13%	Misunderstanding or relying on external tools

As summarized in Table 1, seven students (47%) relied on listing unique arrangements, three (20%) generalized structural patterns, and three (20%) directly applied the multiplication principle. The remaining two students (13%) either misunderstood the problem (DZ) or relied on external tools (BG). These results indicate that while most students were able to engage in combinatorial reasoning at an intuitive level, their strategies reflected varying degrees of formalization—from exhaustive manual listing to emerging multiplicative reasoning.

Discussion

The findings of this study reveal that students employ various initial strategies in solving enumeration problems, reflecting diverse levels of understanding related to permutations and counting principles. The three main strategies identified were: (1) listing all possible arrangements, (2) generalizing patterns of arrangement, and (3) applying the multiplication principle. These findings suggest that students' intuitive approaches reflect early forms of relational understanding, as opposed to purely instrumental reasoning (Skemp, 1978). Recognizing such informal reasoning is essential for designing learning trajectories that connect intuitive thinking with formal combinatorial principles. The development of these strategies aligns with the theoretical framework of Realistic Mathematics Education (RME), where understanding progresses from contextual to formal reasoning through guided reinvention (Solomon et al., 2021).

This classification was guided by previous research on students' combinatorial reasoning (Lockwood, 2013; Salavatinejad et al., 2021). Lockwood (2013) proposed a model of students' combinatorial thinking that illustrates how learners progress from constructing explicit lists of possible outcomes to identifying structural patterns and, eventually, applying formal counting principles such as the multiplication rule. This developmental trajectory reflects a gradual shift from informal to formal reasoning. Similarly, Salavatinejad et al. (2021) found that students initially rely on intuitive strategies such as listing and drawing visual representations to organize possibilities, with only a few able to generalize patterns or correctly use the multiplication principle. Together, these studies provide a strong theoretical basis for the three strategy categories identified in this research—listing unique arrangements, generalizing patterns of arrangement, and using the multiplication principle—as they represent distinct yet connected stages in students' evolving understanding of enumeration problems.

The first strategy, listing all possible arrangements, was employed by the majority of students. This strategy is aligned with Lamanna et al. (2022) that unveil that the majority of students without combinatorics instruction use listing, mostly systematic, to help the enumeration process. The listing strategy corresponds to the situational level of RME, where reasoning remains embedded in the problem context (van den Heuvel-Panhuizen, 2003). This strategy represents an intuitive and concrete approach to problem-solving and is characteristic of the early stage of mathematization in the framework of Realistic Mathematics Education (RME) (Freudenthal, 1991). Lockwood et al. (2015) argue that partial lists of the set of outcomes which created by students led to significant improvements in solving problems, implying that systematic listing worthwhile for students as they learn to count.

The second strategy—generalizing patterns of arrangement—was demonstrated by students who attempted to make a tree diagrams however not complete but they can extrapolate them to solve whole problems. This approach marks a transition toward horizontal mathematization, where students begin to connect contextual problem situations to more structured mathematical representations (van den Heuvel-Panhuizen, 2003). For example, students like FZ and AD showed an intuitive grasp of repeated structural patterns, even though they were not able to articulate them formally. This indicates that students were beginning to construct informal models of mathematical structure, which can serve as a foundation for formal instruction on permutations and tree diagrams (Litwiller & Bright, 2002). This symbolic representation like tree diagrams enables students to perceive how elements can be combined in a systematic manner and help them develop their combinatorial reasoning (Borba et al., 2015).

The third strategy, applying the multiplication principle, reflects a shift toward vertical mathematization—moving from informal representations to formal mathematical procedures (Gravemeijer, 2004). However, it is important to note that most students using this strategy did not demonstrate a deep conceptual understanding of why multiplication was appropriate in the given context. Although the use of the multiplication principle suggests potential for formal reasoning, pedagogical support is needed to guide students toward a full conceptual grasp of the method. The research shows that it is not always natural for students to use the multiplication rule in solving combinatorics problems because it involves sequential steps that tend to be complicated for beginners (Lockwood & Purdy, 2020).

In the specific case of the PIN problem, students' understanding became more complex due to the involvement of digit repetition. Some students limited their responses to nonrepetitive arrangements, while others conflated repetition and non-repetition scenarios. This finding underscores the need for explicit instruction on the distinction between permutations with and without repetition, as recommended by Lockwood (2013) in her study on combinatorial representations.

Two students, DZ and BG, displayed responses that did not align with any of the three major strategies. DZ misunderstood the concept of "route" as a complete tour rather than a sequence of destinations, while BG provided a correct answer but admitted to relying on AI assistance (ChatGPT). The latter case illustrates a growing issue in mathematics education related to academic integrity and the responsible use of digital tools. While technology can be a valuable tool in fostering mathematical thinking, it can also hinder cognitive engagement if used as a substitute for reasoning rather than a support for it (Zbiek et al., 2007).

A particularly meaningful feature of this study is its cultural context. The enumeration problems—especially the Eid al-Fitr visiting scenario—are deeply familiar to Indonesian students. Such culturally resonant contexts align with the RME emphasis on realistic (experientially meaningful) situations (van den Heuvel-Panhuizen & Drijvers, 2014). The use of tourism and holiday activities connects mathematics to students lived experiences, enabling authentic engagement and supporting horizontal mathematization. This underscores that realistic contexts need not be Western or generic; they can and should be locally and culturally grounded to enhance relevance and motivation.

These findings have significant implications for instructional design informed by RME principles. First, students need learning experiences that stimulate natural mathematization processes from contexts that are meaningful to them—such as the tourism scenarios used in this study. Second, teachers play a crucial role in encouraging open exploration of student strategies prior to introducing formal procedures. Third, students' initial strategies should serve as starting points for developing a bottom-up learning trajectory, in accordance with the principle of guided reinvention in RME (Eerde, 2013).

The findings carry several implications for mathematics instruction, particularly in the domain of counting and permutations. The problems in this study provided meaningful contexts that facilitated student engagement and elicited natural strategies. Teachers should integrate culturally relevant and experientially rich contexts that allow students to mathematize real-life situations (van den Heuvel-Panhuizen & Drijvers, 2014). Teachers should create space for students to share and discuss their own problem-solving strategies before introducing formal methods. This aligns with the RME principle of guided reinvention, allowing students to construct mathematical understanding from their own thinking. Solomon et al. (2021) emphasized that guided reinvention requires environments that encourage exploration before formalization.

Although some students' strategies appeared incomplete or incorrect, these attempts can be seen as instances of productive failure (Kapur, 2016), where the struggle to construct solutions independently lays the groundwork for deeper conceptual understanding in subsequent instruction. Instruction should focus on developing students' reasoning about when and why multiplicative structures apply, using tools such as tree diagrams and systematic listing as transitional models (English, 1991; Lockwood, 2013). Teachers also play a crucial role in orchestrating whole-class discussions, connecting informal and formal strategies, and bridging intuitive ideas with canonical mathematical representations (Stein et al., 2008). While digital tools like AI can support learning, their integration should aim to enhance—not replace—student thinking. Educators must carefully monitor how students use such tools to ensure productive engagement (Zbiek et al., 2007).

Conclusion

This study demonstrates that even without formal instruction, students naturally use a range of strategies—listing all possible arrangements, generalizing patterns, and attempting to apply the multiplication principle—reflecting varying levels of informal combinatorial reasoning. While some students relied heavily on trial-and-error or incomplete listing strategies, others began to show signs of structural understanding and pattern generalization. Only a small number attempted formal mathematical reasoning. These findings underscore the importance of recognizing and leveraging students' intuitive thinking as a foundation for meaningful learning trajectories in combinatorics. Teachers should view students' informal approaches not as misconceptions, but as seeds of mathematical understanding that can be nurtured through guided reinvention.

Future research should extend this work in several directions. First, design experiments could be conducted to test learning trajectories that build on students' initial strategies and progressively develop conceptual understanding of counting principles. Second, cross-cultural studies could explore whether similar intuitive patterns emerge in different sociocultural settings, thereby enriching the global understanding of combinatorial reasoning. Third, the integration of digital tools—such as dynamic tree diagram applications or AI-based visualization aids—should be examined for their potential to support students' exploration and deepen their relational understanding of combinatorial structures.

Conflicts of Interest

The authors declare no conflict of interest regarding the publication of this manuscript.

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Author Contributions

Aan Putra: Conceptualization, formal analysis, investigation, writing - original draft, and visualization; **Zulkardi:** Conceptualization, methodology, validation, and writing - review & editing, supervision; **Ratu Ilma Indra Putri:** Methodology, validation, and supervision; **Laswadi:** Writing - original draft, validation, and supervision.

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