



Developing GeoGebra-based digital worksheets to foster conceptual understanding in circle geometry

Putu Friska Anandita, I Wayan Sudiarsa, I Gusti Agung Ngurah Trisna Jayantika *

Mathematics Education Department, Universitas PGRI Mahadewa Indonesia, Bali, Indonesia

* Correspondence: jayantika@mahadewa.ac.id

© The Author(s) 2026

Abstract

Digital tools are increasingly being used in mathematics learning; however, there are few empirical studies that have examined the effect of GeoGebra integration in structured learning media (e.g., digital worksheets) on students' conceptual understanding in particular topics such as circles. This research aims to design and validate a digital worksheet using GeoGebra to improve students' understanding of basic mathematical concepts. This study employed the Research and Development (R&D) approach using the ADDIE model. The subjects of the study were 48 students of Grade XI of a state senior high school in Gianyar, Bali, Indonesia. Data collection was conducted through expert validation, user evaluation, and concept understanding tests. Results showed that the developed digital worksheet was valid (average expert rating was above 4.2), practical (practically applicable on all indicators' evaluation), and effective, which was evidenced by a significant increase in student scores from pre-test ($M = 73$) to post-test ($M = 87$) with an N-Gain score of 0.68. Theoretically, this study contributes to the evidence of GeoGebra-supported conceptual learning in geometry, based on design-based research. Unlike its use as a standalone tool, the current study incorporates GeoGebra in structured worksheets to provide a guided-discovery framework for conceptual understanding.

Keywords: circle topic; fundamental mathematical concepts; GeoGebra-based digital worksheets

How to cite: Anandita, P. F., Sudiarsa, I. W., & Jayantika, I. G. A. N. T. (2026). Developing GeoGebra-based digital worksheets to foster conceptual understanding in circle geometry. *Jurnal Elemen*, 12(2), 356-374. <https://doi.org/10.29408/jel.v12i2.31803>

Received: 24 July 2025 | Revised: 5 October 2025

Accepted: 28 March 2026 | Published: 30 April 2026



Introduction

Understanding fundamental mathematical concepts is a cornerstone of students' mathematical competence. It goes beyond rote procedures and formulas, enabling learners to reason, generalize, and apply mathematical ideas in various contexts. In particular, conceptual understanding allows students to recognize relationships among mathematical structures, which is essential for building advanced mathematical thinking. However, numerous studies have reported that many students struggle with understanding core mathematical concepts, including those related to geometry, such as circles. These difficulties often stem from traditional learning approaches that prioritize procedural tasks over meaningful engagement and visualization of abstract ideas (Nesta et al., 2024; Suryati & Adnyana, 2022).

In response to these challenges, digital learning tools have been increasingly adopted to foster deeper understanding of mathematical concepts. One such approach is the development of interactive digital worksheets integrated with GeoGebra, a dynamic mathematics software. GeoGebra enables real-time manipulation of geometric objects, which can help learners explore, visualize, and internalize abstract mathematical ideas (Puspita et al., 2022; Ziatdinov & Valles, 2022). Previous research has highlighted the effectiveness of GeoGebra-based instruction in enhancing conceptual understanding, motivation, and student engagement in mathematics classrooms. For example, the integration of GeoGebra into learning materials has been shown to support students' active exploration and improve their ability to make meaningful connections between concepts (Nopitria et al., 2022; Sugandi et al., 2020). At the same time, Salifu (2020) and Nugroho et al. (2025) have shown the benefits of GeoGebra in increasing student engagement and visualization skills, these interventions were rarely embedded in structured learning tools like digital worksheet (Nugroho et al., 2025; Salifu, 2020).

Despite the growing body of research supporting GeoGebra and digital worksheet as effective learning tools, there is still limited empirical evidence on how their integration specifically influences students' understanding of fundamental mathematical concepts in particular topics, such as circles. Most existing studies tend to focus on broader mathematics achievement or technology acceptance, leaving a gap in understanding the conceptual impact of these tools on specific mathematical domains (Foo & Ng, 2022; Mollakuqe et al., 2021).

Research emphasizes that conceptual understanding in mathematics develops most effectively through instruction that helps students make sense of relationships among concepts, something that traditional paper-based worksheets often fail to achieve (Tay & Wonkyi, 2018). Meanwhile, Foo and Ng (2022) found that while students are often able to perform calculations involving circles, they demonstrate weak understanding of the underlying concepts, such as the meaning of π , the relationship between diameter and circumference, and the derivation of formulas. Furthermore, students' misconceptions persist because instructional practices often fail to support students in exploring relationships among geometric concepts, resulting in fragmented and superficial understanding (Maison et al., 2022). A previous study demonstrated the validity of GeoGebra-supported guided-inquiry worksheets for teaching circles, though without examining learning gains (Pertwi et al., 2023), while another showed significant

improvement in student outcomes for circle equations through GeoGebra software (Widana & Laksitasari, 2023).

Although previous studies have demonstrated the potential of GeoGebra to support engagement and visualization, these interventions are often implemented as standalone activities rather than being embedded within structured, student-centered learning tools such as digital worksheet. The limited number of studies integrating GeoGebra within comprehensive student-centered learning formats highlights the need for further investigation in this area.

This research is therefore urgent in addressing both a practical and theoretical need: to develop a targeted digital intervention that not only supports students' comprehension of circle concepts but also provides evidence-based guidance for future instructional design. The study holds significant implications for both practitioners and researchers. For educators, the research offers a concrete model of integrating GeoGebra into curriculum materials to improve students' conceptual understanding (Pertiwi et al., 2023). For researchers, it contributes to the ongoing discourse on digital tool integration in mathematics education and opens new directions for exploring specific learning outcomes—especially through a design-based research approach (Ntshangase et al., 2022; Pertiwi et al., 2023).

This study aims to develop an explorative learning design that integrates GeoGebra-based digital media, to examine the validity and practicality of the developed media, and its effectiveness in strengthening students' understanding of fundamental mathematical concepts, specifically on the topic of circles.

Methods

This study adopts a Research and Development (R&D) approach that integrates the systematic stages of the ADDIE model (Analysis, Design, Development, Implementation, and Evaluation) as its operational framework. While the ADDIE model provides a structured process for product development, the broader R&D paradigm guides the study's iterative refinement and theoretical grounding. R&D is particularly suited for educational innovation because it emphasizes the development of practical solutions while simultaneously contributing to theory building through cycles of design, implementation, and reflection (Brinkerhoff et al., 2024). This study adopts a R&D approach to iteratively develop, test, and refine the digital worksheet, with the goal of producing a usable and pedagogically sound learning tool (Pertiwi et al., 2023).

In this study, the R&D orientation is reflected in the continuous feedback obtained from experts, teachers, and students across the ADDIE phases, which informed iterative improvements to the GeoGebra-based digital worksheet. Thus, the study not only produced a validated and effective digital learning tool but also provided insights into how interactive visualization and explorative design support students' conceptual understanding in geometry. This dual contribution—product development and theoretical refinement—aligns with the principles of R&D in real classroom contexts. The ADDIE model is widely recognized for its systematic and flexible structure, making it particularly suitable for the development of instructional media—especially in mathematics education, which demands adaptive and responsive instructional design (Komariah et al., 2024; Martatijana et al., 2023).

The research subjects consisted of 48 students from class XI B2 at SMA Negeri 1 Gianyar, Bali, Indonesia, evenly divided into 24 male and 24 female students. The selection of SMA Negeri 1 Gianyar as the research site was based on several considerations: while the school already had access to instructional media, it had not yet been utilized to its full potential; it also had adequate facilities and infrastructure to support the research activities, and the school administration offered full support for the implementation of this study.

The study was conducted in Grade XI mathematics class at SMA Negeri 1 Gianyar, a public senior high school located in Bali, Indonesia. The school has adequate ICT infrastructure, including computer laboratories and stable internet access. Most students had prior exposure to basic digital learning tools such as Google Classroom and interactive quizzes but had not previously used GeoGebra-based learning media. The mathematics curriculum follows the *Kurikulum Merdeka* framework, which emphasizes student-centered learning, problem-solving, and the use of digital technology to support exploration and reasoning. This context provides a relevant setting for implementing and evaluating the GeoGebra-based digital worksheet designed to enhance conceptual understanding.

Participation in this study was voluntary, and informed consent was obtained from the students and the school prior to data collection. Students' identities were kept anonymous, and all data were treated confidentially and used solely for research purposes. The study was conducted in accordance with ethical standards for educational research.

The type of data used in this research is primary data, obtained directly from the main sources and focusing on students' problem-solving abilities after using the learning media. Quantitative data included the results of validation questionnaires from media experts, subject matter experts, and students' test scores. Meanwhile, qualitative data were gathered from feedback provided by validators, students, and teachers.

The GeoGebra-based interactive digital worksheet developed in this study will be evaluated for its validity and practicality through an integrated process across each phase of the ADDIE model. A detailed explanation of each development phase will be provided in the Table 1 below.

Table 1. Stages and activities of the geogebra-based digital worksheet development process using the ADDIE model

No	Stages	Activity description
1	Analyze	This phase involved conducting a needs assessment and analyzing the curriculum.
2	Design	In this phase, a preliminary design of Geogebra based digital worksheet was created, informed by the results of the needs and curriculum analysis. Additionally, a storyboard for the media was developed.
3	Develop	At this stage, the media content was produced based on the initial design. Product validation was carried out by four experts, including two media experts and two subject matter experts, using the Learning Object Review Instrument (LORI).
4	Implement	During this stage, the Geogebra based digital worksheet was implemented with students. Practicality was

No	Stages	Activity description
		assessed using the User Experience Questionnaire (UEQ), while the effectiveness of the media was measured through a problem-solving ability test.
5	Evaluate	Evaluation activities were embedded throughout all stages of development.

The data in this study were collected using the LORI questionnaire for validity data, the ueq questionnaire for practicality data and the concept understanding ability test for effectiveness data. The LORI questionnaire was utilized to evaluate the validity of the developed product and was implemented during the development stage. This validation process involved two media experts and two subject matter experts. The structure of the LORI questionnaire is presented in the Table 2 and Table 3.

Table 2. LORI dimensions and indicators used to assess content validity of the geogebra-based digital worksheet (material experts)

No	Aspect	Indicator
A	Content Quality	1. Rigour of the material
		2. Accuracy of material
		3. Regularity in presentation of material
		4. Accuracy in the level of detail of the material
B	Learning and Alignment	1. In accordance with the learning objectives
		2. In accordance with the learning activity
		3. In accordance with the assessment in learning
		4. In accordance with student characteristics
C	Feed-back and adaptation	Adaptation or feedback content can be executed by different learners or learning models
D	Motivation	Ability to motivate and attract student attention

Table 3. LORI dimensions and indicators used to assess media design and usability of the geogebra-based digital worksheet (media experts)

No	Aspect	Indicator
A	Presentation Design	Multimedia design (visual and audio) can help improve and streamline learning.
B	Interaction Usability	1. Ease of navigation
		2. Predictable display
C	Accessibility	Ease of access
D	Reusability	Ability to be used in different learning variations and with different students
E	Standards compliance	Adhere to international standard specifications

UEQ questionnaire is used to measure the practicality of the developed media. The indicators of the UEQ questionnaire are as follows in Table 4.

Table 4. UEQ dimensions and operational definitions used to evaluate the practicality of the geogebra-based digital worksheet

No	Indicator	Definition
1	Attraction	The overall impression users have of the product, including whether it is perceived as enjoyable, engaging, or dull.
2	Agility	How easily users can understand and start using the product, including whether it appears intuitive and user-friendly.
3	Efficiency	The degree to which users can perform tasks efficiently, with minimal time and effort when using the product.
4	Accuracy	Users' confidence in the product, reflected in the sense of control, reliability, and consistency it provides.
5	Stimulation	The ability of the product to deliver an enjoyable, immersive, and motivating experience for the user.
6	Novelty	How original and creative the product is perceived to be, and whether its uniqueness makes it engaging rather than monotonous.

Both the Learning Object Review Instrument (LORI) and the User Experience Questionnaire (UEQ) used in this study are standardized instruments that have been widely applied and validated in various educational technology studies. The LORI instrument developed by [Leacock and Nesbit \(2007\)](#) and the UEQ developed by [Schrepp et al. \(2017\)](#) have demonstrated strong psychometric properties, including construct validity and high internal consistency (Cronbach's $\alpha > 0.80$) across multiple international contexts.

In this study, both instruments were used without structural modification, and only minor linguistic adjustments were made to ensure clarity for Indonesian respondents. The translated items were reviewed by two bilingual experts to maintain conceptual equivalence, a process that aligns with the recommendations for cross-cultural adaptation of standardized instruments ([Beaton et al., 2000](#)). The instruments were administered in their original validated formats, with psychometric properties established in prior research.

For the concept understanding ability test, the indicators used for the development of this instrument are as follows in Table 5.

Table 5. Indicators of students' fundamental mathematical conceptual understanding used in the effectiveness test

No	Indicator	Description
1	Identifying concepts	Students are able to recognize, select, or recall relevant terms or mathematical concepts related to the given topic.
2	Explaining concepts in their own words	Students can restate definitions or key ideas of a concept using their own language, demonstrating a deep conceptual understanding.

No	Indicator	Description
3	Providing examples and non-examples	Students are able to give appropriate examples that illustrate a concept and distinguish them from non-examples to clarify conceptual boundaries.
4	Connecting concepts	Students can explain the relationship between one concept and another, either within the same topic or across different topics in mathematics.
5	Applying concepts in new situations	Students are able to apply mathematical concepts to unfamiliar problems or real-life contexts, showing flexibility in understanding.
6	Interpreting various representations	Students can interpret and explain mathematical concepts using different forms of representation (e.g., graphs, tables, symbols, or narratives).

To assess validity and practicality, the data were analyzed quantitatively by computing the average scores obtained. Meanwhile, the effectiveness data were analyzed using the normalized gain (h-gain) formula as follows.

$$N - Gain = \frac{S_{post} - S_{pre}}{S_{max} - S_{pre}} \tag{1}$$

The N-Gain calculation results are then interpreted based on the following criteria in Table 6 below.

Table 6. Interpretation criteria for normalized gain (n-gain) scores in measuring learning effectiveness

N-Gain Score	Category
$g > 0.7$	High
$0.3 \leq g \leq 0.7$	Moderate
$g < 0.3$	Low

To determine the statistical significance of the improvement in students’ conceptual understanding, the pre-test and post-test scores were analyzed using a paired sample t-test with a significance level of 0.05. Prior to conducting the t-test, the normality of the data distribution was examined using the Shapiro–Wilk test. This combination of N-Gain analysis and inferential testing provides both descriptive and statistical confirmation of the effectiveness of the GeoGebra-based digital worksheet. Such an approach is recommended for educational intervention studies with moderate sample sizes (Sarah & Batiibwe, 2024).

Results

The stages of developing geogebra-based digital worksheet using the ADDIE phase model as follows.

Analyze

The initial stage of the ADDIE model is the analysis phase. Based on interviews with teachers and preliminary classroom observations, it was found that students face difficulties in understanding fundamental concepts related to circles—such as tangents, radii, diameters, and central angles—especially when these concepts are presented abstractly in textbooks. Furthermore, teacher-centered instruction limits student engagement during the learning process. Although the school is equipped with adequate ICT facilities, these resources have not been fully utilized to support learning. Therefore, there is a need for a learning medium that offers an interactive and visual experience, helping students develop a deeper and more meaningful conceptual understanding.

Design

Following the needs analysis, the next stage involves designing the exploratory digital worksheet based on GeoGebra. The initial design includes formulating learning outcomes aligned with the Kurikulum Merdeka, determining the sequence of content delivery, incorporating exploratory activities using GeoGebra, and developing instruments for evaluation and reflection. The media design also features a simple yet engaging navigation structure, allowing students to easily access the content via both computers and smartphones. Additionally, the visual design adheres to principles of readability and aesthetics to ensure a comfortable learning experience. The instructional materials are tailored to the characteristics of Phase E and are aligned with learning objectives that emphasize reasoning and conceptual understanding.

In this phase, researchers also developed a lattice for the concept understanding ability test in accordance with the indicators of the theoretical study of this variable. the draft that became the output of this phase was then developed in the next phase, which is develop

Develop

After going through the design stage, and all parts have been made, the next stage is the creation or develop stage. This GeoGebra-based digital worksheet design is made using Canva. The initial display of this learning media displays the title 'Equation of the Outer Tangent Line and Inner Tangent Line of the Circle' with an attractive design and educational illustrations with the theme of Mathematics Learning Module Designed by the Researcher. Underneath are the basic competencies and achievement indicators that emphasise understanding the concept of tangent lines and the use of GeoGebra for visualisation and problem solving. For the initial appearance of Geogebra-based digital worksheet is shown in Figure 1.

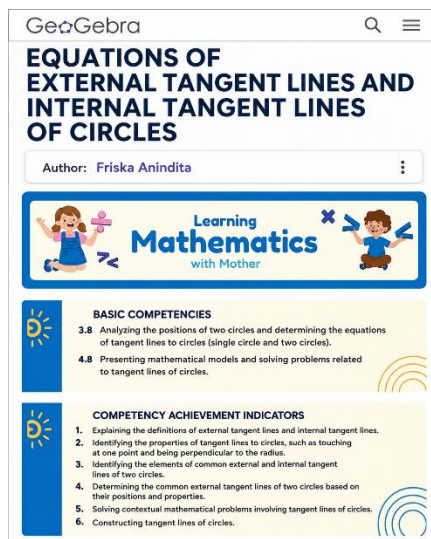


Figure 1. Initial media display

In the next section, instructions for using the media are presented. Instructions for using this media are presented in geogebra 1 media, which discusses the elements of the tangent line of the outer fellowship of two circles, and at the beginning of geogebra 2 media, which discusses the tangent line of the circle. The instructions for using this geogebra-based exploratory digital worksheet contain how to use it, what is produced, and also what the next steps are after trying the geogebra can be shown in Figure 2.

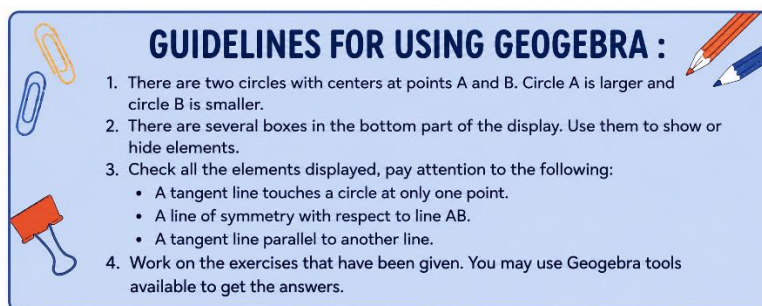


Figure 2. Instructions for using the media

The next display shows interactive learning media using GeoGebra to understand the tangent line of two circles, both outer and inner. Students can observe the visualisation of two circles along with important elements such as the centre point, radius, tangent line, and the line connecting the centre of the circles. Through these instructions and simulations, students are invited to explore the relationship between these geometry elements logically and dynamically. GeoGebra also provides practice problems that can be done directly through the available links to strengthen concept understanding. The Figure 3 below is an exploratory media display that can be accessed on the Geogebra page.

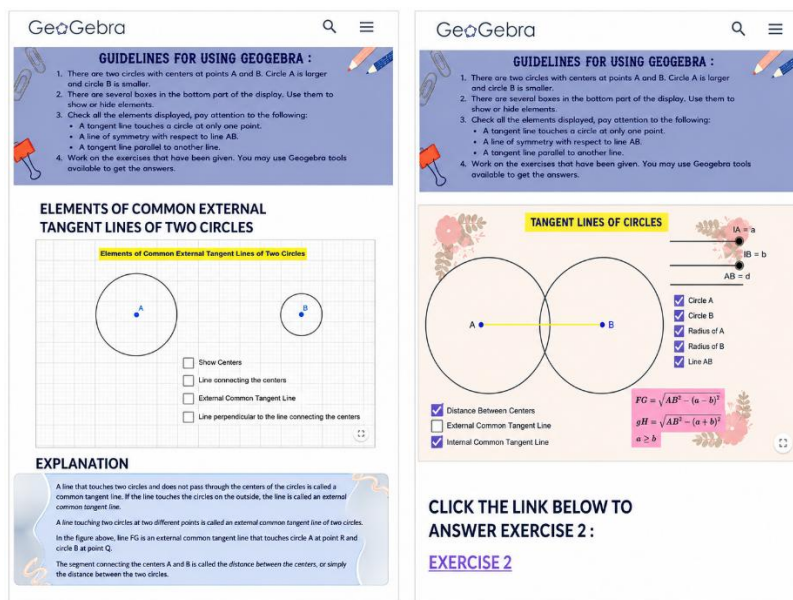


Figure 3. Exploration page view

On the Exercise page there are two questions, namely Exercise question 1 and Exercise question 2. Exercise problem 1 explains more about the elements of the outer tangent of the circle and the definition of tangent, while Exercise problem 2 is more specifically related to the second media where there is also a formula for the tangent line of the outer circle and the tangent line of the inner circle. The Exercise page is the last part of the GeoGebra-based digital worksheet. It is presented in an interactive format with multiple choices and followed by a link to the online exercise, demonstrating a digital learning approach that supports students' active engagement in understanding the material. The example of a practice question show in Figure 4.

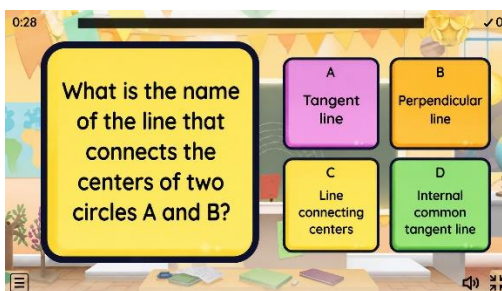


Figure 4. Sample practice questions

After the media was developed, three main types of testing were conducted, namely validity, practicality, and effectiveness. These three aspects are used as the basis for assessing whether the media produced is suitable for use in the technology-based mathematics learning process. The validity test was conducted by two material experts and two media experts. The purpose of this validation is to determine the extent of the feasibility of material content, media suitability with basic competencies, as well as the appearance and navigation of GeoGebra-based digital worksheet.

The recapitulation of the results of the validity test by material experts and media experts is presented as follows in Table 7.

Table 7. Recapitulation of expert validation test results

	Material Expert		Media Expert	
	Average	Category	Average	Category
Validator 1	4.10	Feasible	4.62	Very Feasible
Validator 2	4.30	Very Feasible	4.54	Very Feasible

In addition to the results in Table 7, there are also input and suggestions from both material and media experts which are presented in Table 8 below.

Table 8. Recapitulation of input from material and media experts

Materi Expert		Media Expert	
Validator	Suggestion	Validator	Suggestion
1	<ol style="list-style-type: none"> 1. Pay attention to the writing of mathematical symbols 2. The layout of the material is organised to make it more attractive to students 	1	<ol style="list-style-type: none"> 1. Media made more interesting 2. Revise the appearance and explanation of the material 3. Writing is made neater to make it easier for students
2	Add a variety of questions with higher cognitive levels	2	<ol style="list-style-type: none"> 1. Add an interesting background 2. Shorten the text to make it more interesting

The results of the expert validation test state that the Geogebra-based digital worksheet is feasible to be implemented in learning. Implementation in learning is carried out in the next phase, namely implement.

Implement

To ensure implementation fidelity, the Problem-Based Learning (PBL) model was delivered consistently across three learning sessions. Each session followed the planned phases: (1) problem orientation, (2) group exploration using GeoGebra-based digital worksheet, (3) group discussion and presentation, and (4) reflection and conclusion. The researcher and the classroom teacher jointly facilitated the sessions based on a shared lesson plan to maintain alignment with the intended design. Minor deviations occurred, such as additional time allocation for technical guidance during the first session, but these adjustments were necessary to ensure students' readiness and did not alter the core structure of the PBL implementation.

Implementation of Geogebra-based digital worksheet on learning through PBL learning model. During the implementation phase, both practicality and effectiveness were evaluated. Practicality data were obtained through the User Experience Questionnaire (UEQ), while effectiveness data were gathered using a test on Understanding of Fundamental Mathematical Concepts. A summary of the practicality test results is presented in the following table and figure.

Table 9. UEQ scale score results

UEQ Scales	Mean	Comparisson to benchmark
Attraction	2.63	Excellent
Clarity	2.21	Excellent
Efficiency	2.26	Excellent

UEQ Scales	Mean	Comparisson to benchmark
Accuracy	2.46	Excellent
Stimulation	2.41	Excellent
Novelty	2.35	Excellent

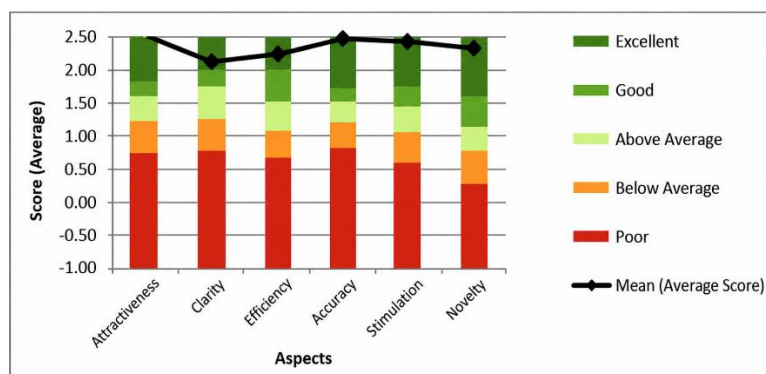


Figure 5. UEQ score average distribution diagram

From the results in table 8 and figure 5, the analysis results show that all UEQ scales are in the ‘Excellent’ category. The highest score was obtained on the attractiveness scale with a score of 2.63, followed by accuracy (2.46), stimulation (2.41), novelty (2.35), efficiency (2.26), and clarity (2.21). The finding of this results is students feel comfortable, interested, and easy to understand the material when using the media, and assess that the appearance and navigation of GeoGebra-based digital worksheet greatly support their learning process. The UEQ results indicated that all six scales exceeded the benchmark values, suggesting a high level of perceived usability

Furthermore, the effectiveness of the media is seen from the improvement of student learning outcomes. The effectiveness test was conducted by comparing the pre-test and post-test scores after students followed the learning using GeoGebra-based digital worksheet. The average student score increased significantly, from 73 in the pre-test to 87 in the post-test. In addition, learning completeness increased from 42% before the use of media to 92% afterwards. To measure the magnitude of the improvement, the N-Gain formula was used, which resulted in a value of 0.68 and fell into the ‘medium to high’ category. The following is a recapitulation of the results of the test of students' ability to understand basic mathematics concepts.

Table 10. Recapitulation of test results of understanding basic mathematics concepts

	Total Score	Number of Students		Average	Percentage of Completion
		Completed	Not Completed		
Pre-Test	3525	20	28	73	42%
Post-Test	4180	44	4	87	92%

A summary of the n-gain test is presented in the following Table 11.

Table 11. Recapitulation of n-gain values

N-gain score	Category
0.68	Moderate effectiveness

Based on the results of the N-Gain data analysis, the average pre-test and post-test scores, and the percentage of student completeness, it can be concluded that the use of Geogebra-based digital worksheet is effective in improving students' understanding of basic mathematics concepts. The N-Gain score of most students was in the moderate to high category, the average score increased from 73 to 87, and the percentage of completeness increased significantly from 42% to 92%. These results indicate that the developed Geogebra-based digital worksheet met the predefined criteria for validity, practicality, and effectiveness.

In addition to the N-Gain analysis, a paired sample t-test was conducted to examine whether the observed improvement from pre-test ($M = 73$, $SD = 7.6$) to post-test ($M = 87$, $SD = 6.9$) was statistically significant. The Shapiro–Wilk test indicated that both pre-test and post-test data were normally distributed ($p > 0.05$), meeting the assumption for parametric testing. The results of the paired t-test revealed a significant difference between pre-test and post-test scores, $t(47) = 10.42$, $p < 0.001$, indicating that students' conceptual understanding improved significantly after using the GeoGebra-based digital worksheet. These findings confirm that the observed improvement is not only practically meaningful but also statistically significant, demonstrating the effectiveness of the developed learning media in enhancing students' conceptual understanding of circle concepts.

Overall, the three aspects of testing-validity, practicality, and effectiveness-show that the developed GeoGebra-based digital worksheet has met the criteria as a learning media that is suitable for use in the classroom. These results support the notion that technology integration, especially the use of GeoGebra in exploratory digital worksheet, can create a more interesting, participatory learning experience, and improve the achievement of student learning outcomes. Thus, this product can be an alternative solution for learning mathematics that is in line with the needs of digital native students in the modern era.

Discussion

The findings of this research indicate that the GeoGebra-based digital worksheet developed for the circle topic is valid, practical, and effective in strengthening students' understanding of fundamental mathematical concepts. These results are grounded in a thoughtful design that responds directly to students' learning needs, particularly the difficulty in grasping abstract concepts such as tangents, radii, and central angles. By integrating GeoGebra into an explorative digital worksheet, students are provided with interactive visualizations that allow them to manipulate mathematical objects dynamically, fostering deeper conceptual understanding. This improvement can be explained through the lens of representational coordination, where students actively connect visual, symbolic, and verbal representations of mathematical concepts. GeoGebra allows students to observe how changes in one representation—such as dragging a point on a circle—simultaneously affect algebraic expressions and geometric properties. This coordination of representations supports conceptual

restructuring, enabling students to move beyond procedural reasoning toward a deeper understanding of geometric relationships. This approach also supports engagement, which is often lacking in traditional instruction (Hayati & Ulya, 2022; Muchlis et al., 2021). This finding is consistent with constructivist perspectives suggesting that learners construct mathematical meaning through interactive manipulation of representations (Duval, 2006).

In addition, embedding this media within a problem-based learning (PBL) framework enhances students' participation by situating learning in meaningful real-world contexts.

These findings align with and reinforce existing literature on the integration of dynamic mathematics software. For instance, Radović et al. (2020) found that GeoGebra enhances students' conceptual understanding and retention, especially in geometry. Similarly, Vonkova and Hrabak (2015) highlighted the ability of ICT-assisted instruction to improve students' reasoning and problem-solving skills. Further supporting these results, Setiawan et al. (2022) and Amalia et al. (2024) demonstrated that the combination of contextual or scientific approaches with GeoGebra increases students' creative thinking and engagement. These learning gains indicate that students were able to internalize key circle concepts through guided exploration rather than relying on memorization or formula-based reasoning.

In addition, this research supports the conclusions of No et al, who demonstrated that GeoGebra-assisted worksheets are particularly effective in helping students understand abstract mathematical topics through guided exploration (Norberg, 2023). Research by Alyani et al. (2022) also found that students' critical thinking is enhanced when they learn mathematics using GeoGebra, especially when supported by well-structured learning environments. Furthermore, Putra et al. (2025) revealed that teachers participating in professional development involving GeoGebra reported increased confidence and competence in designing interactive instruction. The inclusion of user experience analysis in this study adds an affective layer of evaluation, showing that all UEQ scales—such as attractiveness, stimulation, novelty, and clarity—were rated “Excellent”. Muazarah et al. (2025), emphasizing that the media positively impacts not only learning outcomes but also learner motivation and enjoyment.

This study extends existing GeoGebra research by moving beyond isolated evaluations of software use or instructional models toward a more integrated pedagogical framework. While prior studies have predominantly examined GeoGebra's impact on students' achievement or engagement in discrete instructional contexts, the present study embeds GeoGebra within a structured explorative digital worksheet aligned with problem-based learning principles. Furthermore, a bibliometric review by Awaji et al. (2025) identified a growing trend in GeoGebra research but noted the lack of integrated frameworks that address both instructional effectiveness and student experience—precisely what this study offers. Responding to gaps identified in the bibliometric analyses, this study incorporates user experience analysis as an integral component of instructional evaluation, capturing affective and usability dimensions that are often overlooked in GeoGebra research. Previous studies consistently report that GeoGebra enhances students' conceptual understanding and engagement in geometry learning; however, much of this research focuses on general achievement gains or isolated instructional contexts rather than on how structured exploration supports sustained conceptual engagement. In contrast, the present study emphasizes how structured exploration within an digital

worksheet environment supports sustained conceptual engagement, thereby extending earlier findings on GeoGebra's instructional affordances. By integrating technology, pedagogy, and learner experience within a mobile-accessible digital worksheet, this research offers a more holistic model for GeoGebra-supported mathematics instruction that advances both theoretical understanding and practical design of exploratory digital learning environments.

These findings offer significant implications for mathematics education. The GeoGebra-based digital worksheet presented in this study provides a replicable model for developing digital instructional media that are pedagogically robust and technologically engaging. For educators, it offers a way to modernize instruction while maintaining focus on conceptual understanding and problem-solving. For policymakers and curriculum designers, it reinforces the relevance of integrating dynamic mathematics software into the national curriculum as a means to enhance 21st-century skills. Theoretically, this study proposes a set of design principles that underpin a replicable model for developing pedagogically grounded and technologically engaging digital instructional media.

Building on this success, future research may explore the implementation of GeoGebra-based digital worksheet across different mathematical domains, such as trigonometry, algebra, or calculus, to examine their scalability and adaptability. Researchers may also investigate the long-term retention effects or analyze their impact on students with diverse learning styles. Moreover, integrating learning analytics or adaptive features into GeoGebra environments could provide new insights into personalized learning trajectories. Finally, extending this model to different educational levels—including junior high school and higher education—could broaden its impact and offer a deeper understanding of how explorative media can be tailored to diverse learners' needs (Bekene Bedada & Machaba, 2022; Birgin & Uzun Yazıcı, 2021).

Conclusion

The developed explorative GeoGebra-based digital worksheet is valid, practical, and effective. The validity of the product is confirmed through expert evaluations, indicating that the content, construct, and media design meet the established standards. The practicality is demonstrated by positive user feedback obtained from the User Experience Questionnaire (UEQ), suggesting that the GeoGebra-based digital worksheet is user-friendly and easy to navigate. Its effectiveness is evidenced by the improvement in students' understanding of fundamental mathematical concepts, as shown by the comparison between pre-test and post-test results.

Theoretically, this study contributes to mathematics education by demonstrating how explorative learning supported by dynamic visualization can facilitate students' conceptual understanding. The findings reinforce the role of representational coordination, showing that the integration of interactive GeoGebra applets within a structured digital worksheet enables students to actively connect visual, symbolic, and verbal representations. This contribution extends existing instructional design perspectives by illustrating how technology-enhanced exploration can be systematically embedded into digital worksheets to support conceptual restructuring in geometry learning.

From a practical and policy perspective, this study provides implications for curriculum designers and educational policymakers. The GeoGebra-based explorative digital worksheet offers a viable model for integrating dynamic mathematics software into student-centered and technology-enhanced curricula. Incorporating such digital learning tools into curriculum guidelines may support the development of students' conceptual understanding, problem-solving skills, and digital competence in mathematics education.

Nevertheless, several limitations should be acknowledged. It was conducted within a limited scope involving students from a single school, which may affect the generalizability of the findings. Moreover, the implementation duration was relatively short, potentially limiting the observation of long-term learning impacts.

Despite these limitations, the developed GeoGebra-based digital worksheet demonstrates strong potential as a replicable model for designing interactive and explorative digital learning tools in mathematics education. To support replication and adaptation, the detailed structure of the GeoGebra-based digital worksheet, including examples of activities and GeoGebra applet links, is provided in the supplementary file. Future studies are encouraged to implement this model across diverse educational contexts and extended timeframes to further evaluate its scalability, effectiveness, and sustainability in fostering conceptual understanding and problem-solving skills in mathematics.

Acknowledgment

Thank you to Mr Sukendra, Mr Surat, Mr Wiadnyana and Mrs Sunita for contributing as validators to this research. In addition, thanks also to the principal of SMA Negeri 1 Gianyar for accommodating the implementation of this research.

Declarations

- Conflicts of Interest : The authors declare no conflict of interest regarding the publication of this manuscript.
- Generative AI Statement : Generative AI tools, including ChatGPT and Grammarly were utilized in a limited and supportive capacity to assist in preliminary idea development and language refinement. All aspects of the study's conceptualization, analytical processes, interpretation of findings, and synthesis of the literature were conducted independently by the authors and critically verified to ensure academic rigor, accuracy, and integrity.
- Funding Statement : This work received no specific grant from any public, commercial, or not-for-profit funding agency.
- Author Contributions : **Putu Friska Anandita:** Conceptualization, writing - original draft, editing, and visualization; Developing media, Writing - original draft, editing, and visualization; **I Wayan Sudiarsa:** Writing - review & editing, formal analysis, and methodology; **I Gusti Agung Ngurah Trisna Jayantika:** Conceptualization, Validation and supervision.

References

- Alyani, F., Dinda, N., & Putri, S. (2022). Students' mathematical critical thinking using GeoGebra software based on adversity quotient. *Jurnal Pendidikan dan Pengajaran*, 55(3), 562–575.
- Amalia, F. N., Ristiana, M. G., & Afrilianto, M. (2024). Pengembangan LKPD berbasis pendekatan saintifik berbantuan GeoGebra [Development of GeoGebra-assisted student worksheets based on a scientific approach]. *Jurnal Pembelajaran Matematika Inovatif*, 7(4), 611–624. <https://doi.org/10.22460/jpmi.v7i4.21503>
- Awaji, B. M., Khalil, I., & Al-Zahrani, A. A. (2025). A bibliometric study of two decades of GeoGebra research in mathematics education. *Journal of Educational and Social Research*, 15(1), 130–150.
- Beaton, D. E., Bombardier, C., Guillemin, F., & Ferraz, M. B. (2000). Guidelines for the process of cross-cultural adaptation of self-report measures. *Spine*, 25(24), 3186–3191.
- Bekene Bedada, T., & Machaba, M. F. (2022). The effect of GeoGebra on students' abilities to study calculus. *Education Research International*. <https://doi.org/10.1155/2022/4400024>
- Birgin, O., & Uzun Yazıcı, K. (2021). The effect of GeoGebra software-supported mathematics instruction on eighth-grade students' conceptual understanding and retention. *Journal of Computer Assisted Learning*, 37(5), 1431–1445. <https://doi.org/10.1111/jcal.12532>
- Brinkerhoff, M. R., Porter, E., & Anderson, J. (2024). Success in asynchronous courses through increasing students' social presence. *Journal of Higher Education Theory and Practice*, 24(7), 113–121.
- Duval, R. (2006). A cognitive analysis of problems of comprehension in a learning of mathematics. *Educational Studies in Mathematics*, 61, 103–131. <https://doi.org/10.1007/s10649-006-0400-z>
- Foo, C. H., & Ng, E. C. S. (2022). Pembinaan modul berasaskan GeoGebra untuk meningkatkan kefahaman murid bagi topik bulatan tingkatan dua [Development of a GeoGebra-based module to enhance students' understanding of form two circles. *Journal of Science and Mathematics Letters*, 10(2), 67–73.
- Hayati, Z., & Ulya, K. (2022). Developing students' mathematical understanding using GeoGebra software. *Jurnal Ilmiah Didaktika*, 22(1), 134–146.
- Komariah, K., Rufii, & Rohman, U. (2024). Development of Google Sites-based teaching materials for mathematics subjects. *Eduvest: Journal of Universal Studies*, 4(11), 10438–10455.
- Leacock, T. L., & Nesbit, J. C. (2007). A Framework for Evaluating the Quality of Multimedia Learning Resources. *Educational Technology & Society*, 10(2), 44–59.
- Maison, H., M, K., A, D., Sandra, R. O., Yolviansyah, F., & Iqbal, M. (2022). Misconception materials tool: Web-based assessment of buoyancy. *Journal of Education Technology*, 6(2), 237–246.
- Martatiyana, D. R., Usman, H., & Lestari, H. D. (2023). Application of the ADDIE model in designing learning media. *Journal of Education & Teaching Primary School Teachers*, 6(1), 105–109.
- Mollakuq, V., Rexhepi, S., & Iseni, E. (2021). Incorporating GeoGebra into teaching circle properties at high school level and its comparison with the classical method of teaching. *International Electronic Journal of Mathematics Education*, 16(1), 1–11.
- Muazarah, S. F., Qohar, A., & Sisworo, S. (2025). Development of learning media assisted by GeoGebra and interactive games on cone material. *Unnes Journal of Mathematics Education*, 14(1), 12–21.

- Muchlis, E. E., Priatna, N., & Dahlan, J. A. (2021). Development of a web-based worksheet with a project-based learning model assisted by GeoGebra. *Jurnal Riset Pendidikan Matematika*, 8(1), 46–60.
- Nesta, R. A., Susanta, A., & Haji, S. (2024). Pengembangan LKPD berbasis GeoGebra dan problem based learning untuk meningkatkan pemahaman konsep matematika peserta didik kelas VII SMPN 17 [Development of GeoGebra-based student worksheets and problem-based learning to improve students. *mathematical conceptual understanding*]. *Lebesgue: Jurnal Ilmiah Pendidikan Matematika, Matematika dan Statistika*, 5(3), 1423–1440.
- Nopitria, M., Susanta, A., & Koto, I. (2022). Pengembangan lembar kerja peserta didik (LKPD) berbasis discovery learning berbantuan GeoGebra pada geometri ruang kelas IV SD [Development of GeoGebra-assisted discovery learning worksheets for fourth-grade elementary school geometry. *Jurnal KAPEDAS: Kajian Pendidikan Dasar*, 1(2), 206–214.
- Norberg, M. (2023). Young students' meaning-making when working with mathematics textbooks—A multimodal study focusing on the designed and the discovered. *Research in Mathematics Education*, 25(2), 194–218. <https://doi.org/10.1080/14794802.2022.2045624>
- Ntshangase, S. C., Ndlovu, M., & Oladele, J. I. (2022). The effect of dynamic geometry software use for teaching and learning Grade 11 circle geometry. *African Journal of Teacher Education and Development*, 3(1), 1–11.
- Nugroho, H., Desti, D., & Kismiantini. (2025, 2025). *Integrating GeoGebra into project-based learning to improve students' problem-solving skills on circle topics* Proceedings of ISMEI 2024, <https://doi.org/10.2991/978-2-38476-434-1>
<http://dx.doi.org/10.2991/978-2-38476-434-1>
- Pertiwi, S., Mailizar, M., & Elizar, E. (2023). Validity of guided inquiry-based learning instruments with GeoGebra applets for teaching mathematics. *Kalamatika: Jurnal Pendidikan Matematika*, 8(1), 63–76.
- Puspita, R. P., Susanta, A., & Koto, I. (2022). Pengembangan lembar kerja peserta didik menggunakan model problem based learning berbantuan GeoGebra pada geometri kelas V SD [Development of GeoGebra-assisted problem-based learning worksheets for fifth-grade elementary school geometry. *Jurnal KAPEDAS: Kajian Pendidikan Dasar*, 1(2), 215–223.
- Putra, Z. H., Lestari, S., & Siller, H. S. (2025). Elementary school teachers' experiences in engaging with digital technology in teacher professional development: The case of GeoGebra. *Journal on Mathematics Education*, 16(1), 299–320.
- Radović, S., Radojčić, M., Veljković, K., & Marić, M. (2020). Examining the effects of GeoGebra applets on mathematics learning using interactive mathematics textbooks. *Interactive Learning Environments*, 28(1), 32–49. <https://doi.org/10.1080/10494820.2018.1512001>
- Salifu, A. S. (2020). Effect of GeoGebra on pre-service teachers' achievement and perception of circle theorems. *International Journal of Innovative Science and Research Technology*, 5(8), 1–7.
- Sarah, M., & Batiibwe, K. (2024). Application of interactive software in classrooms: A case of GeoGebra in learning geometry in secondary schools in Uganda. *Discover Education*, 3, 179. <https://doi.org/10.1007/s44217-024-00291-8>
- Schrepp, M., Hinderks, A., & Thomaschewski, J. (2017). Construction of a benchmark for the User Experience Questionnaire (UEQ). *International Journal of Interactive Multimedia and Artificial Intelligence*, 4(4), 40–44. <https://doi.org/10.9781/ijimai.2017.445>

- Setiawan, H., Fitriani, N., & Sabandar, J. (2022). Pengembangan bahan ajar matematika SMP berbantuan software GeoGebra dengan pendekatan kontekstual untuk meningkatkan pemikiran kreatif matematis [Development of GeoGebra-assisted junior high school mathematics teaching materials using a contextual approach to improve mathematical creative thinking]. *MaPan: Jurnal Matematika dan Pembelajaran*, 10(2), 299–311.
- Sugandi, A. I., Bernard, M., & Linda, L. (2020). Efektivitas pembelajaran daring berbasis masalah berbantuan GeoGebra terhadap kemampuan penalaran matematis di era Covid-19 [The effectiveness of GeoGebra-assisted problem-based online learning on mathematical reasoning abilities during the Covid-19 era. *AKSIOMA: Jurnal Program Studi Pendidikan Matematika*, 9(4), 993–1004.
- Suryati, K., & Adnyana, I. G. (2022). Pendidikan matematika realistik berbantuan media GeoGebra untuk meningkatkan hasil belajar geometri ditinjau dari kemampuan spasial [Realistic mathematics education assisted by GeoGebra media to improve geometry learning outcomes in terms of spatial ability. *PENDIPA Journal of Science Education*, 6(3), 658–663.
- Tay, M. K., & Wonkyi, T. M. (2018). Effect of using GeoGebra on senior high school students' performance in circle theorems. *African Journal of Educational Studies in Mathematics and Sciences*, 14, 1–17.
- Vonkova, H., & Hrabak, J. (2015). The (in)comparability of ICT knowledge and skill self-assessments among upper secondary school students: The use of the anchoring vignette method. *Computers & Education*, 85, 191–202. <https://doi.org/10.1016/j.compedu.2015.03.003>
- Widana, I. W., & Laksitasari, B. D. (2023). Improving students' learning outcomes on circle equation material using GeoGebra software. *Indonesian Journal of Educational Development*, 4(1), 32–39.
- Ziatdinov, R., & Valles, J. R. J. (2022). Synthesis of modeling, visualization, and programming in GeoGebra as an effective approach for teaching and learning STEM topics. *Mathematics*, 10(3), 398.