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Enhancing vocational students' mathematical problem-solving through contextual teaching factory learning

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Abstract

The discrepancy between classroom mathematics and the demands of the Business and Industrial World often results in less-than-ideal problem-solving ability for vocational high school students. To meet this challenge, learning methods that link mathematics to real-world work contexts are needed. The purpose of this study is to investigate the effectiveness of integrating the Teaching Factory (TeFa) model with a contextual learning approach in teaching relations and functions. A quantitative quasi-experimental method with the pre-test–post-test control group design was employed. The sample was 46 eleventh-grade pharmacy students at Muhammadiyah Al Manaar Vocational High School, Pemalang, selected through purposive sampling and divided equally into experimental and control groups. Data were collected using a validated mathematical problem-solving instrument and analysed using an independent samples t-test. Post-test performances differed significantly between the two groups ($p < 0.05$). The experimental group gained a moderate improvement with an N-gain score of 0.42. Moreover, the Cohen's d value of 0.81 indicated a large effect of the TeFa model compared with conventional instruction. These findings indicate that the integration of TeFa and contextual learning is effective to improve students' mathematical problem-solving abilities while reinforcing the relevance of mathematics learning to real industrial workplace contexts in vocational education.

Keywords: mathematics instruction; quasi-experimental design; vocational education

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Introduction

Vocational High Schools are designed to develop a skilled, competitive workforce capable of competing in the era of globalization. Besides simply reducing unemployment, vocational high schools aim to produce reliable human resources whose competencies must align with the ever-evolving demands of the business and industrial world (Hadam et al., 2017). This alignment requires a specific and industry-based learning framework that ensures the quality of graduates directly reflects professional standards (Ouanhlee, 2023; Sumantri et al., 2017).

The main challenge in modern vocational education lies in synthesizing theoretical knowledge with practical learning experiences that meet the needs of the industry (Arsanti et al. 2021; Peng et al., 2023). Within this framework, mathematical problem-solving skills have emerged as a critical competency, enabling students to draw logical conclusions and express opinions (Rahman, 2019; Siswanto, 2024). Strengthening these skills is crucial to minimizing the gap or discontinuity between the knowledge gained in the classroom and the high demands of the industrial environment.

However, empirical data indicate a significant pedagogical crisis, with PISA 2022 results showing a sharp decline in Indonesian students' problem-solving abilities, with only 18% meeting the minimum mathematics benchmark (OECD, 2020). This quantitative decline is reflected qualitatively in the vocational environment, particularly in the Pharmacy program in Pemalang Regency. Initial field research and interviews with education practitioners revealed a systematic lack of student engagement and motivation, which correlated with low math proficiency. This phenomenon reflects broader academic findings by Jatisunda et al. (2024) and Nurhayati (2019) who argue that mathematical problem-solving abilities in vocational high schools are still underdeveloped.

Further analysis through student questionnaires in the same region highlights the perception of mathematics as an abstract and burdensome subject. This disconnection often leads to passive learning, where students observe without participating (Hanh, 2020; Utomo et al., 2024). This passivity creates a stagnant learning cycle, and low motivation reduces the effectiveness of material delivery, which ultimately further erodes students' ability to solve complex problems in their field of specialization (Muhayyang et al., 2023).

The long-term impact of this passive activity is the inability to apply the fundamentals of theoretical mathematics to diverse industrial challenges (Gupta & Zheng, 2020; Sinaga et al., 2023). To bridge this gap, educators or teachers must turn to situational learning theory, which states that knowledge is best internalized when embedded in an authentic context (Bakker, 2014; Collins et al., 2018; Norainna, 2018). By simulating real-world work environments, vocational schools can transform abstract concepts into functional, professional learning tools.

In response to this need, the integration of adaptive learning offers a flexible pedagogical solution. By giving students control over their learning time, location, and content, this approach fosters the self-directed skills needed for the modern workplace (Bergamin et al., 2012; Fraihat et al., 2022). In the context of Pharmacy, for example, adaptive learning can involve role-playing scenarios where mathematics is applied to dosage calculations or inventory management, through strengthening the connection between theory and professional practice (Fantinelli et al., 2024; Gallagher et al., 2022).

This study proposes a dual-strategy approach that integrates contextual learning with the Teaching Factory (TeFA) model. In the contextual approach, mathematics is demystified by framing it as a tool for trade (Dalby & Noyes, 2015; Duignan et al., 2018; Hall, 2014), while the Tefa model provides a concrete industrial ecosystem within the school. In this integrated model, mathematics is not a separate subject but a cognitive infrastructure for production used for cost analysis, measurement, and process optimization, ensuring students build knowledge through active job simulations (Hadam et al., 2017; Li et al., 2025).

Recent research strengthens this synergy, as studies by Triyono et al. (2025) and Fatimah and Isyanto (2022) show that integrating mathematics with vocational tasks allows students to bridge the gap between abstract principles and practical application. Empirical evidence from Amiyani and Widjajanti (2018) further confirms that such vocational-based instruction significantly improves learning outcomes. Collectively, these findings suggest that real-world applications catalyze engagement, providing the analytical rigor necessary for technical mastery (Chandra et al., 2024; Zega t al., 2024).

Based on this study and its findings, the researcher applied a TeFa-based contextual framework that promotes independent discovery of mathematical concepts aligned with industry standards. Previous research has shown that contextual mathematics improves long-term memory retention (Darmawan et al., 2019; Kaya & Kesan, 2023; Samo et al., 2017) and enhances student agency (Mahmuti et al., 2025). Similarly, the TeFa model has been proven to increase student motivation and improve summative evaluation results by providing an authentic learning environment (Perdana, 2018; Prianto et al., 2025). Although the effectiveness of TeFa in improving technical skills has been proven (Wahjusaputri & Bunyamin, 2022). Its application largely still focuses on managerial aspects, while the contextualization of mathematics is often limited to material used only in the classroom (Mar et al., 2023). This creates a pedagogical gap where students may succeed in structured problems but struggle with the unstructured complexity of real-world production (Hidayati & Wagiran, 2020).

The novelty of this research lies in filling this specific gap by transforming mathematics from a supporting subject into a vocational component, making it a functional and integrated part of the TeFa production ecosystem. Unlike previous models that treated mathematics as a prerequisite for practice, this research examines an integrated workflow where mathematics learning is embedded directly within real-time industrial activities at school, fostering a holistic learning environment that reflects the actual business and Industry World.

Methods

This study employs a quantitative approach using a pretest-posttest control group design. This experimental framework was chosen to measure the impact of integrating the Contextual Approach and the Teaching Factory (TeFa) model on the mathematical problem-solving skills of vocational school students. This design plays a crucial role in evaluating improvements within each group, while also allowing the researcher to control for baseline variables through pre-intervention comparisons. Through this procedure, the researcher can minimize bias and ensure that any statistical differences in outcomes between the experimental and control groups directly reflect the effectiveness of the tested learning model. This study was conducted in June 2025 during the 2025/2026 academic year at a Pharmacy Vocational High School (SMK) in Pemalang Regency. Using purposive sampling at SMK Al Manaar Muhammadiyah Pemalang, two intact classes were selected to minimize disruption to the learning process and ensure participants possessed the requisite foundational knowledge for pharmacy-specific mathematics. Consequently, Class XII Pharmacy 1 was designated as the experimental group, while Class XII Pharmacy 2 served as the control group. Grade 12 students were specifically chosen to align with the advanced competencies and vocational integration required by the Teaching Factory (TeFa) model.

Due to the quasi-experimental design with non-random assignment, the researchers acknowledge inherent limitations regarding internal validity and potential selection bias. To mitigate these concerns and ensure statistical comparability, baseline equivalence was rigorously assessed using independent t-tests on pretest scores before the intervention

Table 1. The independent t-test on the pretest scores

Group	N	Mean	SD	t-value	p-value (sig. 2-tailed)
Experimental	23	59.17	13.016	0.278	0.782
Control	23	58.09	13.517		

The independent t-test conducted on the pretest scores yielded a t-value of 0.278 with a p-value (sig. 2-tailed) of 0.782. Since the p-value exceeds the alpha level of 0.05 ($p > 0.05$), the null hypothesis is accepted, indicating no statistically significant difference between the experimental and control groups' initial abilities. These results demonstrate baseline equivalence between the two groups, thereby mitigating the internal validity threat of selection bias previously identified.

The research instrument consists of five essay questions aligned with Polya's problem-solving indicators. While the pre-test utilized previous mathematics test results, the post-test items underwent rigorous validation. Specifically, the post-test was evaluated for validity, reliability, discriminating power, and difficulty using the Rasch model to ensure its suitability. This model was selected to produce an equal-interval measurement scale, providing accurate data on both participant performance and item quality (Boone et al., 2014; Linacre, 2019; Muhtarom, 2024; Sumintono, 2018). Key analyses included item fit, reliability, and difficulty levels

The mathematical problems in this study are vocational-based, employing George Polya's problem-solving framework as cited by Sepriyanti et al. (2020). This systematic approach assists students in navigating complex tasks; by applying these four stages, learners are guided from the initial understanding of a problem to the conclusion (Suarsana et al., 2019). Furthermore, Polya's methodology offers a practical and organized structure that facilitates a more intuitive learning process (Arifin & Aprisal, 2020; Obiano & Parangat, 2023). The indicators for the problem-solving test instrument are as follows:

Table 2. The indicators for the problem-solving test instrument

No	Indicator	Assessment Rubric
1	Understanding the Problem	Writing down the known and asked information correctly and completely, and elaborating the answer according to the question.
2	Devising a plan	Preparation of a resolution plan accurately and systematically, and connecting with information from the given questions accordingly.
3	Carrying out the plan	Perform calculations accurately and systematically, connecting them with the information provided
4	Look Back	Draw a conclusion based on the conceptual framework used for verification

The data collection for this study was conducted through a structured testing procedure using the vocational-based mathematical instrument previously defined in Table 2. The process was divided into several key stages:

1. Data Collection Procedure
   The primary data were gathered through a pre-test and post-test design to measure the students' initial and final problem-solving abilities. During the administration of the test, students were instructed to solve vocational-context problems by explicitly writing down the steps according to Polya's framework: (1) understanding the problem, (2) devising a plan, (3) carrying out the plan, and (4) looking back. This structured response format ensured that the qualitative process of problem-solving could be quantified accurately. To maintain the authenticity and reliability of the data, the testing process was supervised directly by the researchers and vocational instructors.
2. Data Analysis Techniques
   The analysis of the collected data employed a combination of descriptive and modern psychometric approaches: (a) Scoring Rubric: students' written responses were evaluated using a holistic scoring scale (0–4) based on the indicators in Table 2. This allowed for a detailed assessment of which specific problem-solving stage presented the most significant challenge for vocational students. (b) Rasch Model Analysis: to ensure the instrument's quality and the objectivity of the measurement, the raw scores were processed using the Rasch Model (Many-Facet Rasch Measurement) via Winsteps software. This analysis transformed the ordinal raw data into interval logit scales, providing a more precise estimation of both item difficulty and person ability. This step also allowed for the detection of misfit responses and ensured the unidimensionality of the test instrument. (c) Effectiveness and Improvement: the improvement in students' problem-solving abilities was calculated using the N-Gain score. The N-Gain results were then categorized into high $(g > 0.7)$, medium $(0.3 \leq g \leq 0.7)$ and low $(g < 0.3)$ criteria. And (d) Statistical Significance: to determine if there was a statistically significant difference between the learning outcomes, an Independent Sample T-test was performed. Additionally, Cohen's d effect size was calculated to measure the magnitude of the instructional design's impact on the students' problem-solving proficiency.

Results

This integrated approach links mathematics directly to real-world professional challenges. By shifting from abstract theory to practical application, students better grasp the subject's relevance, boosting motivation and understanding. Central to this is the Teaching Factory (TeFa), where learning transitions into operational practice. Here, students apply contextual knowledge to industrial simulations—covering product planning and cost estimation—positioning mathematics as a vital tool for solving complex production problems.

The assessment instrument, comprising five vocational mathematics essay questions, was validated by four experts: two mathematics lecturers and two vocational teachers. These experts evaluated the instrument based on content coverage, construction, problem-solving indicators, and linguistic quality. Following iterative refinements based on their feedback, all items were deemed suitable for use. The validation results are summarized in the table below:

Table 3. Expert validation results

Item	Validator				Average	Category
	I	II	III	IV		
Item 1	0.85	0.84	0.88	0.90	0.87	Very good
Item 2	0.87	0.86	0.87	0.92	0.88	Very good
Item 3	0.86	0.85	0.88	0.90	0.87	Very good
Item 4	0.90	0.87	0.88	0.93	0.89	Very good
Item 5	0.92	0.87	0.88	0.95	0.91	Very good

The application of the Multifaceted Rasch model in this validation process aimed to minimize subjectivity bias among validators. The results showed that, despite variations in evaluator characteristics—where Validator II demonstrated a higher level of strictness compared to Validator IV, who tended to be more lenient—the Rasch model successfully calibrated the scores, thereby producing an objective estimate of the instrument's ability. This demonstrates that the developed mathematics problem-solving instrument is highly reliable for use as a measurement tool in the Teaching Factory learning context at vocational schools. The following are the validation results using the Rasch Multifaceted Model:

Table 4. The rasch multifaceted model results

Item	Average Score	Category	MFRM Logit	Interpretation
Item 1	0.87	Very Good	0.45	Fit
Item 2	0.88	Very Good	0.52	Fit
Item 3	0.87	Very Good	0.48	Fit
Item 4	0.89	Very Good	0.61	Fit
Item 5	0.91	Very Good	0.78	Strongest Indicator

Based on Table 4, the five items demonstrate high consistency across the four validators' assessments. The results confirm that each item is well-designed and meets established validity standards. This uniform quality suggests that the instrument is robust and reliable throughout. Following expert validation, the instrument was tested on 30 students to evaluate its reliability and item difficulty. The Rasch model was employed to produce an interval scale, providing precise data on participant ability and instrument quality (Staver & Boone, 2014; Linacre, 2019; Muhtarom, 2024; Sumintono, 2018).

Validity

The results of the item validity analysis using the Rasch model are considered very effective because they yield trustworthy validity analysis results (Sari et al., 2016). Here are the results of the content validity analysis using the WinStep program, which yielded output in the form of Misfit Order, as shown in the following table:

Table 5. Item analysis results

No	Item Number	Criteria for Measurement Accuracy			Decision
		Outfit MNSQ	Outfit ZSTD	PT-Meancorr	
1	Number 1	0.96	-0.1	0.34	Invalid
2	Number 2	0.7	-1.2	0.51	Valid
3	Number 3	1.08	0.4	0.43	Valid
4	Number 4	1.23	1.0	0.52	Valid
5	Number 5	0.97	0	0.7	Valid

Based on the Rasch model analysis presented in the table above, item validity was strictly assessed using three criteria: Outfit Mean Square (MNSQ), Outfit Z-Standard (ZSTD), and Point Measure Correlation (Pt Measure Corr). The results indicate that only Item 1 was invalid due to significant statistical inconsistency, failing to contribute consistently to the intended construct. Consequently, this item was removed to ensure that students' mathematical problem-solving abilities are evaluated using a high-quality instrument.

Although removing Item 1 was a very easy task slightly skewing the difficulty distribution the researchers prioritized measurement precision and internal consistency over balance. Retaining invalid items would introduce noise and compromise the accuracy of proficiency scores. To mitigate this imbalance, the remaining four items were recalibrated, confirming they adequately represent a range of cognitive complexity to differentiate student performance within a pharmacy context

Reliability

Reliability is a measure of the consistency of results obtained from a test (Langenfeld et al., 2020). A high reliability value indicates that the items in the test provide stable and dependable results when the same conditions are repeated (Hasnida & Ghazali, 2016). From the overall analysis of the instruments used after item number 1, which was invalid in this study, detailed in the following Summary Statistics in Table 6 and Table 7.

Table 6. Summary Statistic: Measurement of persons

Mean	Standard Deviation	Max	Min	Separation	Person Reliability
11.9	2.1	13.0	3.0	0.67	0.31

Table 7. Summary Statistic: Measurement of item responses

Mean	Standard of Deviation	Max	Min	Separation	Item Reliability
71.4	8.2	81.0	57.0	5.75	0.97

The summary output statistics from Winsteps provide dual measurements: a summary of person (student) measurements and a summary of item responses across all validated items. Based on Rasch analysis, the item reliability score reached 0.97, indicating a very high level of consistency in the item difficulty hierarchy. Conversely, the person reliability score was recorded at 0.31, which is Pharmacy below the conventional threshold of 0.67. This low reliability is acknowledged as a limitation of the study, potentially caused by a narrow range of student abilities or the limited number of items used. Nevertheless, these data were used descriptively to provide an initial overview of the distribution of student abilities during the intervention.

To further evaluate the instrument's performance, the difficulty level of each item was analyzed through an Item Measurement table. In Rasch modeling, the item difficulty level is determined by the logit value, where a higher logit score indicates a more challenging task. A good assessment instrument ideally shows a proportional and balanced distribution of difficulty levels to effectively differentiate between students' varying proficiency levels (Rusiyah et al., 2020). The logit values for those items, ordered from most to least demanding, provide a clear map of the cognitive requirements imposed on students. The specific difficulty levels analyzed for each item are detailed as follows in Table 8.

Table 8. Item difficulty level

No. Item	Logit value	Measure Logit	Interpretation
2	-0.53	$-0.57 <$ Measure logit ≤ 0.00	Moderate
3	1.95	Measure logit > 0.57	Very Difficult
3	1.95	Measure logit > 0.57	Very Difficult
5	0.70	Measure logit > 0.57	Very Difficult

The results of the difficulty-level analysis of the pharmacy vocational-based mathematics test items can be interpreted using logit values, which reflect each item's relative difficulty level on the test. In the table above, the difficult category group is dominated by 2 questions (50%), 1 question in the medium category (25%), and 1 question in the easy category (25%). Analysis reveals a structural imbalance in the instrument, with difficult items dominating at 50% while easy ones comprise only 25%. This disparity suggests a mismatch between student ability and item difficulty, likely prompting inconsistent answers or guessing. Consequently, the low individual reliability reflects the instrument's limited discriminative power rather than actual behavioral inconsistency. Acknowledging these limitations, the author treats the results as exploratory data rather than a basis for formal statistical conclusions. Nevertheless, the instrument will proceed as a posttest for both groups, supported by strong content validity as confirmed by expert validation.

After conducting the research using the previously analyzed test instrument, the posttest results were then presented descriptively to provide a more transparent overview of the impact of the intervention on the effectiveness of the Teaching Factory Model on problem-solving abilities between the two groups. And here is a summary of the research data obtained, presented in Table 9 below.

Table 9. Summary of pre-test, post-test and effectiveness analysis

Group	N	Pre-test (Mean)	Post-test (Mean)	N-gain Score	Category
Experimental	23	59.17	76.26	0.42	Fairly Effective
Control	23	58.09	67.61	0.23	Ineffective

The analysis results show that the experimental class using the Teaching Factory model achieved an N-gain score of 0.42, which falls into the quite effective category. Conversely, the control class using conventional learning only reached a score of 0.23, which is categorized as ineffective. Additionally, to measure the impact of this intervention, Cohen's d was calculated. The result was 0.81, which, according to Cohen's standards, falls into the Large Effect category. These results confirm that although individual measurements have limitations in reliability, overall, the Teaching Factory model intervention has a significant and practically real impact on improving students' abilities.

To strengthen the above exploratory findings, this study triangulated by analyzing written reflections from several students. This qualitative data aims to understand changes in student learning behavior that were not captured by test scores. And to strengthen the exploratory quantitative findings, triangulation was conducted through the analysis of student reflection sheets. The activities provided included simulating pharmaceutical interactions using relationship mapping and identifying mathematical variables in drug dosage calculations. This qualitative data provides strong evidence regarding the cognitive processes experienced by the students.

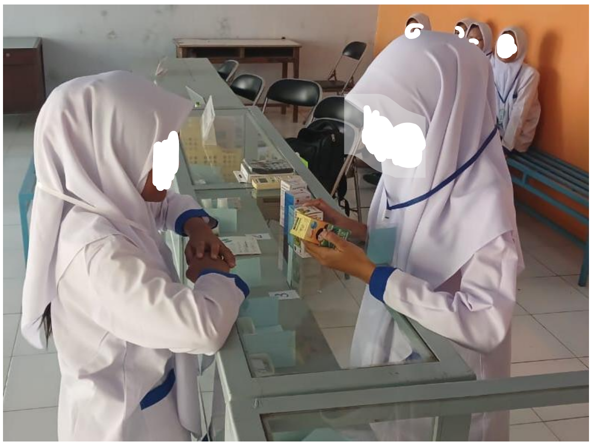

Figure 1. Students simulating pharmacist–client interaction using relation mapping

As illustrated in Figure 1, students participated in a Drug Information Service simulation. Prior to this simulation, students completed a worksheet designed to internalize the concept of mathematical relationships, where drug names served as the domain and drug indications as the codomain. During the activity, students were observed applying this relationship by explaining the function of the medication to customers in a simulated pharmacy environment. Reflective data shows that this relational mapping significantly improved students' clinical awareness. One student provided a conceptual reflection on this process:

> “The systematic mapping of the relationship between drug names and their dosages during patient simulation increased my awareness. This activity allowed me to identify illogical dosage levels exceeding safety limits, ensuring safety in prescribing.”

Figure 2. Students identify mathematical variables in drug dosage calculations.

In Figure 2, the simulation focused on the precision required for pediatric dosage calculations. Students acted as pharmacy technicians processing doctor-provided prescriptions. This activity required the application of the following mathematical function:

$$\text{Child's dose} = \left(\frac{\text{Child's age}}{\text{Child's age}+12}\right) \times \text{Adult dose} \quad (1)$$

In this context, the child's age (n) represents the pre-image variable (independent variable), while the resulting child's dose (D) represents the image variable (dependent variable). By expressing these clinical requirements as mathematical functions where x is the age and $f(x)$ is the calculated dose—students were able to perceive the vital role of mathematical accuracy in patient safety. Student reflections confirmed a cognitive shift from rote arithmetic to functional understanding:

> "I recognized that dosage calculation is a direct application of mathematical variables. By analyzing the relationship between medication concentration and patient requirements, I transitioned from merely performing calculations to understanding how mathematical functions act as a critical safeguard for patient health."

Next are the observation results showing students' ability to solve problems with contextual mathematics learning integrated with the teaching factory model, which are as follows in Table 10.

Table 10. Student observation results

Component	Specific Problem-Solving Activity	Percentage of success	Impact on student ability
Relational logic	Students map the relationship between drug name (domain) and dosage (codomain) in service simulation.	85%	Students are able to independently setect dosage errors through the visualization of mathematical relationships, rather than simply memorizing.
Variable identification	Extract quantitative variables (volume, substance concentration, maximum dose).	87%	Students are able to formulate linear functions or mathematical equations to solve drug dosage calculation problems.
Contextual Computing	Calculating drug usage duration and dosage adjustments based on patient weight/age variables.	83%	Transformation from theoretical mathematics to clinical application; improving calculation accuracy in industrial work stress situations.
Evaluation (Checking)	e-verifying the results of dosage calculations before administering medication to the client.	92%	Forming a Zero Error Mindset; students realize that mathematical errors in pharmacy are potentially fatal to patients.

Table 10 shows how integrating mathematics into the Teaching Factory ecosystem significantly strengthens students' problem-solving abilities. With a Cohen's d value of 0.81 (large effect), confirmed through real-world observations where students no longer see mathematics as a separate subject but rather as an essential tool in pharmaceutical services. Mapping the relationship between drug names and dosages helps students build a strong cognitive structure to prevent medication errors. Additionally, students' ability to identify mathematical variables in drug dosage calculations showed a significantly higher improvement in solving mathematical problems contextually compared to conventional methods. This observational data proves that despite limitations in the reliability of individual tests, the impact of the intervention on students' critical thinking processes and work behavior is highly significant and clearly visible in the field.

Based on the results above, it was found that contextual mathematics learning integrated with the Teaching Factory model is highly recommended for mathematics education in vocational high schools (SMK). The results obtained from effective learning activities support students' ability to develop problem-solving skills. The interview results also indicate that this design shows an increase in student engagement and learning motivation, and substantially affects students' mathematical problem-solving abilities, as well as their critical and creative thinking skills. Then, the advice given was that its implementation requires more mature preparation and adequate resource availability, provides a meaningful learning experience, and makes it easier for teachers to facilitate learning.

Discussion

The integration of contextual learning within the Teaching Factory (TeFa) framework has fundamentally reshaped students' perceptions of mathematics, transitioning from viewing it as a collection of abstract formulas to a vital professional tool. With an N-gain of 0.42 (medium effectiveness) and a Cohen's d effect size of 0.81 (large effect), this model demonstrates a robust capacity to bridge the gap between classroom theory and industrial application. These findings align with the Constructivist Learning Theory, suggesting that mathematical knowledge becomes more meaningful and persistent when constructed within authentic, work-resembling environments (Bakker, 2014; Norainna, 2018). In the pharmacy context, this authenticity acts as a boundary object that allows students to negotiate mathematical meanings between school and workplace practices.

The Rasch analysis provided a nuanced perspective on the instrument's performance. While the person reliability was low (0.31), the high item reliability (0.97) confirms that the difficulty hierarchy of the tasks is stable and replicable (Linacre, 2019). The discrepancy in person reliability indicates that the items were significantly challenging for the respondents' average ability, rather than suggesting inconsistent student behavior. This person-item mismatch is common in specialized vocational tasks where students encounter high-level cognitive demands for the first time. Retaining the instrument based on expert validation is methodologically sound, as it ensures that the cognitive standards remain aligned with the rigorous requirements of the pharmaceutical industry, where precision is non-negotiable (Ansel, 2010).

A pivotal finding is the students' enhanced ability to transform real-world variables into mathematical models. In the Drug Information Service simulation, students successfully converted age variables (n) into precise dosage formulas. This transition from word problems to mathematical modeling signifies a shift toward higher-order thinking skills (HOTS). According to Frejd and Arleback (2024) mathematical modeling in vocational education is not merely about calculation but about functional thinking the ability to see relationships between variables in a professional system. The 92% success rate in the "Looking Back" stage of Polya's model further confirms that students did not just solve the problems but verified their results against the reality of patient safety.

Furthermore, the fear of error in pharmaceutical calculations served as a powerful intrinsic motivator. This finding resonates with the concept of Professional Accountability in vocational mathematics, where the stakes of the real world—such as patient health—drive meticulousness (Geist et al., 2018). Triangulation with qualitative data suggests that problem-solving skills develop most rapidly when students face "ill-defined" industrial challenges that require both technical proficiency and logical reasoning.

In conclusion, aligning the mathematics curriculum with vocational competencies through TeFa integration has a profound impact on student engagement. While the implementation requires significant resource preparation and interdisciplinary collaboration between math and vocational teachers, the output—students who possess both technical skills and strong mathematical logic—is essential for the modern workforce. As an implication, this model is highly recommended for vocational institutions aiming to produce graduates capable of making informed, data-driven decisions in high-stakes professional environments (Bakker, 2014).

Conclusion

This study concludes that the implementation of the Teaching Factory (TeFa) model integrated with a contextual approach has a positive impact on improving students' mathematical problem-solving abilities in the sample of Pharmacy Vocational High Schools studied. This finding theoretically contributes to the theory of situated cognition by demonstrating that transforming abstract mathematical concepts into real-world applications can strengthen students' cognitive structures in understanding the functional relevance of mathematics. Although the research instrument has limitations in individual reliability due to the high difficulty level of the questions, the significant comparison of N-gain improvements indicates that this model is effective in increasing technical accuracy while fostering a mindset of meticulousness (zero error mindset) that is crucial in the healthcare field.

However, it should be noted that these findings are limited to the scope of the pharmacy vocational program at the current research site, so generalizing the results should be done with caution in different vocational contexts. As a contribution to the development of vocational education science, synergy between the mathematics curriculum and industrial practice in schools is highly recommended to create a more meaningful learning experience. For future research, it is recommended to conduct multi-site studies to validate the effectiveness of this model across various vocational fields, as well as longitudinal studies to observe students' problem-solving skills retention after they fully enter the workforce.

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Declarations

Conflicts of Interest	:	The authors state that there are no conflicts in the publication of this manuscript. Additionally, the authors have also resolved ethical issues, including plagiarism, violations, data fabrication and/or falsification, duplicate publication and/or submission, and redundancy.
Generative AI Statement	:	Generative AI tools, such as Grammarly and Microsoft Copilot, were employed solely for language editing and minor phrasing

enhancements. All conceptualization, analysis, and scholarly content were independently developed and verified by the authors.

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