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Designing a learning trajectory for cube nets using the *Engklek* game: An educational design research study within realistic mathematics education

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Abstract

The tendency to teach geometry mechanically highlights the need for mathematics instruction grounded in students' daily contexts. This study designs a Learning Trajectory (LT) for cube nets using the engklek game—a traditional Indonesian hopping game—as a realistic context, framed within Realistic Mathematics Education (RME), and implemented through educational design research (validation study type). Conducted at one school in Kudus, the study involved 21 Grade 5 students across three phases: preparing for the experiment, designing the experiment (pilot and teaching experiment), and retrospective analysis. The resulting LT comprises four sequenced activities aligned with RME's modeling levels—situational, model-of, model-for, and formal—through which students progressed from informal understandings (e.g., engklek kapal terbang as a cube net) to formal knowledge: recognizing eleven distinct cube nets organized into four pattern categories (1-4-1, 2-3-1, 2-2-2, and 3-3). This LT contributes to the expansion of instructional design in mathematics and provides teachers with a culturally responsive, game-based pedagogical tool that bridges students' informal experiences with formal geometric concepts.

Keywords: cube nets; design research; *Engklek* game; learning trajectory; realistic mathematics education

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Introduction

Geometry is one of the most important topics in mathematics, so it starts to be taught in elementary school. The importance of studying geometry is in line with the fact that geometry has important connections and roles in various contexts of everyday life. Geometry is often seen as just the study of points, lines and shapes, but it actually has a significant role in other fields such as music and architecture (Facciaroni et al., 2023). The concept of spatial geometry, for instance, is evident in architectural composition and design, including the construction of traditional houses and mosques (Chen & Ja'Faruddin, 2021; Fauzi et al., 2022; Moradzadeh & Ebrahimi, 2020; Zuliana et al., 2023). Moreover, geometric principles are embedded in cultural artifacts such as ritual equipment, traditional foods, musical instruments, batik patterns, traditional dances, and traditional games (Fauzi et al., 2025; Putra & Ramdhani, 2025; Rahmi et al., 2025; Salma et al., 2022; Sulistyawati et al., 2025; Zuliana & Nugroho, 2025). This variety of findings shows that geometry is widely utilized by people in everyday life, both consciously and unconsciously. Understanding of geometry and its application plays an important role in various contexts of everyday life (Sunzuma & Maharaj, 2022).

Although geometry has a connection to the context of everyday life and plays an important role in it, like other mathematical topics, it also turns into an abstract topic in the scope of formal education (Alangui, 2017; Riswari et al., 2023; Rizki et al., 2023). This leads to a tendency to teach topics in mathematics mechanically, which is only memorizing various formulas, so that the learning process is not connected to the cultural context and daily experiences of students (Prahmana et al., 2025). Geometry is becoming difficult for students to understand because it is taught in a very abstract way (Sari et al., 2023). Most students have difficulty identifying and classifying geometric objects (Grigaliūnienė et al., 2025). There are claims that many students are unaware of mathematics in everyday life, have low critical thinking in mathematics, and experience mathematical anxiety, so that students tend to avoid learning mathematics and tasks related to it (Altakhyneh, 2020; Amanda et al., 2024; Tarkar et al., 2022). In addition, traditional approaches have not been able to address these challenges, making it difficult for teachers to find the right teaching strategies in geometry (Kamalodeen et al., 2021; Sunzuma & Maharaj, 2019). Some of these claims reveal a real need for a learning approach that can connect abstract concepts in geometry with students' daily experiences and cultural contexts. Utilizing cultural contexts has good potential to change students' perceptions of the mathematics learning process (Fauzi et al., 2023).

Realistic Mathematics Education (RME) is a learning approach that can connect abstract concepts in geometry with students' daily experiences and cultural context. This is in line with the beginning of the development of the RME approach, which is based on Hans Freudenthal's idea that mathematics is a human activity (Van den Heuvel-Panhuizen, 2003). Mathematics is a human activity, namely the activity of finding and solving problems, as well as the activity of organizing a subject matter from real problems, with the aim that it can be more easily understood through an axiomatic approach (Freudenthal, 1971). Freudenthal's view intends to emphasize that mathematics is both an activity and a human product, and intends to integrate and make the rich context of reality a source of learning mathematics (Gravemeijer & Terwel, 2000; Treffers, 1993). This idea means that the RME approach utilizes a real context or one that can be reached by students' imagination as a starting point for learning (Van den Heuvel-Panhuizen & Drijvers, 2020). From the use of this context, students are involved in constructing and discovering their own mathematical knowledge through mathematical modeling from concrete to abstract levels (Freudenthal, 2002). As stated by Gravemeijer (1994), the levels of mathematical modeling in RME include the situational stage, referential or model-of stage, general or model-for stage, and formal mathematics stage. The implication is that RME can bridge between students' experiences and cultural context with formal mathematical knowledge (Gravemeijer & Doorman, 1999).

Engklek game is one of the cultural contexts that is very close to students' daily lives. *Engklek* is a traditional Indonesian game that can be played individually or together by jumping with one foot on a checkered pattern drawn on the ground or on the floor. The pattern drawn usually forms what is known as a *kapal terbang* pattern, consisting of six squares. This pattern is identical to a cube net pattern. In line with findings Zayyadi et al. (2023), which showed that the *engklek* game utilizes several mathematical concepts in it, namely addition and subtraction, numbers, lines and angles, 2-dimensional shapes, and comparison. *Engklek* also contains the concepts of probability, congruence, geometric transformations, sets, permutations, and distances (Afghohani et al., 2024; Anisa et al., 2023).

The richness of mathematical concepts demonstrates the great potential of the *engklek* game to be integrated into mathematics learning. Integrating games into mathematics learning can have a positive impact (Erşen & Ergül, 2022). Using games has a wide-range effect on student achievement, making it important to use them in the teaching process (Karakoç et al., 2022). Using games in mathematics learning can support students to improve their basic and advanced skills in mathematics (DePascale & Ramani, 2025). Using games in mathematics learning can increase student motivation and reduce their mathematical anxiety (Chen et al., 2022). In addition, using games has a moderate to large effect on cognitive, social, emotional, motivational, and engagement outcomes (Alotaibi, 2024).

Several previous studies have proven the use of the *engklek* game as a context for learning mathematics. Supriadi (2019) found that the *engklek* game can optimize the creative thinking ability of mathematics, geometry thinking, and algebra of elementary school students. Sholihah et al. (2022) using the context of *engklek* to develop student worksheets that are effective for teaching the cube and rectangular pyramid. Another study found that the context of the *engklek* game can facilitate elementary school students' understanding of the concept of fractions (Fauzi et al., 2023). Study by Ginting and Rakhmawati (2024) using RME and *engklek* game, which found a significant effect on students' problem-solving abilities. Arisetyawan et al. (2025) found that the context of the *engklek* game with a 2-2-2 pattern could be used to teach the concept of cube nets in mathematics learning at elementary school. However, there is a gap regarding the lack of Learning Trajectory (LT) designs specifically for 3D geometry visualization using traditional games.

This study attempts to address these gap by focusing on designing LT within the framework of the RME approach for cube nets using the *engklek* game. Nuraida and Amam (2019) state that LT has an initial design in the form of a conjecture, namely Hypothetical

Learning Trajectories (HLT). Simon (1995) introduced HLT as a design that encompasses activities, objectives, and conjectures about the learning processes. The HLT design will be validated during the experiment so that the following is obtained Actual Learning Trajectory (ALT) or called Learning Trajectory (LT). LT consists of three parts, namely mathematical objectives, developmental paths through which children progress to achieve those objectives, and a series of instructional activities or tasks that gradually increase in difficulty (Clements & Sarama, 2009). LT can be used to link student-centered learning with an understanding of how students think (Wilson et al., 2015). LT has relevance to support a better understanding of student learning, effective teaching strategies, and guide the design of better standardized lessons (Clements et al., 2023; Ellis et al., 2016).

Thus, this study aims to design a Learning Trajectory (LT) for cube nets using the *engklek* game as a realistic context in learning. The novelty of this study is focus on LT for 3D vizualitation which is integrate RME and traditional games. Because it utilizes the framework of the RME approach, students will be involved in the discovery of concepts and patterns of cube nets through mathematical modeling from concrete to abstract levels, so it is hoped that students will gain a more structured and comprehensive understanding, and gain a meaningful understanding of the relationship between mathematics and traditional games that are close to their daily lives. The resulting LT is also expected to provide teachers with a deep understanding of mathematics learning practices integrated with traditional games.

Methods

This research implements an educational design research with a type of validation study to design a Learning Trajectory (LT) through the RME approach that utilizes the *engklek* game as a realistic context in teaching the concept of cube nets. The validation study type was chosen because it not only emphasizes the development of a theoretical framework for learning interventions, but also emphasizes the validation of the theoretical framework, to create an LT that is proven to improve the quality of education (Bakker, 2018; Van den Akker, 1999). Based on this, this research not only designs the proposed LT but also validates the design and its application for learning the concept of cube nets.

This research was conducted at one school which is located in Kudus, involved 21 Grade 5 students. This research was conducted with ethical considerations. The research location was selected with the permission of the school. In addition, the participants involved in this research volunteered and agreed to participate in the research while maintaining their privacy and anonymity. Researchers act as teacher during the research process, while classroom teacher become partner who help researchers review students' understanding and thought processes during learning activities. The stages of research design in this study follow the stages by Gravemeijer (2006) which consists of three phases, namely preparing for the experiment, design experiment, and retrospective analysis.

The preparing for the experiment phase has the main objective of establishing the HLT that will be elaborated and refined (Gravemeijer, 2006). In this phase, the research team conducted a comprehensive literature review on the topic of cube nets according to the elementary school curriculum, the RME approach, and the *engklek* game. In addition, the research team conducted interviews and field observations at the school to evaluate the mathematics learning process, students' prior knowledge and abilities, as well as the learning environment, which includes facilities and infrastructure that allow to support the learning process. From the comprehensive review, interviews, and observations, the research team designed Hypothetical Learning Trajectories (HLT) for learning cube nets through the RME approach with the context of the *engklek* game. HLT consists of three main components: activities, objectives, and hypothesized learning processes that serve as predictions of how students' thinking and understanding will develop.

The design experiment phase focuses on implementing and refining the HLT that was developed in the preparing for the experiment phase (Gravemeijer, 2006). This phase consisted of two sub-phases: the pilot experiment and the teaching experiment. During the pilot experiment, the Hypothetical Learning Trajectory (HLT) was implemented for a group of 5 students. The pilot experiment aimed to review any revisions or refinements that may be needed to the HLT. Furthermore, the revised or improved results were implemented in a teaching experiment with 21 students. During the teaching experiment, data were collected through trustworthiness strategies with triangulation and various methods, including observation, interviews, field notes, and documentation in the form of photos and video recordings.

The retrospective analysis phase aims to provide a thorough understanding of the implementation of the HLT and its impact (Gravemeijer, 2006). The retrospective analysis stage involves data analysis techniques that are oriented towards a comprehensive, longitudinal, and cyclical approach (Bakker, 2018). This means that the various data that have been collected are systematically and comprehensively analyzed to review the comparison of the implementation of the HLT with the original design proposal, which is done through data coding, reflection, and interpretation (Prahmana & Kusumah, 2016). In this study, the constant comparative method by Glaser and Strauss (1967) is used to code data procedures. The constant comparative method involves researchers continuously sorting the collected data, analyzing, and classifying information (Kolb, 2012). Based on the analysis data process, ALT or LT will be obtained, which has been empirically tested for use in learning and teaching practices.

Results

A comprehensive review of relevant literature and primary school curriculum, as well as the results of observations and interviews in schools, became the basis for designing the Hypothetical Learning Trajectory (HLT) of cube nets in this study. Observations revealed a gap in the utilization of realistic contexts in mathematics teaching, especially in geometry teaching. Further interviews with teachers reinforced this, that teachers focus more on directly teaching abstract concepts such as various formulas in geometry. This gap in the integration of realistic contexts in mathematics teaching causes students to have mathematical knowledge without a strong foundation, because students are not involved in constructing mathematical knowledge through experiences of their cultural and daily contexts. Interviews with students confirm this, that in learning mathematics, they are used to being directed to memorize various formulas and their use in practice problems, without them knowing about the relationship and benefits of these formulas in their daily lives. Further interviews with students revealed that they understood the *engklek* game because they often played it. However, they stated that their mathematics learning in class had never utilized games, especially the *engklek* game as a learning context. Therefore, there is a strong basis to design the iceberg as in Figure 1 as the foundation of the learning process with the RME approach, then developed into HLT in the form of Table 1.

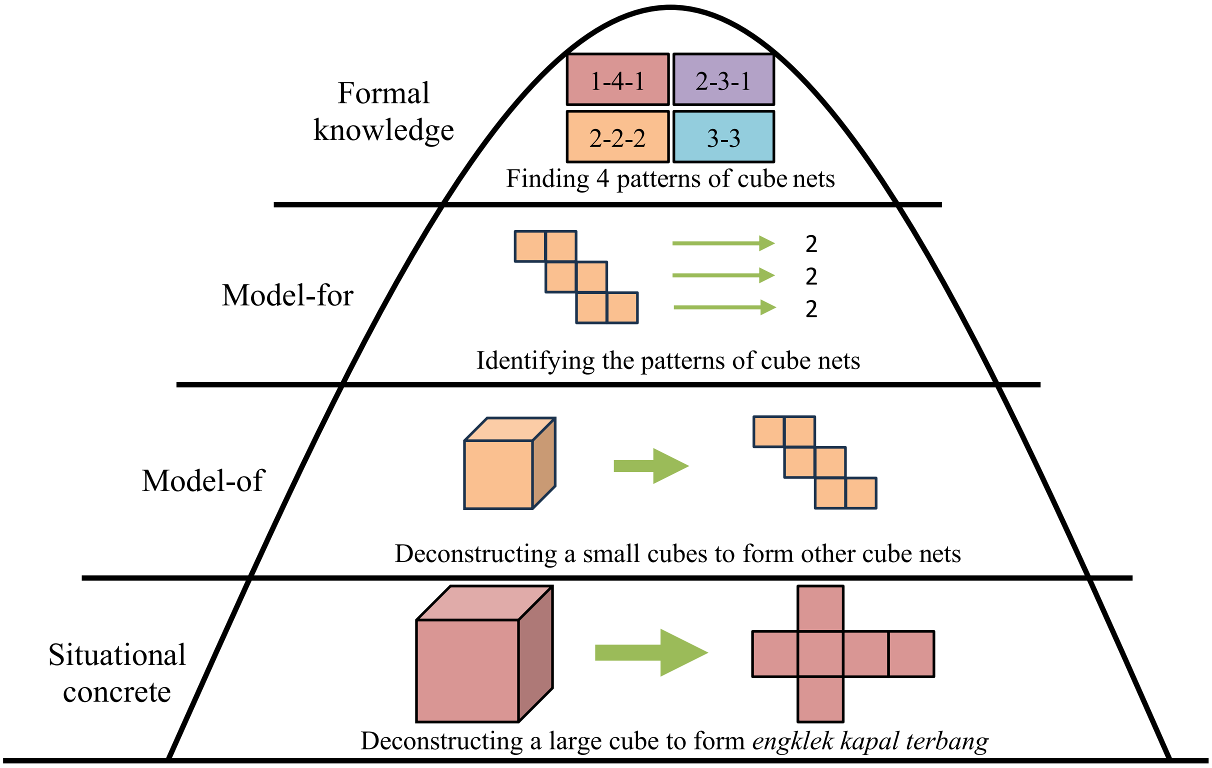


Figure 1. Iceberg of cube nets using the *engklek* game

Table 1. HLT of cube nets using the *engklek* game

RME stage	Activity	Learning objective	Predicted student response	Teacher support strategy
Situational concrete	Deconstructing a large cube to form *engklek kapal terbang.*	Students are able to explain the relationship between the large cube and the *engklek kapal terbang.*	1. Students explain that the *engklek kapal terbang* is the sides of a large cube. 2. Students may find it confusing to explain the relationship between the large cube and the *engklek kapal terbang.*	1. The teacher gives verbal appreciation to students who can explain correctly. 2. The teacher guides the students to find the relationship between the large cube and the *engklek kapal terbang.*

RME stage	Activity	Learning objective	Predicted student response	Teacher support strategy
Model-of	Deconstructing a small cubes to form other cube nets.	Students are able to form a cube nets through deconstructing small cubes.	1. Students deconstruct a small cube into a cube net shape. 2. Students deconstruct the small cube but do not form a cube net.	1. The teacher asks the students to prove that the nets can indeed become cubes again. 2. The teacher guides the students in the process of deconstructing the small cube to form the right cube net.
Model-for	Identifying the patterns of cube nets.	Students are able to identify the net pattern formed from the deconstruction activity of the small cube in the previous stage.	1. Students use the strategy of counting the number of squares in each row that make up the net. 2. Students may be confused about the strategy to identify the pattern of the net.	1. The teacher asks what patterns are formed and asks the students to remember the patterns they have obtained. 2. The teacher guides the students to identify the net pattern correctly
Formal knowledge	Finding 4 patterns of cube nets.	Students are able to find 4 types of cube net patterns through the activity of grouping various forms of cube nets that have been obtained	1. Students group the shapes of the cube nets according to 4 types of patterns, namely 1-4-1, 2-3-1, 2-2-2, and 3-3. 2. Students group the shapes of cube nets, but not according to the type of pattern	1. The teacher asks the students to count the number of nets formed in each type of pattern. 2. The teacher guides the students to remember the cube net patterns that have been obtained and group them with the same pattern

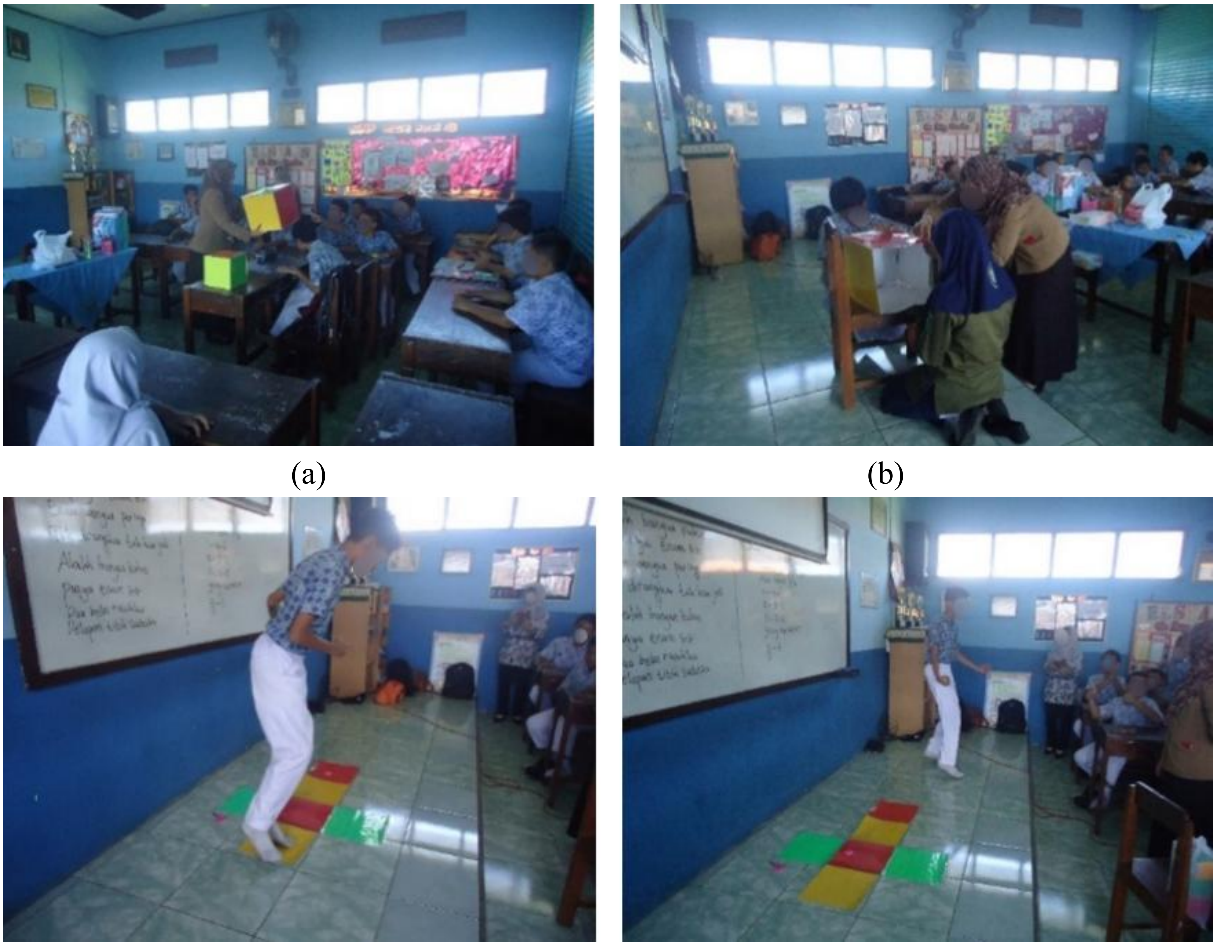
Figure 1 is an iceberg of learning cube nets using the *engklek* game. This iceberg is a metaphor for the learning structure in the RME approach, where students' formal knowledge has a strong foundation from meaningful learning experiences through a realistic context or concrete situation. This learning structure is formed through several learning activities that utilize the mathematical modeling stage in RME, starting from situational concrete, model-of, model-for, to formal knowledge. This iceberg was then developed into a Hypothetical Learning Trajectory (HLT) design as shown in Table 1.

Table 1 is the HLT for cube nets using the *engklek* game, which contains activities, objectives, and predictions of student responses during learning. The flow of learning activities starts from the situational concrete stage, which proposes a realistic situation as a starting point for learning through the activity of deconstructing a large cube to form an *engklek kapal terbang,* which can then be played by students. This stage facilitates contextualization while providing a strong foundation for students' understanding of the concept of cube nets with the *engklek* game that they often play in everyday life. The next stage is model-of, where in this stage, students are involved in simple mathematical modeling activities, namely by deconstructing small cubes to form other cube nets. This stage facilitates students' understanding that it turns out that the shape of the cube net is not only the shape of a *kapal terbang* found in the *engklek* game, but there are also other shapes. More complex mathematical modeling is found in the model-for stage, when students are required to find strategies in order to identify the pattern of the cube net formed from the activity of deconstructing small cubes in the previous stage. Finally, at the formal knowledge stage, students have constructed their formal mathematical knowledge about the 4 cube net patterns and the number of net shapes in each pattern.

The HLT design was implemented in a pilot experiment involving 5 students. The findings from the pilot experiment revealed that no modifications were needed to the HLT because students were able to respond as predicted and fulfill the learning activity objectives. However, the researcher made improvements to the large cube media used in the situational concrete stage. Initially, the size of the large cube was still not sufficient, so that when it was deconstructed into the form of *engklek kapal terbang*, students found it difficult to play with it. Based on this problem, the researcher increased the size of the cube used at the situational concrete stage in order to maximize the contextualization experience during the learning process. In addition, it is necessary to optimize the time for activities at the model-of and model-for stages, so that it is expected to provide better effectiveness towards the formal knowledge stage. Furthermore, after some adjustments were made, the HLT was implemented in a teaching experiment involving a larger number of students, as many as 21 students, where the findings from the retrospective analysis of learning activities are described as follows.

Activity 1: Deconstructing a large cube to form *engklek kapal terbang*

The first activity was deconstructing a large cube to form *engklek kapal terbang*. After the researcher opened the learning and prayer activities together, the researcher told the students that they would learn the cube nets. As shown in Figure 2 (a), the researcher started the activity by using the large cube media that had been previously prepared, and asked students to recall what shapes were in front of them and what their characteristics were. In this case, the students can well recognize that the shape is a cube that has 6 square sides, 12 ribs, and 8 corners. Next, the researcher told the students that the large cube would be deconstructed to form the *engklek kapal terbang* that they could later play with together. As shown in Figure 2 (b), the students found it quite difficult to deconstruct the large cube to form the *engklek kapal terbang,* so the researcher assisted them. After forming the *engklek kapal terbang*, students played the *engklek* game as shown in Figure 2 (c). In Figure 2 (d), students can be seen appreciating their friends who completed the *engklek* game, proving that the realistic context in the form of the *engklek* game in mathematics learning can encourage students' active participation and bring a fun classroom atmosphere.



(a) (b) (c) (d)

Figure 2. (a) Introducing the large cube; (b) Deconstructing the large cube; (c) Playing the *engklek kapal terbang*; (d) Students appreciate their friends

After students played the *engklek* game, the researcher guided students to enter the initial concept of cube nets, namely by asking about the relationship between the *engklek kapal terbang* and the large cube. The dialog excerpt below shows the students' thinking process in understanding the relationship between the *engklek kapal terbang* and the large cube.

Researcher	*: "We know that the shape of the engklek kapal terbang originates from a large cube. What, then, do you think is the relationship between the engklek kapal terbang and the large cube?"*
Student 1	*: "The number of squares in the engklek kapal terbang is six, which corresponds to the six faces of the large cube."*
Researcher	*: "That is correct. Is there another perspective?"*
Student 2	*: "All of the squares are of equal size."*
Researcher	*: "So, what can you infer from these observations?"*
Student 3	*: "If we fold the figure and reassemble it, the engklek kapal terbang will form a large cube, just as before."*
Researcher	*: "Precisely. This configuration is referred to as a net, and the engklek kapal terbang represents one of the possible nets of a cube."*

The dialog excerpt illustrates how the researcher employed a series of guiding questions to scaffold students' reasoning toward an initial understanding of cube nets. This instructional strategy enabled students to recognize that the *engklek kapal terbang* represents the faces of a large cube. Through the process of comparing the shape of the *engklek kapal terbang* with the defining characteristics of a cube, students identified that it consists of six squares of equal size. Building on this observation, they concluded that when the *engklek kapal terbang* is folded and reassembled, it forms a cube. At this stage, students acquired a preliminary understanding of the concept of cube nets, which the researcher reinforced by confirming that the *engklek kapal terbang* constitutes one of the possible nets of a cube. This foundational insight serves as an essential basis for further exploration of other cube net configurations in subsequent learning activities.

Activity 2: Deconstructing a small cubes to form other cube nets

The second activity involves students in mathematical modeling simply through the activity of deconstructing small cubes to form the other cube nets. As shown in Figure 3 (a), students are enthusiastic in their efforts to deconstruct the small cube with their friends, proving that this stage can build collaborative skills in students. Meanwhile, Figure 3 (b) shows students who successfully deconstruct the small cube into a net shape and try to compare it with the net shape produced by another friend behind them.

(a) (b)

Figure 3. (a) Deconstructing the small cube; (b) Deconstruction results forming the cube net

In this activity, the researcher has a crucial role in testing the accuracy of the findings of the cube nets formed by the students. In this case, the researcher asked questions to trigger students to put forward their arguments. The dialog excerpt below shows how the students in Figure 3 (b) put forward their arguments to prove that the nets formed are cube nets.

Researcher : *"Are you certain that the cube nets you found are correct?*
Student 1 : *"Of course, Ma'am."*
Researcher : *"Why are you so certain?"*
Student 2 : *"When these nets are folded, they reconstruct the cube as before, with each face of identical color positioned opposite its corresponding face. For instance, the red-colored face aligns with the other red-colored face."* (Following this explanation, the students provided a demonstration by folding the nets they had constructed, thereby reassembling the cube).

The dialog excerpt demonstrates the accuracy with which students articulated arguments to support their findings regarding cube nets. Initially, they drew upon their prior understanding, asserting that the nets they had identified could be folded to reconstruct a cube. To strengthen this claim, they reasoned that faces of identical color would align with one another, and they substantiated this explanation through a clear demonstration. The researcher extended the inquiry beyond a single student, posing the same question to others in order to test the validity of their proofs. For students who encountered difficulties, the researcher provided guidance to facilitate their reasoning. Moreover, the researcher encouraged students to compare the various cube nets they had discovered, thereby fostering an understanding that multiple variations of cube nets exist..

Activity 3: Identifying the patterns of cube nets

The third activity involves students in more complex mathematical modeling, which requires students to use appropriate strategies to identify the cube net patterns they find. Students engage in discussions with their friends to identify the cube net patterns they find from deconstructing the small cubes in the previous activity. The success of students in this pattern identification activity can be seen in Figure 4 (a), which shows a student showing a cube net with a **1-4-1** pattern, while Figure 4 (b) shows a student showing a cube net with a **2-2-2** pattern. The same success was also achieved by other students who found cube nets with patterns **2-3-1** and **3-3**.

(a) (b)

Figure 4. (a) Students show cube net of pattern **1-4-1**; (b) Students show cube net of pattern **2-2-2**

The important role of the researcher in this activity is to trigger students to express the problem-solving strategies they use in identifying the cube net patterns they find. Students are required to be able to explain how they concluded the pattern formed. The dialog excerpt below shows the success of students in Figure 4 (b) in explaining their problem-solving strategy to the researcher.

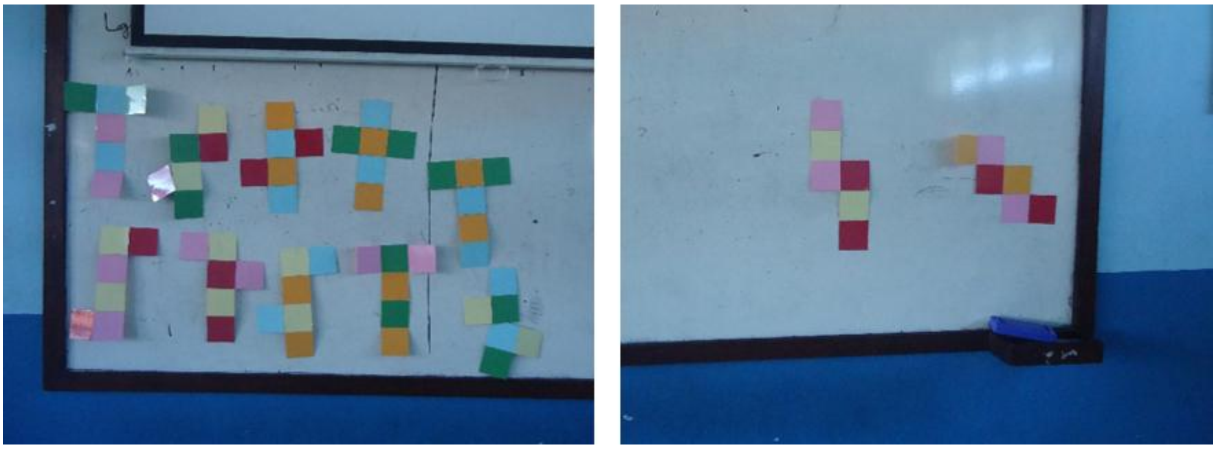
Researcher	*: "What patterns can be identified in your cube nets?"*
Student 1	*: "Two-Two-Two (2-2-2), Ma'am."*
Researcher	*: "How did you derive that pattern?"*
Student 2	*: "By counting the number of squares in each row. In this net, the first row consists of two squares, the second row also consists of two squares, and the third row likewise consists of two squares."*

The dialog excerpt above shows that students are able to determine the right strategy in solving the problem. They can identify the cube net pattern they found by counting the number of squares in each row. This proves the students' accuracy in observing and identifying the cube net pattern they get, and shows the sequence of students' thinking in solving problems to conclude. However, some students found it difficult to immediately use strategies like this, so they were unable to identify the cube net patterns they found. In this case, the researcher intervened by directing students to observe their cube nets, then guided them to count the number of squares in each row, and finally concluded what patterns were in their cube nets.

Activity 4: Finding 4 patterns of cube nets

The fourth activity is the last activity, which involves students in concluding their formal mathematical knowledge through the activity of finding 4 patterns of cube nets. To arrive at this formal mathematical knowledge, students grouped the various shapes of cube nets that they had obtained. Cube net shapes with the same pattern are placed in one group. It can be seen in Figure 5 (a) that students are very enthusiastic about grouping the cube net patterns they get by pasting them on the board. Figure 5 (b) shows some students who are confused to group their cube nets, so the researcher assists them so that they can paste the cube nets they get into the right pattern group. As a result, they obtained pattern groups **1-4-1** and **2-3-1** shown in Figure 5 (c), and pattern groups **2-2-2** and **3-3** shown in Figure 5 (d).

(a) (b)



(c) (d)

Figure 5. (a) Students grouping cube nets; (b) Researcher helping students with difficulties: (c) Group of cube nets with patterns **1-4-1** and **2-3-1**; (d) Group of cube nets with patterns **2-2-2** and **3-3**

The role of the researcher in this activity is to guide the class discussion so that students arrive at the formalization of their mathematical knowledge. Students are guided to conclude what patterns are owned by the cube nets by analyzing the results of the grouping they have done. In this case, the researcher uses a strategy by asking questions that can lead students to the final conclusion. During the class discussion, students actively answered questions and expressed their opinions. This shows that the class discussion process encourages students to participate, dare to argue, and improve academic performance that can bridge them to arrive at formal mathematical knowledge about cube nets. The dialog excerpt below illustrates the class discussion process that took place.

Researcher : *"Now, examine carefully all the cube nets that you have grouped. How many categories of patterns do you think exist?"*

Student 1 : *"There are four, Ma'am."*

Researcher : *"That is correct. Can anyone identify the specific pattern groups?"*

Student 2 : *"Yes, Ma'am. The groups are one-four-one **(1-4-1)**, two-three-one **(2-3-1)**, two-two-two **(2-2-2)**, and three-three **(3-3)**."*

Researcher : *"Good. Now, how many cube nets are contained within each pattern group, and what is the total?"*

Student 3 : *"There are **nine** nets in the one-four-one **(1-4-1)** pattern, **one** in the two-three-one **(2-3-1)** pattern, **one** in the two-two-two **(2-2-2)** pattern, and **one** in the three-three **(3-3)** pattern. That makes **twelve** in total, Ma'am."*

Researcher : *"Are you certain? Look again carefully. Within the one-four-one **(1-4-1)** pattern, some of the nets are identical, so the number you mentioned is not accurate."*

Student 4 : *"Then, what should the total be, Ma'am?"*

Researcher : *"The total number of distinct cube nets is **eleven**. It is mathematically established that a cube possesses exactly **eleven** unique nets. Nets are considered identical if one can be rotated or reflected to coincide with another. Specifically, there are **six** nets in the one-four-one **(1-4-1)** pattern, **three** nets in the two-three-one **(2-3-1)** pattern, **one** net in the*

> *two-two-two **(2-2-2)** pattern, and **one** net in the three-three **(3-3)** pattern. Therefore, there are **two** nets in the two-three-one **(2-3-1)** pattern that you have not yet identified."*

The dialog excerpt above shows the students' thinking process to draw conclusions and gain formal mathematical knowledge about the concept of cube nets. This thinking process occurred with the guidance of the researcher, who asked a number of questions. Students were able to answer the questions well, so they could correctly state that there were 4 patterns of cube nets, which they knew from the grouping of cube net patterns on the board. However, when students specify the number of shapes in each pattern, there is a slight error in the formal knowledge of the correct number of cube nets. This error did not occur due to students' analysis error, but occurred because there were several findings of cube nets with the same shape in pattern **1-4-1,** and they were only able to find 1 shape in pattern **2-3-1** (see Figure 5 (c)). In this case, the researcher intervened by informing the students of the shortcomings of the cube net shape findings that had been made by the students, then the researcher described 2 other net shapes in the **2-3-1** pattern that had not been successfully found by the students. Finally, students get formal math knowledge correctly, that there are **four** patterns of cube nets which include: **six** shapes in pattern **1-4-1**, **three** shapes in pattern **2-3-1**, **one** shape in pattern **2-2-2**, and **one** shape in pattern **3-3**, so the total number of cube nets is **eleven**.

Discussion

A student Learning Trajectory (LT) is obtained in learning the concept of cube nets. Prediction of student responses or thought processes proved accurate, making it possible to use questions that guide the deepening of student understanding and achieve the expected learning objectives. Through four learning activities, the Learning Trajectory (LT) formed is in accordance with the modeling stage in the RME approach, which is mutually sustainable, starting from the situational concrete level, the model-of level, the model-for level, to the formal knowledge level.

At the situational concrete level, students gain an initial understanding of the concept of cube nets. This initial understanding is obtained with a starting point from a realistic context in the form of the *engklek* game. The use of game contexts allows students to use an engaging way to explore mathematical concepts and their relevance to real life (Putri et al., 2025). Through the *engklek* game, students engaged in exploring the relationship between the *engklek* game pattern and cubes. They discovered that the *engklek* game pattern consists of 6 square shapes of the same size. They then understand that these characteristics are the same as the characteristics of the sides of a cube, so that when folded, it will form a cube as before. They then concluded that the *engklek* game pattern is a form of cube net. This shows the important role of realistic contexts that can make it easier for students to understand the relevance between concepts in mathematics and the context of their daily lives. In line with the emphasis on the need for pedagogies that are relevant to daily and cultural contexts in enhancing student engagement and understanding (Kolovou, 2022; Mark & Id-Deen, 2022). The use of everyday contexts to start learning mathematics is useful to help students understand the relevance and benefits of mathematics, encourage active student participation, and maintain student interest during the learning process (Murniati et al., 2025; Zuliana et al., 2025).

Students' initial understanding of the shape of the cube net is strengthened at the model-of level. At this level, students engage in simple mathematical modeling to explore the shape of other cube nets through the activity of deconstructing small cubes. This exploration process strengthens the interaction between students because they are required to work together to find various forms of cube nets. High learning motivation is seen when students actively work with their group mates and compare their work with other groups. In line with the claim Rawani et al. (2023) that interaction and cooperation between students allows for good quality learning. Reinforced by the findings that the RME approach, which involves interaction between students, has a direct effect on student learning motivation (Fauzi et al., 2024). Evidently, in this study, the interaction and cooperation carried out by students succeeded in making them able to find various forms of cube nets and prove their findings by presenting logical mathematical arguments. This achievement is inseparable from the strategic questions posed to students, so as to encourage them to think critically and logically to prove the accuracy of the shape of the cube net they found. As the statement Prahmana et al. (2025) that teachers' strategic questioning effectively encourages students to reflect on, articulate, and deepen their understanding.

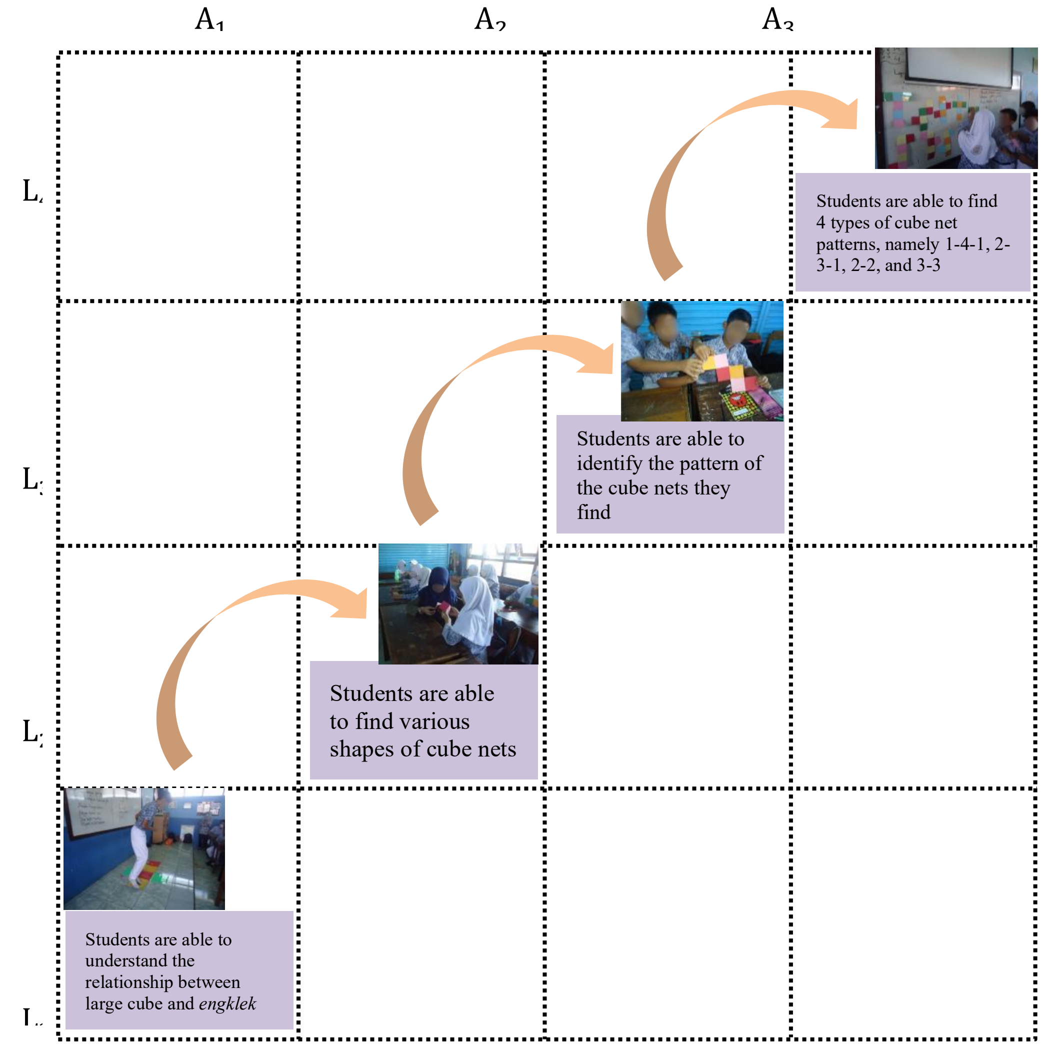
After students get various forms of cube nets, at the model-for level, students are involved in more complex mathematical modeling. This level requires students to find the right strategy in solving problems in the form of identifying patterns of cube nets. In this study, through collaboration, students proved to be able to determine the right strategy, namely by calculating the number of squares in each row of the cube nets they got. The accuracy of this strategy proves that the RME approach encourages students to reason logically in identifying the cube net pattern they get. In line with claims that collaboration can involve students in establishing consensus about problem solutions (Battista, 1999). This finding is also similar to several other findings that the RME approach can increase students' logical reasoning capacity in modeling to improve problem-solving skills (Ginting & Rakhmawati, 2024; Windari & Amir, 2024). For students who had difficulty determining the strategy to identify the pattern of the cube nets, targeted guidance was given to them so that they could identify the right pattern. In line with the emphasis that students who struggle in learning need to be given more guidance to guide them in understanding a mathematical concept and its application (Listiawati et al., 2023). This kind of guidance is important because the feedback provided can lead students to a more mature thought process in solving problems (Rahmadi et al., 2024).

At the formal knowledge level, students gain formal knowledge about cube net patterns. At this level, they group different shapes of cube nets based on the same pattern, analyze them, and engage in class discussions. During the class discussion, they engage in reasoning activities to reach a conclusion and communicate it. In line with the findings of Palinussa et al. (2021), the RME approach can improve students' mathematical reasoning and communication. In addition, thinking and understanding strategies emerge from within students themselves through questioning and answering, discussion, reflection, cooperation, and evaluation (Prahmana et al., 2023). Evidently, in this study, students were able to infer formal mathematical knowledge about cube nets. They can understand that there are 4 patterns of cube nets, which include: **6** shapes in pattern **1-4-1**, **3** shapes in pattern **2-3-1**, **1** shape in pattern **2-2-2**, and **1** shape in pattern **3-3**, so that the total number of cube nets is **11**. It is mathematically established that all possible nets for a cube, eleven nonidentical or noncongruent nets in total (Jeon, 2009; Pavlovičová et al., 2022; Pavlovičová & Švecová, 2015; Sahrudin et al., 2022).

Learning Trajectory (LT) findings in this study show how students' thinking processes through the RME stage from the situational concrete level, the model-of level, the model-for level, to the formal knowledge level. In line with claims that LT has consistently proven to improve student achievement in learning (Clements & Sarama, 2025). Findings in this study show that students can construct their own formal mathematical through the process of deconstructing large cubes into *engklek*, then gradually rediscovering the cube nets in a guided manner. In the RME principle, the process utilizes *guided reinventation* so allows students to assume the knowledge they acquire is their own knowledge for which they are responsible (Gravemeijer & Terwel, 2000). This process also involves *progressive mathematization* that guides students in thinking from one level to the next level (Treffers, 1987a). Mathematization consists of horizontal mathematization, which is concerned with translating the real world into symbols, and vertical mathematization, which is concerned with organizing mathematical knowledge that produces concepts or strategies in abstract mathematical symbols (Treffers, 1987b). In the findings of this study, students are guided to engage in horizontal mathematization through the *engklek* game, which then leads to vertical mathematization regarding the 4 patterns of cube nets. The *engklek* pattern (*situation-specific model*) evolved into abstract pattern notation (*general model*). In line with *emergent modelling* by Gravemeijer (2007) who has the idea that students start with modeling their own informal mathematical activity, which is expected to gradually develop into a model for more formal mathematical reasoning. In summary, the Learning Trajectory (LT) findings in this study show the development of students' thinking processes that occur through a series of activities in the frame of the modeling level in the RME approach, as shown in Figure 6.

The modeling level in the RME approach that frames the LT in this study shows how the realistic context in the form of the *engklek* game can support students' conceptual understanding in the cube nets material. RME bridges the integration of cultural context in mathematics learning both explicitly and implicitly, so it can create a learning atmosphere that supports students' mathematical achievement in geometry materials (Nugroho et al., 2025). The use of the *engklek* game as a context in mathematics learning, especially in this case geometry, can have a beneficial impact. The game context can address learning difficulties, cultural preservation, and support mathematical process skills (Pawartani et al., 2024; Qirom & Juandi, 2023). The *engklek* game can foster creativity and intelligence, improve motor development, and improve students' personality development (Kamid et al., 2021; Kristanto & Wibowo, 2023). The findings in this study are reinforced by several studies that also use the context of games to design LT, for example, for length and volume measurement materials, probability materials, and multiplication materials (Rahayu et al., 2022; Wijaya et al., 2021; Zakaria & Dewantara, 2024).

Thus, this study successful designing Learning Trajectory (LT) to teach the concept of cube nets. The LT findings in this study are actual and empirically tested which show that to teach the concept of cube nets, instruction can start from a realistic context as a strong foundation, then gradually progress towards abstraction. The *engklek* game used as a realistic context in the LT design was able to bridge students' informal and formal mathematics experiences, improve their academic performance, and create meaningful learning. Teachers can utilize the LT findings in this study to expand their instructional strategies in teaching cube nets, offer context-rich learning close to students' daily lives, and encourage students to have an awareness that mathematics is a human activity.



L_1 = Level 1 (Situational concrete)
L_2 = Level 2 (Model-of)
L_3 = Level 3 (Model-for)
L_4 = Level 4 (Formal knowledge)

A_1 = Activity 1 (Deconstructing a large cube to form *engklek kapal terbang*)
A_2 = Activity 2 (Deconstructing a small cubes to form other cube nets)
A_3 = Activity 3 (Identifying the patterns of cube nets)
A_4 = Activity 4 (Finding 4 patterns of cube nets)

Figure 6. Learning trajectory of cube nets using the *engklek* game

Conclusion

This study successfully designing a Learning Trajectory (LT) for cube nets using the *engklek* game. This is an important contribution to expanding instructional design in mathematics that integrate with traditional games. The LT design in this study consists of four activities—deconstructing a large cube to form *engklek kapal terbang*, deconstructing a small cubes to form other cube nets, identifying the patterns of cube nets, and finding 4 patterns of cube nets—in the frame of the modeling level in the RME approach that can bridge students' informal and formal mathematics experiences. The LT design in this study has been proven to effectively guide students in understanding the concept of cube nets, so can be utilized by teacher as a source for mathematics teaching strategies in cube nets material. Utilizing LT in this study can create a learning atmosphere that encourages students' awareness of the relevance of mathematics to their daily lives.

Nevertheless, there are several limitations in this study that need to be considered. First, this study conducted at single class and single school, so requires adjustments for transferability. Second, this study only led to students' formal knowledge without involving a comprehensive evaluation quantitatively, specifically pre-assessment and post-assessment, so it cannot quantify learning gains. Third, social validity—teacher usability—not evaluated. Further research is recommended to address these limitations and expand the context of traditional games in designing Learning Trajectory (LT) on other 3D geometry topics.

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Declarations

Conflicts of Interest	:	The authors declare no conflict of interest regarding the publication of this manuscript.
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