



Tracing elementary students' learning trajectories of surface area in museum-based activities

Dina Octaria, Zulkardi *, Ratu Ilma Indra Putri, Cecil Hiltrimartin

Department of Mathematics Education, Sriwijaya University, South Sumatra, Indonesia

* Correspondence: zulkardi@unsri.ac.id

© The Author(s) 2026

Abstract

Understanding the surface area of three-dimensional shapes remains challenging for elementary students, particularly in distinguishing between area and perimeter and in connecting two-dimensional representations to three-dimensional objects. This study aims to trace elementary students' learning trajectories on the surface area of cubes and cuboids through museum-based activities to support spatial literacy development. The study employed a design research methodology with a validation study approach, consisting of preliminary design, a pilot experiment, a teaching experiment, and a retrospective analysis. Participants were 44 fifth-grade students from an elementary school in Palembang, Indonesia. Data were collected through classroom observations, students' worksheets, video recordings, and interviews, and analyzed qualitatively by comparing the Hypothetical Learning Trajectory (HLT) with the Actual Learning Trajectory (ALT). The results show that the museum context supported students' spatial visualization, reasoning, and communication by enabling them to identify geometric attributes of artefacts, construct and validate nets, and derive surface area formulas through guided reinvention. The targeted use of Augmented Reality (AR) further supported students during critical transitions between two-and three-dimensional representations. The synthesis of the HLT and ALT yielded a validated Learning Trajectory, which was abstracted into a Local Instructional Theory (LIT) for teaching surface area in elementary geometry.

Keywords: learning trajectories; museum-based learning; PjBL; PMRI; solid geometry; spatial literacy; three-dimensional nets

How to cite: Octaria, D., Zulkardi, Putri, R. I. I., & Hiltrimartin, C. (2026). Tracing elementary students' learning trajectories of surface area in museum-based activities. *Jurnal Elemen*, 12(1), 241-262. <https://doi.org/10.29408/jel.v12i1.33240>

Received: 1 December 2025 | Revised: 8 January 2026

Accepted: 20 January 2026 | Published: 19 February 2026



Introduction

Elementary school students frequently experience difficulties in understanding the surface area of cubes and cuboids, particularly when instruction emphasizes memorization of formulas rather than conceptual understanding. Several studies have shown that students who can recall surface area formulas often fail to understand the mathematical principles underlying these formulas (Juniati & Siswono, 2019; Kim, 2020; Hussein & Csíkos, 2023). When asked to explain how surface area formulas are derived, students commonly demonstrate misconceptions by adding edge lengths or confusing surface area with volume (Tan Sisman & Aksu, 2016; Chiphambo & Mtsi, 2021; Putri & Nurlaelah, 2022). These findings indicate that students exhibit varying levels of spatial thinking, largely due to insufficient conceptual foundations or meaningful spatial understanding.

Museum-based learning offers a promising context for addressing these challenges because it provides authentic three-dimensional objects that students can directly observe, manipulate, and interpret. The Sultan Mahmud Badaruddin II Museum in Palembang contains various cubes-and cuboid-shaped artifacts, such as storage jars, glass cabinets, and historical showcases, which can serve as meaningful starting points for learning surface area concepts. Learning in museum environments aligns with principles of learning by doing and contextual learning, which emphasize active engagement and real-world experiences. Previous studies have shown that museum-based learning enhances student engagement, supports spatial representation skills, and fosters connections between mathematical learning and cultural identity (Çil et al., 2016; Chatterjee et al., 2016; El Bedewy et al., 2024).

Research on geometry learning consistently reports multiple conceptual and procedural difficulties encountered by students when studying surface area. Students often struggle to distinguish between area and perimeter measurements, misinterpret measurement concepts, and apply formulas incorrectly (Tan Sisman & Aksu, 2016). Procedural errors frequently arise from incorrect calculations, inappropriate formula application, and an overreliance on memorization without conceptual understanding (Putri & Nurlaelah, 2022; Lehmann, 2023; Imami & Wafa, 2023). Difficulties in visualizing and interpreting nets further hinder students' ability to connect two-dimensional representations with three-dimensional objects (Nanga & Hurit, 2020; Sahrudin & Budiarto, 2022). In addition, problems related to mathematical language and terminology often lead to misunderstanding of problem instructions and geometric relationships (Chiphambo & Mtsi, 2021). Students with weaker spatial abilities tend to make more errors in tasks involving nets and spatial transformations, highlighting the close relationship between surface area understanding and spatial literacy (Winarti et al., 2012; Widjaja & Vale, 2021).

Spatial literacy is widely recognized as a fundamental competency for the 21st century, as it enables individuals to understand, reason about, and communicate spatial relationships using multiple representations (De Lange, 2003; Moore-Russo et al., 2013; Ramful et al., 2017). In geometry learning, spatial literacy supports students in linking three-dimensional objects with two-dimensional representations, particularly through the construction and interpretation of nets of cubes and cuboids. Research indicates that students develop stronger spatial literacy when they engage in learning activities that combine visual exploration, manipulative tasks,

and open-ended problem-solving (Ha & Fang, 2018; Medina Herrera et al., 2019; Gilligan-Lee et al., 2023; Herrera et al., 2024). These findings suggest the need for instructional approaches that guide students along structured learning paths, allowing them to discover mathematical patterns rather than merely apply formulas.

From a design research perspective, learning trajectory analysis provides a framework for examining how students' mathematical understanding develops over time. Learning trajectories describe students' progression from informal reasoning toward more formal mathematical concepts through carefully designed instructional sequences (Gravemeijer & Cobb, 2006; Bakker, 2018). By comparing hypothetical learning trajectories with actual learning trajectories observed in classroom settings, researchers can identify key transitions in students' thinking and refine instructional designs. In the context of surface area learning, such analysis helps reveal how students move from concrete interactions with objects and nets toward symbolic representations and generalization.

Although existing research has provided valuable insights into students' difficulties with surface area, most studies have been conducted in traditional classroom environments using standard instructional materials. There remains a lack of research that explicitly traces elementary students' learning trajectories of surface area through museum-based activities, particularly within the Indonesian educational context. Moreover, few studies integrate museum-based learning with the Indonesian Realistic Mathematics Education (PMRI) framework and Project-Based Learning (PjBL), both of which emphasize contextual learning, student exploration, and reflection (Bakait et al., 2021; Rehman et al., 2025). This gap highlights the need for research that examines how culturally grounded learning environments can support students' conceptual development and spatial literacy.

Therefore, this study aims to design and trace elementary students' learning trajectories of surface area through museum-based activities using a design research approach. Specially, the study investigates how students construct surface area concepts of cubes and cuboids through planned interaction with artifacts from the Sultan Mahmud Badaruddin II Museum and how these learning trajectories support the development of spatial literacy. The findings are expected to contribute to the development of a Local Instructional Theory (LIT) for teaching surface area in elementary geometry and to enrich the literature on museum-based learning in mathematics education.

Methods

Research design

The study employed a validation study type of design research, which aims to develop and refine instructional theories through iterative cycles of design, implementation, and analysis (Gravemeijer & Cobb, 2006; Bakker, 2018). The primary objective of the research was to design and validate a learning trajectory for teaching the surface area of cubes and cuboids to elementary students. The learning trajectory was initially formulated as a Hypothetical Learning Trajectory (HLT) and subsequently refined into a validated Learning Trajectory (LT) based on classroom implementation and analysis of students' learning processes.

The design research consisted of three interconnected phases: (1) preliminary design, (2) design experiment, and (3) retrospective analysis (Gravemeijer & Cobb, 2006). These phases formed a cyclic and cumulative process, in which findings from each phase informed revisions in subsequent iterations. This recursive structure enabled theoretical insights and empirical classroom data to develop simultaneously, thereby strengthening the validity of the resulting learning trajectory (Bakker, 2018). Figure 1 illustrates the cyclic structure of the design research methodology employed in this study, highlighting the recursive relationship between the preliminary design, design experiment, and retrospective analysis phases.

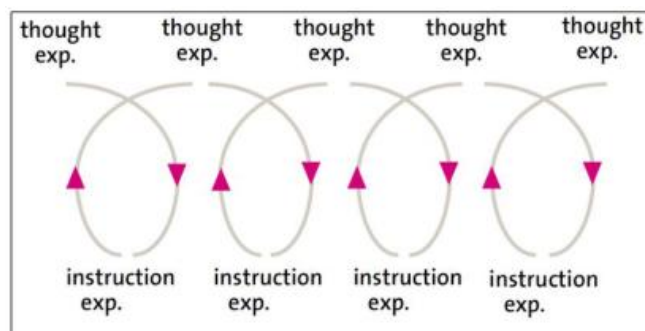


Figure 1. Cyclic structure of the design research methodology (adapted from Gravemeijer & Cobb, 2006)

During the preliminary design phase, an extensive literature review was conducted focusing on students' difficulties in learning surface area, spatial literacy development, museum-based learning, and the theoretical foundations of Indonesian Realistic Mathematics Education (PMRI) and Project-Based Learning (PjBL) (Moore-Russo et al., 2013; Sung et al., 2015; Rahayu & Putri, 2021; Bakait et al., 2021; Sudirman et al., 2023). This phase resulted in the formulation of an initial HLT grounded in the cultural context of the Sultan Mahmud Badaruddin II Museum in Palembang (Octaria et al., 2025).

Design experiment

The design experiment phase consisted of two stages: a pilot experiment and a teaching experiment. The pilot experiment was conducted to examine the feasibility, clarity, and practicality of the designed learning activities. Feedback obtained during this stage was used to revise and improve the HLT prior to broader classroom implementation.

The teaching experiment aimed to test the revised HLT in a full classroom setting and to observe the Actual Learning Trajectory (ALT) that emerged during instruction. The teaching experiment was conducted over five instructional meetings (each lasting 2×35 minutes). The first meeting focused on museum observation through video to introduce cube-and cuboid-shaped artifacts. The second meeting involved an on-site museum visit where students measured selected artifacts. The third meeting focused on the construction of cube and cuboid nets and the exploration of surface area concepts supported by Augmented Reality (AR) to facilitate students' visualization of the transformation between two-dimensional nets and three-

dimensional objects. The fourth meeting addressed volume concepts using AR-based representations to strengthen students' understanding of three-dimensional structures. The fifth meeting involved the construction of miniature artifacts and group presentations to consolidate students' learning.

Instructional design and hypothetical learning trajectory

The HLT was designed to support students' understanding of surface area through museum-based activities involving cube-and cuboid-shaped artifacts from the Sultan Mahmud Badaruddin II Museum. The instructional design was grounded in PMRI principles, particularly the use of meaningful real-world contexts and the progression from model-of to model-for abstraction, and supported by Project-Based Learning (PjBL) to promote collaborative inquiry, hands-on exploration, and reflection (Bakait et al., 2021; Tresnawati et al., 2021; Palupi et al., 2025). The HLT consisted of six main learning stages: (1) contextual orientation and introductory mini-project, (2) observation and measurement of museum artifacts, (3) exploration of cube and cuboid nets, (4) derivation of surface area formulas, (5) miniature artifact construction, and (6) presentation and reflection.

Table 1 presents a condensed overview of the HLT, focusing on learning goals, core activities, and anticipated student thinking. Interpretations of students' actual responses and learning outcomes are intentionally excluded from this section and are presented in the Results section through ALT analysis.

Table 1. Hypothetical learning trajectory (HLT) for the surface area of cube and cuboid

Learning stages	Learning goals	Core learning activities	Anticipated students' thinking
Contextual orientation	Recognize cubes and cuboids in real contexts	Observing museum artifacts via video and discussion	Students identify objects resembling cubes and cuboids based on visual features
Observation and measurement	Identify dimensions and elements of solids	Measuring length, width, height of selected artifacts	Students begin distinguishing faces, edges, and vertices
Net explorations	Connect 2D nets with 3D objects	Constructing and folding cube and cuboid nets	Students explore different net configurations
Formula derivation	Generalize surface area patterns	Identifying and grouping face areas	Students recognize patterns leading to surface area formulas
Miniature construction	Apply surface area concepts	Building proportional artifact models	Students relate measurements to model construction
Presentation & reflection	Communicate mathematical reasoning	Presenting models and explaining strategies	Students articulate surface area reasoning verbally

Integration of Augmented Reality (AR)

Augmented Reality (AR) was integrated as a supporting instructional tool to enhance students' spatial visualization, particularly during the exploration of cube and cuboid nets. Previous research has shown that students often struggle to mentally transform two-dimensional nets into three-dimensional objects, which is a key component of spatial literacy (Moore-Russo et al., 2013). Within the design research cycle, AR was introduced during the design experiment phase

as part of the instructional intervention. AR applications allowed students to visualize how nets fold into three-dimensional solids, thereby supporting the visualization and reasoning dimensions of spatial literacy. During retrospective analysis, students' interactions with AR-supported representations were examined to determine how the toll contributed to refining the HLT. Thus, AR functioned not as a standalone intervention, but as an embedded component aligned with the iterative refinement of the learning trajectory.

Participants and research context

The study was conducted at a public elementary school in Palembang, South Sumatra. Participants included 44 fifth-grade students (aged 10-11 years) and their mathematics teacher. The pilot experiment involved 14 students representing varied academic ability levels, while the teaching experiment involved 30 students from a different class. Participants were selected using purposive sampling to ensure diversity in academic ability and students' readiness to engage in museum-based learning activities. The classroom teacher played an active role in implementing the instructional design, facilitating discussions, guiding student exploration, and coordinating museum visits in collaboration with the researcher.

Data collection and analysis

Data were collected using multiple qualitative methods, including students' worksheet (LKPD), classroom observation notes, video recordings, photographs, and semi-structured interviews with students and the teacher. These data sources were used to document students' learning processes, interactions, and emerging spatial reasoning. Data analysis followed a retrospective analysis approach (Gravemeijer & Cobb, 2006), consisting of data reduction, data display, and conclusion drawing. The analysis focused on comparing the HLT with the ALT observed during classroom implementation to identify critical transitions in students' understanding. Particular attention was given to students' spatial literacy development across visualization, reasoning, and communication dimensions. The resulting validated LT served as the foundation for developing a Local Instructional Theory (LIT) for teaching surface area using museum-base contexts.

Ethical considerations

Ethical approval was obtained from the school administration prior to data collection. Written informed consent was secured from the participating teacher, students, and students' parents. Participants were informed about the purpose of the study and their right to withdraw at any stage without consequences. All data were anonymized, and pseudonyms were used to protect participant confidentiality. The collected data were used solely for academic and research purposes.

Results

The learning process followed the Hypothetical Learning Trajectory (HLT), which was designed as a sequence of activities to support students' spatial literacy development through the Sultan Mahmud Badaruddin II Museum context. In design research terms, the pilot experiment provided evidence for refining instructional supports and sequencing, while the teaching experiment enacted the revised HLT in a full classroom setting and revealed students' Actual Learning Trajectory (ALT) as it emerged through their response, strategies, and difficulties during instruction. This section traces how the HLT was enacted, how the ALT unfolded across the implementation, and how both were synthesized into a validated Learning Trajectory (LT). The analysis is organized into five core learning activities and highlights key learning shifts across spatial visualization, spatial reasoning, and spatial communication.

Overview of the HLT-ALT transition

Overall, the teaching experiment largely followed the designed HLT. However, several refinements became salient during enactment. First, compared to the pilot phase, students benefited from stronger early scaffolding during contextual orientation, particularly through teacher prompts that directed attention to edge length, face uniformity, and proportional differences. Second, difficulties in coordinating 2D-3D representations during net construction-identified in the pilot experiment-motivated the integration of Augmented Reality (AR) as targeted visualization scaffolding in the teaching experiment. Third, during retrospective analysis, several operational steps (e.g., formula derivation and reflection) were interpreted as part of broader conceptual phases, supporting the synthesis of a validated LT and contributing to the formulation of the Local Instructional Theory (LIT) presented in the subsequent section.

Activity 1: Contextual orientation and introductory mini-project

The teaching experiment began with video-based contextual orientation using footage of the Sultan Mahmud Badaruddin II Museum, highlighting geometric forms, with particular emphasis on cube-and cuboid-shaped artefacts. Unlike the pilot phase, the teacher provided more deliberate visual and verbal scaffolding by pausing the video at key moments and directing students' attention to shape attributes (e.g., edge equality, face shape, and proportions). In the pilot phase, most students referred to three-dimensional artefacts simply as "boxes". During the teaching experiment, however, teacher prompts such as "*Do all the edges seem equal?*" and "*Do the side faces appear longer?*" supported students in distinguishing cubes and cuboids based on geometric attributes rather than overall appearance.



Figure 1. Museum artifacts highlighted during video-based contextual orientation

A brief excerpt illustrates the transition from appearance-based identification to attribute-based reasoning:

- T : *“Look carefully at the edges of this artefact. Do they all appear to be the same length?”*
- S1 : *“Yes, Sir. The edges look equal, so this one is a cube”*
- T : *“Now compare it with the other artefact. What makes it different?”*
- S2 : *“That one is longer on one side, Sir, so it is a cuboid”*
- (*T = teacher; S1, S2 = students*)

This interaction indicates growth in spatial visualization, as students began attending to internal geometric structure rather than relying solely on global appearance. When the teacher asked how much paper would be needed to cover an artefact, students described the surface as *“the outside part that can be seen and touched,”* suggesting an emerging but informal conception of surface area grounded in tangible experience. The teacher also posed a capacity question to distinguish *“outer covering”* from *“inner space”*, helping students anticipate the conceptual boundary between surface area and volume for later activities.

Students then selected one artefact for a miniature project. Compared to the pilot phase—where selection tended to reflect aesthetic preference—students in the teaching experiment increasingly provided geometry-related reasons (e.g., equal sides, longer side faces), reflecting growth in spatial communication. Finally, students produced front, top, and side-view sketches. In the pilot experiment, many students drew perspective-like 3D sketches even when asked for orthogonal view. During the teaching experiment, the teacher applied a pause-and-zoom strategy using the video to highlight how an object’s appearance changes from different viewpoints, improving the accuracy of students’ representations, although minor perspective elements still appeared for some students.

HLT-ALT note (Activity 1)

The HLT anticipated a progression from informal recognition toward attribute-based identification. The ALT revealed that this shift required explicit comparative prompts and teacher-led attention focusing during video observation.

Key learning shift (Activity 1)

Students moved from intuitive, appearance-based identification toward recognizing cubes and cuboids using geometric attributes, indicating early development of spatial visualization and communication.

Activity 2: Observation and measurement activities at the museum

The second activity, students visited the Sultan Mahmud Badaruddin II Museum to measure the artefacts they had selected. They measured length, width, and height; identified faces, edges, and vertices; and compared their project artefact with other museum objects. The goal was to deepen students' spatial visualization and reasoning through direct engagement with real objects. During measurement, students employed different strategies depending on object size. For large artefacts such as glass cabinets, students used decomposition by measuring the object in smaller segments step-by-step using a tape measure.



Figure 2. Students conducting measurements of museum artefacts using decomposition strategies

These estimations indicate developing spatial reasoning, particularly decomposing a complex object into measurable parts. Compared to the pilot phase, students in the teaching experiment showed improved accuracy in interpreting decimal measurements. The excerpt below illustrates this shift:

S1 : *“Sir, the measurement goes a bit past 14. So what does that mean?”*

T : *“Look at the smaller marks that follow. If it falls between the major numbers, write it according to its position”*

S1 : *“Oh, so it’s 14.5 cm, Sir”*

Students also demonstrated improved understanding in identifying faces, edges, and vertices. Errors found in the pilot phase (e.g., labeling faces as edges) decreased after structured small-group discussion. When observing other artefacts without measuring them, students increasingly distinguished cubes and cuboids based on proportional relationships among dimensions rather than overall visual impressions.

HLT-ALT note (Activity 2)

While the HLT expected measurement to support dimensional reasoning, the ALT showed that students needed explicit attention to measurement conventions (e.g., starting point, reading decimals) to stabilize dimension-based comparisons that would later be required in net construction.

Key learning shift (Activity 2)

Students progressed from imprecise measurement and identification toward more systematic spatial reasoning and visualization through direct interaction with real objects.

Activity 3: Exploring nets of cubes and cuboids (manual exploration with AR scaffolding)

Learning step 3 focused on constructing cube and cuboid nets based on measurements obtained in Activity 2. In the pilot phase, the entire process was completed manually using grid paper. Several students could draw individual squares or rectangles but mispositioned faces, producing nets that could not be folded into complete solids. The most frequent error in cube nets was placing faces in arrangements that prevented closure (e.g., misplacing a “top” face adjacent inappropriately), reflecting incomplete understanding of face adjacency.

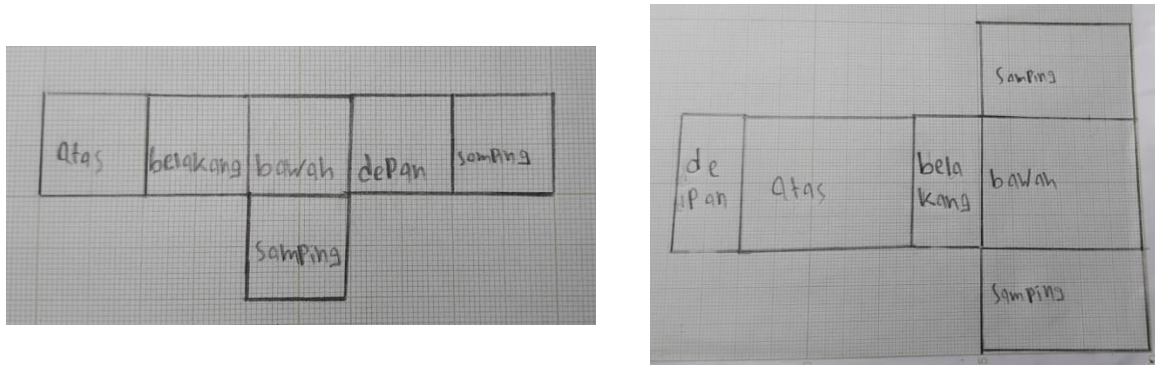


Figure 3a. Original student-produced cube and cuboid nets



Figure 3b. Annotated English version highlighting mispositioned faces

In the teaching experiment, AR was introduced as a targeted scaffolding tool to support students’ visualization of face relationships and to validate whether their nets were foldable. Importantly, AR did not replace manual exploration; rather, it was used when students experienced spatial breakdown (e.g., “the net won’t close”) to help them inspect face adjacency and orientation.

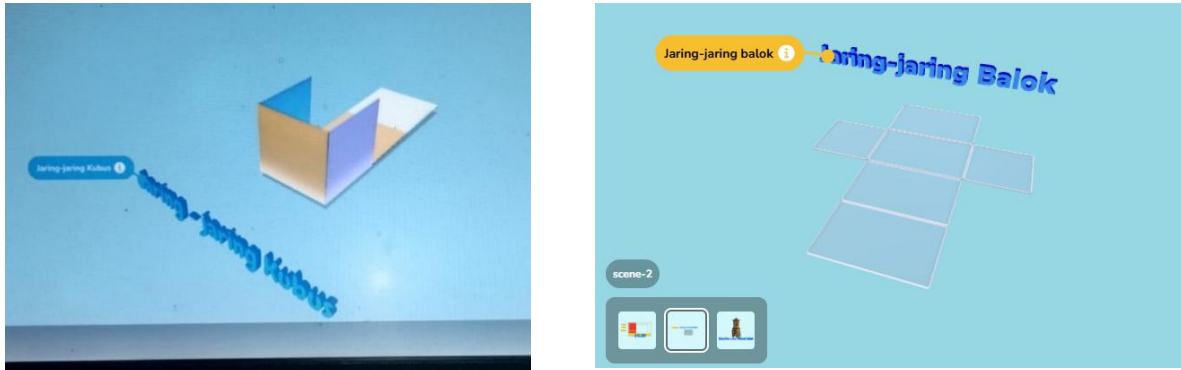


Figure 4. AR-supported validation of cube and cuboid nets

The interaction below illustrates how AR supported students' revisions:

- S1 : "Sir, our net won't close"
 T : "Look at the AR model. If the base is here, which face becomes the top?"
 S1 : "Oh.. ours is in the wrong place. It should be here, not there"

After validating cube nets, students modified their nets to form cuboid nets using measured artefact dimensions. In the pilot phase, common errors involved interchanging length, width, and height or producing rectangles with incorrect proportions. In the teaching experiment, AR helped students recognize that cuboids have three pairs of congruent faces (top-bottom, front-back, left-right) and that dimensional relationships determine net structure. Students' revisions became more purposeful, shifting from trial-and-error to intentional adjustment based on face pairing and proportional reasoning. After validation, students cut and folded their nets; errors in assembly (nets not closing due to misaligned faces) were reduced compared to the pilot phase, consistent with students' increased ability to anticipate folding outcomes.

From a spatial literacy perspective, Activity 3 supported: (a) spatial visualization, by coordinating 2D nets with 3D solids; (b) spatial reasoning, by understanding face adjacency, pairing, and proportionality; and (c) spatial communication, as students increasingly used language such as "closing," "meeting," and "length-width-height" to justify revisions.

HLT-ALT note (Activity 3)

The HLT anticipated that manual net construction would foster 2D-3D coordination. The ALT showed that some students required additional visualization scaffolding; this refinement positioned AR as conditional support at a critical transition point in the validated LT.

Key learning shift (Activity 3)

Student moved from unstructured net construction toward purposeful validation of spatial relationships, with AR functioning as a targeted visualization scaffolding.

Activity 4: Deriving the surface-area formulas for cubes and cuboids

Learning step 4 guided students to derive formulas for the surface area of cubes and cuboids through exploration of their constructed nets. Students reopened their folded nets, calculated

the area of each face, and identified emerging patterns through small-group discussion. In the pilot phase, several students confused area with perimeter or counted only one row of grid squares. For example, some students traced boundaries and treated “outside lines” as area, indicating that the unit structure of area was not yet stable. In the teaching experiment, the teacher provided minimal but focused prompts (e.g., “How many grid squares cover this surface?”), supporting students in treating area as coverage rather than boundary length.

Students then derived the cube surface-area pattern. In the teaching experiment, students recognized more quickly that all six faces are congruent and proposed multiplying the area of one face by six, yielding $6s^2$. When working with the cuboid net, some students initially multiplied all three dimensions (a volume-like structure), reflecting ongoing confusion between surface area and volume. Through guided reinvention, students identified three pairs of congruent faces (lw , lh , wh) and formulated $2(lw + lh + wh)$.

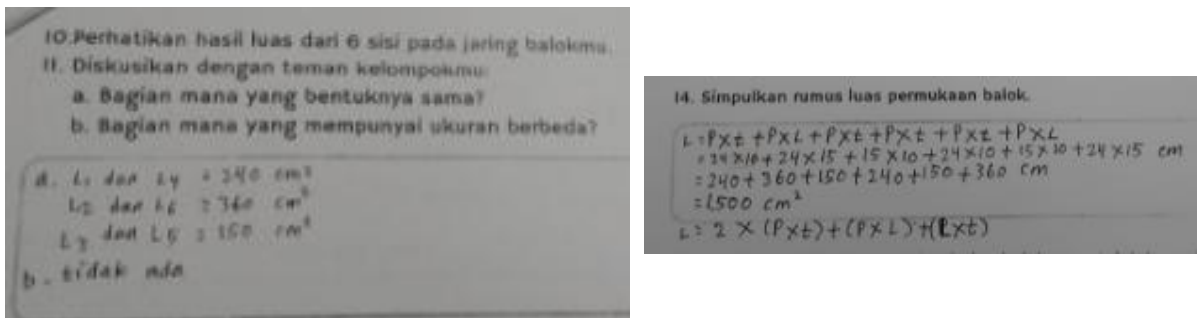


Figure 5a. Students identifying paired faces on nets to derive surface-area formulas (Original student work written in Indonesian)

- | | |
|---|---|
| <p>10. Observe the areas of the six faces in your cuboid net.</p> <p>11. Discuss with your group members:</p> <p>a. which faces have the same shape?</p> <p>b. which have different dimensions?</p> <p>a. L_1 and $L_4 = 240 \text{ cm}^2$
 L_2 and $L_6 = 360 \text{ cm}^2$
 L_3 and $L_5 = 150 \text{ cm}^2$</p> <p>b. None</p> | <p>14. Formulate a conclusion regarding the formula for the surface area of a cuboid</p> $L = l \times h + l \times w + l \times h + l \times h + l \times h + p \times w$ $= 24 \times 10 + 24 \times 15 + 15 \times 10 + 24 \times 10 + 15 \times 10 + 24 \times 15 \text{ cm}$ $= 240 + 360 + 150 + 240 + 150 + 360 \text{ cm}$ $= 1.500 \text{ cm}^2$ $L = 2 \times (l \times h) + (l \times w) + (w \times h)$ |
|---|---|

Figure 5b. Students identifying paired faces on nets to derive surface-area formulas (English-translated and annotated version)

HLT-ALT note (Activity 4)

The HLT anticipated pattern-based reinvention of surface-area formulas through nets. The ALT confirmed this trajectory but showed that distinguishing area from perimeter remained a critical conceptual hurdle, requiring carefully constrained teacher prompts rather than procedural instruction.

Key learning shift (Activity 4)

Students transitioned from procedural errors toward structural reasoning about surface area as the sum of all faces, indicating strengthened spatial reasoning.

Activity 5: Miniature artefact project and presentation

The fifth activity served as an application and consolidation phase. Students used their nets to construct miniature museum artefacts using ice-cream sticks, translating two-dimensional measurements into three-dimensional structures. This activity required proportional reasoning, measurement precision, and the ability to coordinate representations. In the pilot experiment, some students relied on visual estimation when cutting sticks or did not adjust stick lengths according to net measurements, resulting in disproportional models. In the teaching experiment, students more consistently referred back to net dimensions, re-measured before cutting, and verified fit during construction. Teacher scaffolding remained light but strategic, for example, asking students how they knew a stick length matched a net edge. Students' responses increasingly referenced the net structure as a guide, indicating that the 2D-3D correspondence had become more stable.



Figure 6. Student-constructed miniature artefacts and group presentations

During presentations, students linked surface-area understanding to construction decisions. For instance:

S1 : *“This face is the biggest, Sir, because its length and width were the largest on the net”*

Although explanations were sometimes informal, students demonstrated improved spatial communication by connecting calculations to physical construction. Discussions about scaling also suggested emerging proportional reasoning (e.g., recognizing that “*bigger*” objects require “*more covering*”), which indicates continued development of spatial reasoning beyond formula use.

HLT-ALT note (Activity 5)

The HLT anticipated that the miniature project would consolidate learning by connecting nets, measurements, and surface structure. The ALT revealed that students' success depended on

stabilizing the link between net dimensions and physical construction, which improved after the strengthened net-validation phase.

Key learning shift (Activity 5)

Students integrated surface-area concepts into physical model construction and verbal explanation, reflecting coordinated development across visualization, reasoning, and communication.

Synthesis: From HLT to ALT and validated LT

Across the five activities, the ALT revealed refinements to the initial HLT, particularly the need for stronger early scaffolding and the strategic use of AR during critical visualization phases. These refinements were synthesized into a validated LT emphasizing five essential conceptual phases rather than operational steps. This synthesis is summarized in the LIT table presented in the subsequent section.

Overall learning trajectory summary

Overall, students' learning trajectories progressed from intuitive visual recognition toward structured geometric reasoning and communication. Museum-based activities provided meaningful contexts that anchored students' interpretations in experientially real objects, while AR functioned as targeted scaffolding during moments when students struggled to coordinate 2D-3D representations. Together, these elements supported the development of spatial literacy and validated the designed learning trajectory.

Discussion

The findings of this study indicate that learning activities grounded in the contextual features of the Sultan Mahmud Badaruddin II Museum, when integrated within a Hypothetical Learning Trajectory (HLT) and the principles of PMRI, effectively supported the development of elementary students' spatial literacy across the domains of spatial visualization, spatial reasoning, and spatial communication. Rather than emphasizing the correct application of formulas, the learning trajectory positioned students' understanding as an evolving process shaped by interaction with meaningful contexts. These findings reinforce the design research perspective that conceptual learning emerges through guided reinvention, in which students progressively reconstruct mathematical ideas through carefully sequenced activities (Gravemeijer & Cobb, 2006; Abrahamson, 2009; Bakker, 2018). In this study, museum artefact functioned as experientially real contexts that anchored abstract geometric ideas in tangible objects, thereby supporting a gradual transition from informal intuition to formal mathematical understanding (Pier et al., 2019; Nathan et al., 2021).

The effectiveness of the museum context can be interpreted through its role as a learning ecology that integrates cultural relevance, embodied interaction, and geometric structure (Lin & Lu, 2024; Xie et al., 2025). Artefacts in the Sultan Mahmud Badaruddin II Museum are not

only culturally meaningful but also structurally rich in cubic and cuboid forms, enabling students to observe, compare and measure authentic three-dimensional objects. This setting supported horizontal mathematization by allowing students to initially conceptualize surface area as “*the outer part that can be seen and touched*” before formalizing it as the accumulation of face areas (Stylianidou & Nardi, 2019; Lazarinis, 2025). Unlike decontextualized textbook problems, the museum environment required students to coordinate perception, physical action, and reflection (Alberto et al., 2022; Kolovou, 2022). In this respect, the findings extend prior museum-based learning studies (Çil et al., 2016)(Chatterjee et al., 2016) by demonstrating that museums can support the systematic development of spatial literacy when embedded within a coherent learning trajectory rather than used as isolated enrichment activities.

Across the learning trajectory, the observed development of students' spatial visualization reflects a shift from perceptual noticing toward invariant-based geometric attributes such as edge equality, face congruence, and proportional relationships, and to coordinate front, top, and side views with three-dimensional forms. Initial difficulties-such as producing perspective drawings when orthogonal views were required-are consistent with well-documented challenges in elementary geometry learning (Wallace et al., 2001; Vander Heyden et al., 2017). Rather than being treated as errors to be eliminated, these difficulties served as productive starting points for instructional scaffolding. Through guided observation, comparison of artefacts, and discussion, students refined their ways of seeing, indicating a conceptual reorganization of spatial visualization.

Students' spatial reasoning developed through activities involving measurement, net construction, and pattern generalization. During the pilot phase, some students relied on surface-level strategies, such as perimeter counting or visual estimation, which reflected incomplete coordination of representations. During the teaching experiment, however, students increasingly integrated measurements, nets, and symbolic expressions, enabling them to generalize surface area formulas for cubes and cuboids. This progression supports theoretical arguments that meaningful geometric understanding arises when learners reconstruct relationships across multiple representations rather than engage in isolated procedural practice (Rau et al., 2014; Hankeln & Prediger, 2025). From a design research perspective, these shifts highlight how learning trajectories can be used to anticipate and respond to students' representational challenges over time.

Spatial communication emerged as a central mechanism linking visualization and reasoning. Context-driven prompts encouraged students to articulate their ideas using everyday language before transitioning to formal mathematical terminology. This progression aligns with PMRI's principle of progressive formalization, in which informal explanations function as bridges toward formal mathematical discourse (Zulkardi, 2002; Karadag, 2017). Through project presentations, students increasingly justified their decisions by explicitly referring to surface-area calculations and dimensional relationships, indicating a strengthening of mathematical argumentation and communicative competence.

The role of Augmented Reality (AR) in this study warrants critical consideration. AR was not positioned as a primary instructional medium nor intended to replace hands-on exploration. Instead, it functioned as a targeted visualization scaffolding during moments when students

struggled to coordinate two-dimensional and three-dimensional representations, particularly in validating net structures. Its effectiveness depended on prior manual construction and teacher mediation; without these elements, AR alone would risk reinforcing visual dependence rather than conceptual understanding. This finding supports research suggesting that digital tools are most pedagogically effective when used selectively to complement embodied learning experiences rather than substitute them (Gilligan-Lee et al., 2023). At the same time, the results caution against uncritical adoption of AR, emphasizing that its educational value lies in *how and when* it is integrated within a learning trajectory.

Retrospective analysis further revealed that the initial six operational steps of the HLT were conceptually consolidated into five essential phase in the validated learning trajectory. This refinement reflects a shift from activity-level design toward principle-level theorization, which is a central objective of design research (Simon et al., 2018; Brubaker & Cash, 2025). To make this theorization explicit and transferable, the synthesis of the HLT and ALT was abstracted into a Local Instructional Theory (LIT), as summarized in Table 2. The LIT captures not only the sequence of learning phases but also the instructional focus, anticipated student thinking, and associated spatial literacy domains, thereby providing a theoretically grounded framework that can inform future instructional design.

Table 2. Local instructional theory (LIT) for learning surface area of cubes and cuboids through a museum context

LIT phase	Instructional focus	Core learning activities	Anticipated student thinking	Spatial literacy domains
Contextual orientation	Activating geometric meaning in real contexts	Observing museum artefacts; guided comparison of shapes	Informal recognition shifts toward attention to geometric attributes	Visualization; Communication
Observation and measurement	Establishing dimensional structure	Measuring length, width, and height; identifying faces, edges, and vertices	Developing relational understanding of dimensions	Visualization; Reasoning
Net exploration (with AR scaffolding)	Coordinating 2D-3D representations	Constructing and validating nets; conditional AR support	From trial-and-error to intentional face pairing	Visualization; Reasoning; Communication
Structural reasoning	Generalizing surface-area patterns	Identifying face areas; pairing congruent faces; deriving formulas	Structural reasoning replaces perimeter-based reasoning	Reasoning
Application and communication	Consolidating concepts through construction	Building miniatures; presenting and justifying decisions	Linking calculations to physical construction and explanation	Visualization; Communication

Several limitations of the study should be acknowledged. The research involved a relatively small sample drawn from a single local context, which limits direct generalization. Moreover, the cultural specificity of the museum setting suggests that further studies are needed to examine how the proposed LIT functions in different cultural or institutional environments. As is characteristic of design research, the aim of this study was not statistical generalization but theoretical transferability, which should be explored through replication and adaptation in other contexts (Maxwell, 2021). In addition, the use of AR was limited to specific phases of the learning trajectory; future research could investigate longitudinal implementations and alternative forms of digital scaffolding.

Despite these limitations, the findings offer important implications for international mathematics education. They suggest that persistent difficulties in three-dimensional geometry can be addressed by designing coherent learning trajectories grounded in authentic contexts, supported by conditional digital scaffolding, and oriented toward the development of spatial literacy rather than formula memorization. More broadly, this study demonstrates how design research can generate locally grounded yet theoretically transferable instructional knowledge, contributing to the advancement of geometry education across diverse educational systems.

Conclusion

This study demonstrates that the Sultan Mahmud Badaruddin II Museum provides a productive and meaningful context for tracing and supporting elementary students' learning trajectories on the surface area of cubes and cuboids. By integrating museum-based activities within the PMRI and Project-Based Learning (PjBL) frameworks, the designed learning trajectory enabled students to progressively develop spatial literacy across the domains of spatial visualization, spatial reasoning, and spatial communication.

The findings show that students were able to move beyond formula memorization toward conceptual understanding through structured engagement with real artefacts, net construction, and application in miniature-building tasks. Through retrospective analysis, the synthesis of the Hypothetical Learning Trajectory (HLT) and the Actual Learning Trajectory (ALT) resulted in a validated Learning Trajectory that was subsequently abstracted into a Local Instructional Theory (LIT) for teaching surface area in elementary geometry. This LIT represents a theoretically grounded and empirically informed contribution to design research in mathematics education, particularly in the domain of three-dimensional geometry learning.

From a practical perspective, this study suggests that elementary geometry instruction can be strengthened by situating learning in authentic cultural contexts and by designing coherent learning trajectories that foreground spatial literacy rather than procedural calculation alone. Teachers may support students' understanding of three-dimensional concepts by incorporating contextual observation, manipulation of real objects, and representation-based activities such as drawing views, constructing nets, and comparing surface structures. The findings also indicate that digital tools, such as Augmented Reality (AR), can be pedagogically valuable when used selectively as targeted scaffolding during critical visualization phases, rather than as stand-alone instructional media.

In terms of future research, further studies are needed to examine the scalability and adaptability of the proposed Local Instructional Theory across different schools, regions, and cultural contexts. Longitudinal investigation could explore how sustained exposure to context-rich learning trajectories influences the development of students' spatial literacy over time. Additionally, future research may extend the proposed LIT to other geometry topics, such as volume, similarity, or geometric transformations to investigate its broader applicability within elementary and middle school mathematics curricula.

Acknowledgment

The authors sincerely acknowledges the mathematics teacher and fifth-grade students for their active participation and meaningful engagement during the classroom implementation. Appreciation is also extended to the partnering elementary school for their permission and cooperation throughout the research process. The author wishes to thank the experts who contributed valuable insights during the Focus Group Discussion (FGD), which significantly enriched the quality and refinement of the learning design.

Declarations

- Conflicts of Interest : The authors declare no conflict of interest.
- Generative AI Statement : No Generative AI Used:
No generative artificial intelligence (AI) tools were employed in any aspect of this research or the preparation of the manuscript.
AI Used for Limited, Non-Substantive Support:
Generative AI tools, such as grammarly and Microsoft Copilot, were employed solely for language editing and minor phrasing enhancements. All conceptualization, analysis, and scholarly content were independently developed and verified by the authors.
- Funding Statement : The authors gratefully acknowledge Universitas PGRI Palembang for its academic support throughout the research process. This study was supported by the 2025 Domestic Doctoral Completion Scholarship awarded by the Center for Higher Education Funding and Assessment (PPAPT), Ministry of Higher Education, Science and Technology of the Republic of Indonesia.
- Author Contributions : **Dina Octaria:** Conceptualization, investigation, writing – original draft, editing, and visualization; **Zulkardi:** Supervision, formal analysis, and methodology; **Ratu Ilma Indra Putri:** Writing – review & editing, validation. **Cecil Hiltrimartin:** Resources and project administration.

References

- Abrahamson, D. (2009). Embodied design: Constructing means for constructing meaning. *Educational Studies in Mathematics*, 70(1), 27–47. <https://doi.org/10.1007/s10649-008-9137-1>
- Alberto, R., Shvarts, A., Drijvers, P., & Bakker, A. (2022). Action-based embodied design for mathematics learning: A decade of variations on a theme. *International Journal of Child-Computer Interaction*, 32. <https://doi.org/10.1016/j.ijcci.2021.100419>
- Bakait, M., Tajudin, N. M., Masri, R., & Awi, M. (2021). Development of SPARE method by integrating the project-based learning and realistic mathematics education for mathematics learning. *Review of International Geographical Education Online*, 11(4), 918–927. <https://doi.org/10.33403/rigeo.8006806>
- Bakker, A. (2018). Design research in education. In *Design research in Education: A practical guide for early career researchers* (pp. 3–22).
- Brubaker, E. R., & Cash, P. J. (2025). Theorizing in design research: A systematic review of theory development. *Proceedings of the ASME Design Engineering Technical Conference*, 4. <https://doi.org/10.1115/DETC2025-167743>
- Chatterjee, H. J., Hannan, L., & Thomson, L. (2016). An introduction to object-based learning and multisensory engagement. In *Engaging the Senses: Object-Based Learning in Higher Education* (pp. 1–18). <https://doi.org/10.4324/9781315579641-5>
- Chiphambo, S. M., & Mtsi, N. (2021). Exploring grade 8 students' errors when learning about the surface area of prisms. *Eurasia Journal of Mathematics, Science and Technology Education*, 17(8), 1–10. <https://doi.org/10.29333/EJMSTE/10994>
- Çil, E., Maccario, N., & Yanmaz, D. (2016). Design, implementation and evaluation of innovative science teaching strategies for non-formal learning in a natural history museum. *Research in Science and Technological Education*, 34(3), 325–341. <https://doi.org/10.1080/02635143.2016.1222360>
- De Lange, J. (2003). Mathematics for literacy. *Quantitative Literacy: Why Numeracy Matters for Schools and Colleges*, February, 75–90.
- El Bedewy, S., Lavicza, Z., & Lyublinskaya, I. (2024). STEAM practices connecting mathematics, arts, architecture, culture and history in a non-formal learning environment of a museum. *Journal of Mathematics and the Arts*, 18(1–2), 101–134. <https://doi.org/10.1080/17513472.2024.2321563>
- Gilligan-Lee, K. A., Hawes, Z. C. K., Williams, A. Y., Farran, E. K., & Mix, K. S. (2023). Hands-On: Investigating the role of physical manipulatives in spatial training. *Child Development*, 94(5), 1205–1221. <https://doi.org/10.1111/cdev.13963>
- Gravemeijer, K., & Cobb, P. (2006). Design research from a learning design perspective. In *In Educational Design Research* (pp. 29–63). Routledge.
- Ha, O., & Fang, N. (2018). Interactive virtual and physical manipulatives for improving students' spatial skills. *Journal of Educational Computing Research*, 55(8), 1088–1110. <https://doi.org/10.1177/0735633117697730>
- Hankeln, C., & Prediger, S. (2025). Language is essential for avoiding surface translations: Associations of students' spontaneous use of meaning-related phrases for explicating structures with conceptual understanding of multiplication. *Educational Studies in Mathematics*, 120(1), 57–79. <https://doi.org/10.1007/s10649-025-10414-z>
- Herrera, L. M. M., Ordóñez, S. J., & Ruiz-Loza, S. (2024). Enhancing mathematical education with spatial visualization tools. *Frontiers in Education*, 9(1229126). <https://doi.org/https://doi.org/10.3389/feduc.2024.1229126>
- Hussein, Y. F., & Csikos, C. (2023). The effect of teaching conceptual knowledge on students' achievement, anxiety about, and attitude toward mathematics. *Eurasia Journal of*

- Mathematics, Science and Technology Education*, 19(2).
<https://doi.org/10.29333/ejmste/12938>
- Imami, A. I., & Wafa, N. (2023). Students' error in the Van Hiele levels of geometric thinking. *AIP Conference Proceedings*, 2706. <https://doi.org/10.1063/5.0120258>
- Juniati, D., & Siswono, T. Y. E. (2019). Examining prospective teachers' belief and pedagogical content knowledge towards teaching practice in mathematics class: A case study. *Journal on Mathematics Education*, 10(2), 185–202. <https://doi.org/10.22342/jme.10.2.7326.185-202>
- Karadag, Z. (2017). Euclidean exploration of geometry in islamic art. In *Transdisciplinarity in Mathematics Education: Blurring Disciplinary Boundaries* (pp. 3–24). https://doi.org/10.1007/978-3-319-63624-5_1
- Kim, H.-J. (2020). Concreteness fading strategy: A promising and sustainable instructional model in mathematics classrooms. *Sustainability (Switzerland)*, 12(6), 1–18. <https://doi.org/10.3390/su12062211>
- Kolovou, M. (2022). In search of assessment shifts in embodied learning science research: A review. *Journal of Science Education and Technology*, 31(2), 246–257. <https://doi.org/10.1007/s10956-021-09952-x>
- Lazarinis, F. (2025). Embedding math problems in cultural city tours to increase student engagement and inclusion. *Education Sciences*, 15(12). <https://doi.org/10.3390/educsci15121683>
- Lehmann, T. (2023). Learning to calculate surface area: A focus on strategy choice. *Research in Mathematics Education*, 25(3), 301–322. <https://doi.org/10.1080/14794802.2022.2081991>
- Lin, L., & Lu, L. (2024). Research on the design of multisensory interactive experiences in museums based on embodied cognition. *Communications in Computer and Information Science*, 2119 CCIS, 204–211. https://doi.org/10.1007/978-3-031-61966-3_23
- Maxwell, J. A. (2021). Why qualitative methods are necessary for generalization. *Qualitative Psychology*, 8(1), 111–118. <https://doi.org/10.1037/qup0000173>
- Medina Herrera, L., Castro Pérez, J., & Juárez Ordóñez, S. (2019). Developing spatial mathematical skills through 3D tools: Augmented reality, virtual environments and 3D printing. *International Journal on Interactive Design and Manufacturing*, 13(4), 1385–1399. <https://doi.org/10.1007/s12008-019-00595-2>
- Moore-Russo, D., Viglietti, J. M., Chiu, M. M., & Bateman, S. M. (2013). Teachers' spatial literacy as visualization, reasoning, and communication. *Teaching and Teacher Education*, 29(1), 97–109. <https://doi.org/10.1016/j.tate.2012.08.012>
- Nanga, M. Y., & Hurit, R. U. (2020). Analysis of spatial ability of class VIII students Institute Indonesia Yogyakarta Problem Based Learning on topic of cuboids and cubes. *Journal of Physics: Conference Series*, 1470(1). <https://doi.org/10.1088/1742-6596/1470/1/012059>
- Nathan, M. J., Schenck, K. E., Vinsonhaler, R., Michaelis, J. E., Swart, M. I., & Walkington, C. (2021). Embodied geometric reasoning: Dynamic gestures during intuition, insight, and proof. *Journal of Educational Psychology*, 113(5), 929–948. <https://doi.org/10.1037/edu0000638>
- Octaria, D., Zulkardi, Putri, R. I. I., & Hiltrimartin, C. (2025). Ethnomathematical insights from the geometric architecture of the Sultan Mahmud Badaruddin II museum. *Mosharafa: Jurnal Pendidikan Matematika*, 14(1), 1–22. <https://doi.org/https://doi.org/10.31980/mosharafa.v14i1.3008>
- Palupi, R., Mikanda, T. S., Gufitaningrum, G. R., Cahyaningrum, F. I. N., Dani, A. H. R., Rejeki, S., Marsudi, S., & Kartini, N. H. (2025). Contexts in the content of geometry: A review of ethnomathematics studies. *AIP Conference Proceedings*, 3142(1).

- <https://doi.org/10.1063/5.0262163>
- Pier, E. L., Walkington, C., Clinton, V., Boncoddò, R., Williams-Pierce, C., Alibali, M. W., & Nathan, M. J. (2019). Embodied truths: How dynamic gestures and speech contribute to mathematical proof practices. *Contemporary Educational Psychology*, 58, 44–57. <https://doi.org/10.1016/j.cedpsych.2019.01.012>
- Putri, D. U., & Nurlaelah, E. (2022). Analysis of student errors in answering geometry problem based on Newman Theory in term of Van Hiele level. *AIP Conference Proceedings*, 2566. <https://doi.org/10.1063/5.0117129>
- Rahayu, P., & Putri, R. I. I. (2021). Project-based mathematics learning: Fruit salad recipes in junior high school. *Journal on Mathematics Education*, 12(1), 181–198. <https://doi.org/https://doi.org/10.22342/jme.12.1.13270.181-198>
- Ramful, A., Lowrie, T., & Logan, T. (2017). Measurement of spatial ability: Construction and validation of the spatial reasoning instrument for middle school students. *Journal of Psychoeducational Assessment*, 35(7), 709–727. <https://doi.org/https://doi.org/10.1177/0734282916659207>
- Rau, M. A., Alevén, V., & Rummel, N. (2014). Sequencing sense-making and fluency-building support for connection making between multiple graphical representations. *Proceedings of International Conference of the Learning Sciences, ICLS*, 2(January), 977–981. <https://www.scopus.com/inward/record.uri?eid=2-s2.0-84937706493&partnerID=40&md5=455f8b6267ef6d2ae3af5b7a3e03e9b0>
- Rehman, N., Huang, X., & Mahmood, A. (2025). Enhancing mathematical problem-solving and 21st-century skills through PjBL: A structural equation modelling approach. *Educational Studies*. <https://doi.org/10.1080/03055698.2025.2514691>
- Sahrudin, A., & Budiarto, M. T. (2022). Epistemic action of junior high school students with low spatial ability in constructing cube nets. *International Journal of Educational Methodology*, 8(2), 221–230. <https://doi.org/10.12973/ijem.8.2.221>
- Simon, M. A., Kara, M., Placa, N., & Avitzur, A. (2018). Towards an integrated theory of mathematics conceptual learning and instructional design: The Learning Through Activity theoretical framework. *Journal of Mathematical Behavior*, 52, 95–112. <https://doi.org/10.1016/j.jmathb.2018.04.002>
- Stylianidou, A., & Nardi, E. (2019). Tactile construction of mathematical meaning: benefits for visually impaired and sighted pupils. *Proceedings of the International Group for the Psychology of Mathematics Education*, 3, 343–350. <https://www.scopus.com/inward/record.uri?eid=2-s2.0-85143507557&partnerID=40&md5=9031f418e0f00488b885ca68aeb9c816>
- Sudirman, Kusumah, Y. S., & Martadiputra, B. A. P. (2023). Evaluation design for 3D geometry learning using augmented reality. *AIP Conference Proceedings*, 2734(1). <https://doi.org/10.1063/5.0156052>
- Sung, Y.-T., Shih, P.-C., & Chang, K.-E. (2015). The effects of 3D-representation instruction on composite-solid surface-area learning for elementary school students. *Instructional Science*, 43(1), 115–145. <https://doi.org/10.1007/s11251-014-9331-8>
- Tan Sisman, G., & Aksu, M. (2016). A Study on sixth grade students' misconceptions and errors in spatial measurement: Length, area, and volume. *International Journal of Science and Mathematics Education*, 14(7), 1293–1319. <https://doi.org/10.1007/s10763-015-9642-5>
- Tresnawati, N., Saleh, I., & Wardani, S. (2021). Science batik Ciwaringin: The implementation of Ethno-STEM PjBL model in learning biotechnology at PGSD students. *Journal of Physics: Conference Series*, 1842(1). <https://doi.org/10.1088/1742-6596/1842/1/012063>
- Vander Heyden, K. M., Huizinga, M., Raijmakers, M. E. J., & Jolles, J. (2017). Children's representations of another person's spatial perspective: Different strategies for different

- viewpoints? *Journal of Experimental Child Psychology*, 153, 57–73. <https://doi.org/10.1016/j.jecp.2016.09.001>
- Wallace, J. R., Allan, K. L., & Tribol, C. T. (2001). Spatial perspective-taking errors in children. *Perceptual and Motor Skills*, 92(3 PART 1), 633–639. <https://doi.org/10.2466/pms.2001.92.3.633>
- Widjaja, W., & Vale, C. (2021). Counterexamples: Challenges faced by elementary students when testing a conjecture about the relationship between perimeter and area. *Journal on Mathematics Education*, 12(3), 487–506. <https://doi.org/10.22342/jme.12.3.14526.487-506>
- Winarti, D. W., Amin, S. M., Lukito, A., & Van Gallen, F. (2012). Learning the concept of area and perimeter by exploring their relation. *Journal on Mathematics Education*, 3(1), 41–54. <https://doi.org/10.22342/jme.3.1.616.41-54>
- Xie, Y., Shen, M., & Furuya, N. (2025). Genotypes and spatial configurations of public art museums in Japan. *Journal of Asian Architecture and Building Engineering*. <https://doi.org/10.1080/13467581.2025.2526197>
- Zulkardi. (2002). *Developing a learning environment on realistic mathematics education for Indonesian students teachers*. University of Twente, Enschede.