Jurnal Elemen, 12(1), 263-276, January 2026
https:/doi.org/10.29408/jel.v12i1.33325

e- ISSN: 2442 - 4226

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Improving university students' geometrical understanding of derivatives through graphic visualization

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Abstract

This study addresses university students' persistent difficulties in developing geometrical thinking, defined as the ability to interpret slopes, understand tangent behavior, and coordinate graphical and symbolic representations, when learning derivative concepts. The study investigates changes in students' geometrical thinking following instruction supported by graphic visualization. A one-group pretest–posttest quasi-experimental design was conducted with 30 students in mathematics education, limiting causal inference due to the absence of a control group. Data were collected using a six-item open-ended geometrical thinking test and structured classroom observations, which served as triangulation. The results showed significant improvement after the visualization-based instruction. The average pretest score increased from 48.12 to 67.51, with a normalized gain (N-gain) of 0.39, indicating moderate improvement. A paired-sample t-test confirmed a statistically significant difference ($p < 0.001$). Although the effect size was large ($d = 4.17$), this finding should be interpreted cautiously given the study design and sample size. Observation data supported these results by revealing improved graphical reasoning, representational coordination, and student engagement. These findings indicate the potential of graphic visualization to support students' geometrical thinking in derivative learning, rather than to demonstrate definitive instructional effectiveness.

Keywords: calculus; derivative concepts; geometrical thinking; graphic visualization; mathematical representation.

How to cite: Khotimah, Lestari, I., Agus, R. N., Oktaviyanthi, R., Putri, S. P., & Ünver, S. K. (2026). Improving university students' geometrical understanding of derivatives through graphic visualization. *Jurnal Elemen*, 12(1), 263–276. https://doi.org/10.29408/jel.v12i1.33325

Received: 7 December 2025 | Revised: 8 January 2026
Accepted: 22 January 2026 | Published: 19 February 2026

Introduction

The derivative is widely recognized as a foundational concept in calculus because it formalizes the notion of rate of change and serves as a central tool for modelling, optimization, and analysis in scientific, engineering, and economic contexts (Thompson, 2011; Stewart, 2016; Bressoud et al., 2016; Tall & Katz, 2014). Despite its importance, extensive research consistently shows that many university students approach derivatives primarily as procedural rules rather than meaningful mathematical ideas, resulting in limited conceptual and geometric understanding (Thompson, 2011; Bressoud et al., 2016). Previous studies report that students often struggle to interpret the derivative as a slope, connect local rates of change with global graph behavior, and coordinate symbolic, graphical, and contextual representations (García-García & Dolores-Flores, 2021; Zandieh et al., 2023; Zandieh & Rasmussen, 2010). As a consequence, students may perform symbolic procedures correctly but fail to explain their graphical meaning or conceptual implications.

In the Indonesian context, similar difficulties have also been reported, where abstract presentation and symbol-heavy instruction encourage students to rely on memorized formulas rather than conceptual reasoning (Umam, 2024). This evidence indicates that derivative learning challenges are not only global but also locally relevant, reinforcing the need for instructional approaches that strengthen students' conceptual and geometrical understanding.

A central factor underlying these difficulties is the limited development of geometrical thinking. In calculus, geometrical thinking encompasses the ability to interpret slopes, visualize tangent-line behavior, and coordinate algebraic, graphical, and verbal representations when reasoning about derivative concepts. When these representational connections are weak, students may be able to compute derivatives correctly but fail to explain what the derivative indicates about a function's behavior or why a symbolic result is graphically meaningful (García-García & Dolores-Flores, 2021; Nerinckx et al., 2021; Zandieh & Rasmussen, 2010). The use of multiple representations, including graphs, diagrams, and symbols, is considered essential to support higher-order thinking and conceptual integration in mathematics learning (Nerinckx et al., 2021; Schoenherr & Schukajlow, 2023).

Visualization has long been recognized as a powerful tool in mathematics learning (Arcavi, 2003; Schoenherr & Schukajlow, 2023; Khotimah, 2018). Arcavi (2003) emphasized that visualization enables learners to construct meaning, test conjectures, and communicate mathematical ideas, making it particularly relevant for developing geometrical thinking. More recent empirical evidence reinforces this view: visualization tools significantly support conceptual understanding across mathematical domains when aligned with instructional goals (Schoenherr & Schukajlow, 2023; Swidan et al., 2025). A meta-analysis also reported that visual representations improve mathematical achievement, especially when students receive explicit guidance in interpreting visuals (Fiorella & Mayer, 2018).

In calculus education, visualization-supported reasoning has been shown to foster deeper understanding of the derivative. Ozaltun-Celik (2021) documented how students' quantitative reasoning evolved through intensive graph analysis. García-García and Dolores-Flores (2021) demonstrated that tasks involving the sketching of functions and their derivatives reveal how

students connect different representational forms. At a technological level, dynamic geometry systems can foreground geometric meaning and support reasoning about multiple facets of the derivative concept.

Recent research has expanded these visualization approaches by exploring emerging technologies. For example, augmented reality (AR) environments have been investigated for their potential to help learners explore function–derivative relationships through embodied interaction and real-time graphical feedback, offering rich visual and spatial experiences that may support conceptual meaning-making (Swidan et al., 2025; Swidan et al., 2025; Zhang & Lin, 2024). AR has also been documented to enhance student engagement, motivation, and interaction with mathematical structures, further illustrating its potential value in complex domains such as calculus (Ibáñez & Delgado-Kloos, 2018; Qomario et al., 2022). Technology-supported visualization tools such as GeoGebra have also been shown to support students' mathematical literacy and conceptual understanding (Khotimah, 2018).

Although visualization has been widely studied, relatively few investigations conceptualize geometrical thinking as an integrated construct that simultaneously includes gradient visualization, derivative interpretation, and representation coordination (Swidan et al., 2025). Most existing studies examine these components separately, making it difficult to characterize students' holistic geometrical reasoning development. Furthermore, limited research employs hierarchical visualization tasks that systematically guide students from graphical observation to conceptual explanation. This study addresses these gaps by integrating explicit geometrical thinking indicators within a structured, classroom-feasible visualization-based instructional design.

For preservice mathematics teachers, strong geometrical thinking is especially critical, as it supports clear visual explanations and helps prevent the transmission of graph-based misconceptions to future students (Umam, 2024). Strengthening geometrical thinking during teacher preparation is therefore essential for improving the quality of calculus instruction.

Responding to these challenges, the present study investigates the effectiveness of a graphic-visualization-based instructional approach in improving mathematics education students' geometrical thinking about derivatives. The intervention emphasizes three core components: visualizing gradients and slopes on function graphs, interpreting derivatives as rates of change, and coordinating algebraic, graphical, and contextual representations.

Accordingly, this study addresses the following research question: How effective is graphic visualization-based instruction in improving students' geometrical thinking, as indicated by their ability to visualize gradients, interpret derivative meaning, and coordinate algebraic, graphical, and contextual representations?

Methods

This study employed a quantitative approach using a one-group pretest–posttest quasi-experimental design to examine the effectiveness of graphic visualization-based instruction in enhancing students' geometrical thinking on the concept of derivatives. This design was selected because it allows for the measurement of changes in learning outcomes before and

after the intervention within the same group of participants, making it appropriate for classroom-based instructional research where random assignment is not feasible (Uygun & Cesur, 2024).

Although this design allows the measurement of learning changes within the same participants, it is subject to internal validity threats such as testing effects and maturation. To minimize these threats, the intervention was implemented within a short instructional period, and instructional activities were consistently aligned with the study objectives.

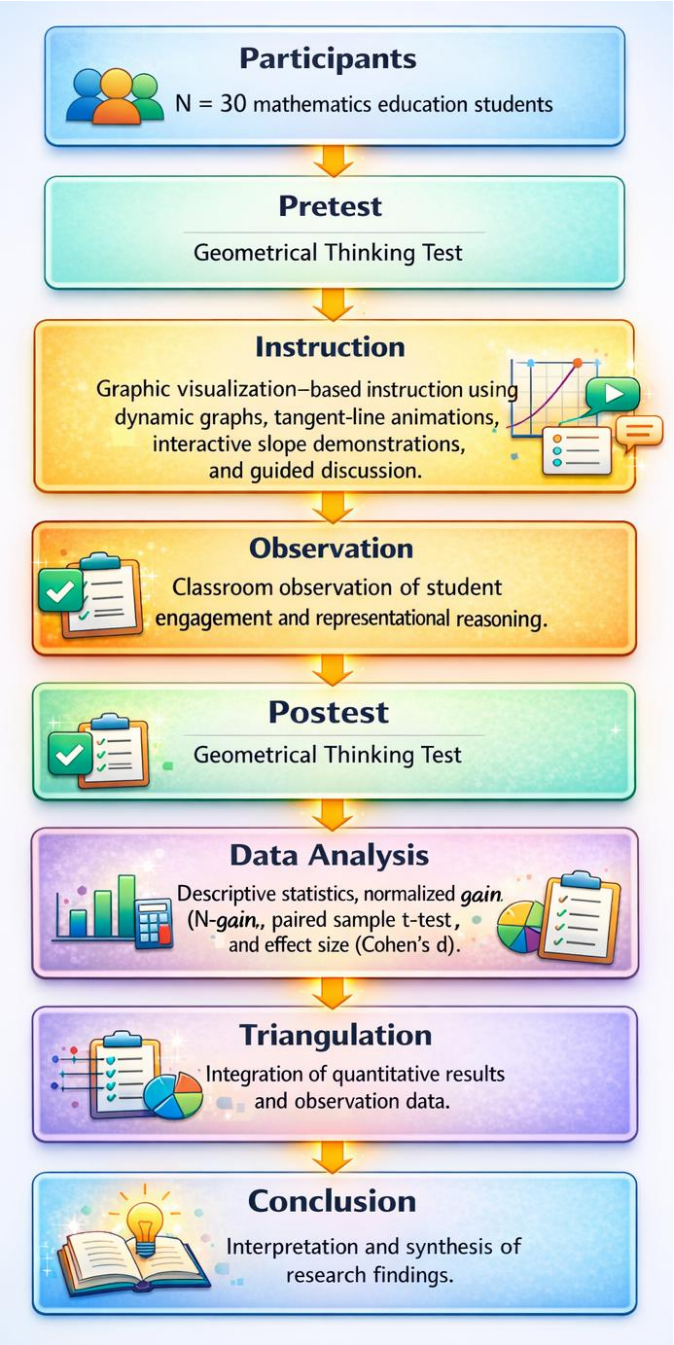

Figure 1. Research flow diagram of the one-group pretest–posttest quasi-experimental design with observational triangulation

The participants were 30 undergraduate students enrolled in a Calculus course in a mathematics education program at a private university in Indonesia. The sampling technique used was purposive sampling, with the primary consideration that participants represented a homogeneous academic background and had previously studied basic functions and algebra. Purposive sampling is commonly used in educational research to ensure coherence between participant characteristics and the research objectives (Robinson, 2024).

Three instruments were employed in this study. The primary instrument was a Geometrical Thinking Test administered as both pretest and posttest, consisting of six open-ended items designed to measure students' geometrical thinking related to derivative concepts. The items assessed visualization of gradients, interpretation of the derivative at a point, coordination between algebraic and graphical representations, analysis of derivative graphs, geometric interpretation of the limit definition of the derivative, and transformation between symbolic and graphical forms. Each item was scored using a 0–4 rubric emphasizing conceptual accuracy, representational reasoning, and clarity of explanation, yielding a maximum score of 24.

Table 1. Scoring rubric

Score	Criteria
4	Accurate sketch, correct interpretation, clear explanation, and proper representational coordination.
3	Minor inaccuracies but conceptual understanding is evident.
2	Partial understanding with limited explanation.
1	Incorrect interpretation with minimal explanation.
0	No response or fundamental misunderstanding.

The same Geometrical Thinking Test items were used in both the pretest and posttest. To reduce potential recall bias, the order of items was rearranged, and students were not informed that identical items would appear in the posttest. The open-ended nature of the items required conceptual explanation rather than simple recall.

A function $f(x)$ is shown in the graph below.

a) Sketch the tangent line at $x = 2$.
b) Estimate the slope of the tangent line and explain its meaning in relation to the function's behavior.
c) Describe how the slope changes as x increases.

Figure 2. Sample geometrical thinking test item

Content validity was ensured through expert review by two calculus lecturers and one mathematics education specialist. Reliability analysis yielded a Cronbach's alpha coefficient of 0.87, indicating high internal consistency. In addition to the test, an observation sheet was used to document students' engagement during the intervention, capturing their attention to visual explanations, participation in graph-based discussions, and exploration of visualization tasks through a structured checklist for consistency. In addition to the research instruments, the study employed instructional learning materials incorporating graphic visualization tools,

including dynamic graphs, tangent-line animations, and interactive slope demonstrations. These materials were used to support the instructional intervention and were designed to align with the geometrical thinking indicators and conceptual objectives of the study.

Table 2. Examples of instrument items before and after content validation

Stage	Item Description
Before validation	Draw the derivative graph of the function and explain it.
After validation	Draw the derivative graph of the function and explain how the slope of the original function changes across different intervals.
Revision rationale: The item was refined to emphasize geometrical interpretation of slope variation and representational coordination.	

The study was conducted over three instructional sessions during the derivatives unit, and the graphic visualization–based intervention was implemented across all three sessions. Each session lasted approximately 90 minutes and followed a structured instructional focus. In the first session, students explored the relationship between function graphs and slopes by observing how gradients change along a curve using dynamic graphs. The instruction emphasized interpreting slope as a local rate of change and identifying increasing and decreasing intervals visually.

In the second session, students focused on tangent-line behavior and derivative interpretation at a point. Through tangent-line animations, students examined how the derivative represents instantaneous rate of change and related symbolic derivative values to graphical features of the function. In the third session, students coordinated multiple representations by connecting algebraic derivative expressions with derivative graphs and predicting function behavior based on derivative information. Guided discussions were conducted to strengthen representational integration and conceptual reasoning.

Students completed a pretest before the first session and a posttest after the third session using the Geometrical Thinking Test. Structured classroom observations were conducted throughout all sessions to document student engagement and representational reasoning, providing qualitative support for the quantitative findings.

Data were analyzed using both descriptive and inferential statistical techniques. Descriptive statistics, including mean, standard deviation, and score distribution, were used to summarize pretest and posttest performance. Raw scores from the six-item test (0–24) were converted to a 100-point scale and categorized into low (0–33), moderate (34–66), and high (67–100) levels of geometrical thinking, based on commonly applied interpretation standards in instructional effectiveness research (Marufi et al., 2021; Rifki et al., 2023). To measure the magnitude of improvement, normalized gain (N-gain) was computed, following common practice in instructional effectiveness research to quantify conceptual change relative to the maximum possible gain (Marufi et al., 2021). Inferential analysis employed a paired sample t-test to determine whether differences between pretest and posttest scores were statistically significant, accompanied by the calculation of Cohen's *d* to assess practical effect size, in accordance with methodological recommendations for quasi-experimental designs involving repeated measures with the same participants (Rifki et al., 2023). Finally, observation data were

analyzed to triangulate the quantitative findings, particularly to illuminate how students engaged with visualization tools and how these engagements supported their reasoning about derivative concepts.

Results

Descriptive statistics of pretest and posttest scores

Before the intervention, students' geometrical thinking levels were generally classified as low to moderate. After the graphic visualization–based instruction, students demonstrated a consistent increase in their test scores. The rise in the posttest mean score indicates a notable improvement in their ability to visualize gradients, interpret derivatives, and coordinate symbolic-graphical representations.

Table 3. Descriptive statistics of pretest and posttest scores (N = 30)

Statistic	Pretest	Posttest
Mean	48.12	67.51
Standard Deviation	9.61	9.42
Raw Score (0–24)	11.55	16.20
Minimum Score	30.87	45.36
Maximum Score	65.79	86.84

The mean score increased by 19.39 points, while standard deviations remained comparable, indicating stable score dispersion before and after the intervention. All students obtained higher posttest scores, suggesting uniform improvement across participants.

Normalized-gain analysis

Normalized gain (N-gain) was calculated to examine the magnitude of improvement relative to the maximum possible gain.

Table 4. Summary of students' N-gain scores

N-gain Statistic	Value
Mean N-gain	0.39
Category	Moderate
Minimum N-gain	0.18
Maximum N-gain	0.61

Approximately 70% of students achieved moderate N-gain levels, while the remaining students were in the low–moderate range. No negative gains were observed, indicating that all students benefited from the instructional intervention.

Paired sample t-test

Prior to inferential testing, normality of difference scores was examined using the Shapiro–Wilk test and indicated normal distribution ($p > .05$). Therefore, a paired-sample t-test was considered appropriate.

Table 5. Results of paired sample t-test

Parameter	Value
t-value	22.82
Degrees of Freedom (df)	29
p-value	< 0.001
Effect Size (d)	4.17

The results revealed a statistically significant improvement in students' geometrical thinking scores. Although the effect size was very large, this finding should be interpreted cautiously due to the one-group design and the focused instructional context.

Changes in geometrical thinking categories

Students' raw scores (0–24) were classified into three categories: Low (0–8), Moderate (9–16), and High (17–24).

Table 6. Distribution of geometrical thinking categories before and after intervention

Category	Pretest (N)	Posttest (N)
Low	4 (13%)	0 (0%)
Moderate	26 (87%)	15 (50%)
High	0 (0%)	15 (50%)

A clear shift toward higher performance levels was observed. Half of the students reached the high category after the intervention, and none remained in the low category.

Individual score comparison

To provide a clearer overview of students' progress, the results from Table 5 were grouped into meaningful categories. The recap includes: 1) distribution of raw score increases, 2) transition of category levels, and 3) summary statistics for score improvement.

Table 7. Summary of individual score changes

Range of Score Increase	Number of Students	Percentage
+2 to +4 points	8 students	26.7%
+5 to +7 points	14 students	46.7%
+8 to +10 points	6 students	20.0%
More than +10 points	2 students	6.6%

Most students improved by 5–7 points, indicating moderate conceptual gains, while a smaller group achieved larger improvements, showing stronger benefits from the intervention.

Observation findings supporting quantitative results

Classroom observations revealed clear behavioral changes that supported the quantitative results. Before the intervention, students tended to rely predominantly on algebraic manipulation and frequently hesitated when asked to interpret graphs or relate them to symbolic expressions. Their explanations were generally formula-oriented and showed limited visual reasoning.

During the visualization-based instruction, students increasingly used graphical features such as slope variation, tangent-line behavior, and curvature to justify their reasoning. Many

students referred spontaneously to dynamic graphs when explaining whether a function was increasing or decreasing at particular intervals. This indicates improved representational fluency, which is a key component of geometrical thinking.

Students also demonstrated stronger coordination between symbolic and graphical representations. Instead of relying solely on symbolic conditions such as $f'(x) = 0$, they first anticipated critical points from the graph and then verified them algebraically. This bidirectional reasoning pattern was rarely observed before the intervention.

In addition, student engagement and collaborative reasoning increased noticeably. Learners frequently discussed graphical patterns with peers, pointed to specific visual features, and negotiated their interpretations during group work. Across all sessions, students showed reduced hesitation when interpreting graphs and greater willingness to explore multiple solution paths using visualization tools. These behavioral shifts complement the quantitative findings by showing that graphic visualization not only improved test performance but also transformed how students approached and reasoned about derivative concepts.

Discussion

The findings of this study indicate that graphic visualization–based instruction is associated with meaningful improvement in students' geometrical thinking, particularly in interpreting slopes, understanding tangent behavior, and coordinating symbolic and graphical representations. These results are consistent with previous research emphasizing the role of dynamic visualization tools in mathematics learning. The findings support previous research on dynamic visualization tools for mathematics learning, such as GeoGebra-based instruction for derivative understanding (Nguyen & Greefrath, 2025) and augmented reality environments that facilitate embodied mathematical experiences (Sinclair, 2025). Recent meta-analytic evidence further confirms that visualization interventions significantly enhance conceptual understanding and engagement in mathematics learning contexts (Bock et al., 2023).

The improvement observed is consistent with international research emphasizing the role of visualization in fostering conceptual coherence. Visualization allows students to detect structural relationships within functions and derivatives that often remain implicit in purely symbolic instruction (Fiorella & Mayer, 2022; Bock et al., 2023). In line with García-García and Dolores-Flores (2021), the present study confirms that coordination between a function and its derivative graph plays a critical role in students' conceptual understanding. The observed shift from symbol-dominated reasoning toward graph-based justification indicates a transition from procedural to more connected mathematical thinking, as also reported in recent calculus visualization studies (Zhang & Lin, 2024).

However, not all students benefited to the same extent. Students who demonstrated larger gains tended to show stronger initial engagement with graphical features and greater willingness to explore multiple representations. In contrast, students with smaller gains often continued to rely on symbolic manipulation and required more time to internalize visual–conceptual connections. This variation suggests that prior representational experience, learning disposition, and confidence in visual reasoning may mediate the effectiveness of visualization-

based instruction. These findings align with Clemente et al. (2022) and Zandieh et al. (2023), who emphasize that representational coordination develops progressively and is sensitive to individual cognitive differences.

From a theoretical perspective, the results support visualization as a mechanism for enhancing representational coordination, reducing cognitive load, and promoting dialogic reasoning. By externalizing relationships between slope, tangent behavior, and function shape, visualization makes abstract calculus concepts perceptually accessible and cognitively manageable (Fiorella & Mayer, 2022). Furthermore, visualization provides a shared representational space that facilitates discussion, argumentation, and collective meaning-making, which are essential processes in conceptual learning (Santos-Trigo et al., 2024).

The scalability of this instructional approach is also noteworthy. The intervention relied on relatively simple and accessible visualization tools, such as dynamic graphs and tangent-line animations, rather than advanced technological infrastructure. This suggests that visualization-based instruction can be implemented in larger and more diverse classroom settings, including resource-limited contexts. Similar conclusions have been reported in recent comparative studies on 2D and immersive visualization environments (Zhang & Lin, 2024). Nevertheless, scaling to larger cohorts may require additional instructional scaffolding, such as guided worksheets, peer discussion structures, or digital learning management integration.

For preservice mathematics teachers, the implications are particularly significant. Developing strong geometrical thinking is essential for explaining calculus concepts visually and for addressing students' graph-based misconceptions. The observed improvement indicates that visualization-based instruction may serve as an effective component of teacher preparation programs by strengthening future teachers' representational competence and pedagogical confidence (Şahin & Kendal, 2023).

Despite these contributions, several limitations must be acknowledged. The absence of a control group restricts causal interpretation, and the relatively small and homogeneous sample limits generalizability. Moreover, the intervention focused on two-dimensional visualizations; future studies comparing 2D, 3D, and immersive visualization environments such as VR or AR could provide deeper insight into which visualization features most strongly support conceptual learning (Cheong et al., 2023; Zhang & Lin, 2024). Longitudinal designs are also recommended to examine the sustainability of geometrical thinking development across multiple calculus topics.

Overall, this study contributes theoretically by conceptualizing geometrical thinking as an integrated construct involving gradient visualization, derivative interpretation, and representational coordination. Practically, it demonstrates that accessible graphic visualization strategies can meaningfully support conceptual learning in calculus. Although the findings should be interpreted cautiously, they extend current visualization research and offer a pedagogically grounded pathway for improving calculus instruction in higher education.

Conclusion

This study examined changes in mathematics education students' geometrical thinking following graphic visualization–based instruction on derivative concepts. The findings indicate observable improvements in students' abilities to visualize gradients, interpret tangent-line behavior, and coordinate symbolic and graphical representations. These improvements should be interpreted as indicative learning progress rather than definitive instructional effectiveness due to the one-group pretest–posttest design.

Classroom observations further supported these results by revealing increased reliance on visual reasoning and more active engagement in representational discussions. Together, the quantitative and qualitative findings suggest that graphic visualization can provide meaningful learning support for developing geometrical thinking in calculus contexts.

From a theoretical perspective, this study contributes to calculus education research by conceptualizing geometrical thinking as an integrated construct involving gradient visualization, derivative interpretation, and representational coordination within visualization-supported learning environments. This perspective strengthens the understanding of how representational integration supports conceptual calculus learning.

Despite these encouraging outcomes, the findings should be interpreted cautiously due to design limitations, limited sample size, and short intervention duration. Future research is recommended to employ control-group or longitudinal designs, refine geometrical thinking assessment instruments, and explore the sustainability of learning outcomes across multiple calculus topics and instructional settings.

Overall, this study indicates that well-structured graphic visualization represents a promising instructional approach for supporting geometrical thinking in calculus, particularly in resource-limited contexts. The integration of visualization practices into preservice teacher education may further strengthen future teachers' representational competence and pedagogical preparedness.

Acknowledgment

The authors would like to express their sincere gratitude to the students of the Mathematics Education Department, Universitas Serang Raya, who participated in this study. Their cooperation and engagement were essential to the successful completion of this research. The authors also thank colleagues and reviewers for their constructive feedback, which significantly contributed to the improvement of this manuscript.

Declarations

Conflicts of Interest : The authors declare no conflict of interest regarding the publication of this manuscript. In addition, the authors have completed the ethical issues, including plagiarism, misconduct, data fabrication and/or falsification, double publication and/or submission, and redundancies.

Generative AI Statement : Generative AI tools were used only to improve language clarity and grammar. All scientific content and interpretations remain the full responsibility of the authors.

Funding Statement : This publication was supported by a publication grant for reputable journals for the 2025 fiscal year from the Ministry of Higher Education, Science, and Technology, Republic of Indonesia [Grant Number 087/SPK/C3/DT.05.00/BP/2025]

Author Contributions : **Khotimah:** Conceptualization, methodology, formal analysis, and writing – original draft; **Indri Lestari:** Validation, resources, writing – review & editing, and project administration; **Ria Noviana Agus:** Data curation, investigation, software, and writing – review & editing; **Rina Oktaviyanthi:** Supervision, validation, resources, and writing – review & editing; **Shila Panca Putri:** Visualization, and documentation support; **Semiha Kula Ünver:** Theoretical guidance, supervision, and final manuscript review.

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