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Integrating desmos and *songket* motifs: A PMRI-based learning trajectory for rotation

Puja Teressa Fawensi, Zulkardi, Ely Susanti *, Meryansumayeka

Faculty of Teacher Training and Education, Sriwijaya University, South Sumatra, Indonesia

* Correspondence: ely_susanti@fkip.unsri.ac.id

Abstract

This study is motivated by students' low conceptual understanding and spatial reasoning in rotation, as well as the limited integration of local cultural contexts with digital technology in mathematics learning. It aims to design and validate a PMRI-based Hypothetical Learning Trajectory (HLT) integrating Songket Mak Raje and Desmos to develop students' conceptual understanding and spatial reasoning on rotation. The study employs a design research method of the validation study type, involving 6 students in the pilot experiment and 36 students in the teaching experiment. The main instrument is a Desmos-based E-LAS. Data were collected through observation, interviews, and analysis of students' work documents and analyzed qualitatively using thematic and retrospective approaches. The results show that the developed HLT is valid and effective in developing students' conceptual understanding and spatial reasoning. Retrospective analysis produced a Local Instructional Theory (LIT) as a refinement of the HLT, emphasizing gradual scaffolding, explicit articulation of rotation properties through guided inquiry before formal generalization, and early introduction of non-contextual problems with geometric constraints to foster spatial reasoning. This study implies a model for integrating ethnomathematics and digital technology, a PMRI-based LIT as a guide for instructional design, and a practical and validated Desmos-based E-LAS prototype.

Keywords: desmos; ethnomathematics; learning trajectory; PMRI; rotation; *songket*

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Introduction

The ability to understand concepts and spatial reasoning are very important mathematical competencies for students to master (Adams et al., 2023; Lakin et al., 2024; Lestari et al., 2022). Spatial reasoning functions as a cognitive tool that enables students to form mental representations of mathematical concepts (Harris, 2023), while conceptual understanding provides a logical framework and principles that enrich and guide spatial reasoning. However, various studies reveal that the ability to understand concepts and spatial reasoning is still a challenge for students, including in geometric transformation material such as rotation (Fowler et al., 2022; Husna & Masduki, 2023; Trisna et al., 2022). The research by Sunariah & Mulyana (2020) also highlights that students often have difficulty contextualizing geometric transformations such as rotations and tend to rely on procedural understanding rather than conceptual understanding, which implies low spatial reasoning abilities among students. In addition, many students consider mathematics to be too abstract and unrelated to their lives, causing them to lose interest in learning and find it difficult to understand the concepts (Permita et al., 2022; Putra et al., 2022). Teaching methods that rely too heavily on lectures also exacerbate the situation, where teachers actively provide information and students are not encouraged to develop their own understanding (Goodwin, 2024).

In response to these challenges, the Realistic Mathematics Education (RME) approach, known in Indonesia as *Pendidikan Matematika Realistik Indonesia* (PMRI) offers a relevant pedagogical perspective. This approach emphasizes that mathematics learning must start from real and meaningful situations for students, so that it can bridge the gap between formal mathematical abstraction and students' intuitive understanding (Zulkardi & Putri, 2019). In this context, the richness of Indonesia's local culture offers authentic and largely untapped learning resources (Khasanah et al., 2025; Siligar et al., 2025). In line with this, Fitriadi et al. (2024) revealed that integrating cultural contexts and traditional practices into education through an ethnomathematics approach has proven beneficial for students' conceptual understanding, learning experiences, and overall academic growth. One example is Songket Mak Raje from Muara Enim Regency, South Sumatra Province. This songket motif is inspired by the reliefs of centuries-old Semende traditional houses. In addition to representing valuable cultural heritage, the motif contains complex and symmetrical geometric patterns, making it a potentially realistic and meaningful initial context for learning rotation. In line with this principle, Koerunnisa et al. (2025) emphasize that the realistic context in RME or PMRI is not limited to the real world but also includes situations that can be imagined and are relevant to students' learning experiences.

The integration of digital technology in mathematics learning has been recognized as an important catalyst for improving the quality of education (Abdykerimova et al., 2025; Wang et al., 2024). Dynamic geometry platforms such as Desmos Geometry Tool offer an interactive, visual, and exploratory learning environment that supports the development of conceptual understanding and spatial reasoning in a more optimal way (Dorel, 2023; Leung et al., 2024; Mediana Jr & Dio, 2025). To support teaching and learning activities, Desmos provides Amplify Classroom as a virtual classroom feature. Amplify Classroom offers integrative advantages over similar platforms, where dynamic geometry exploration activities using Desmos Geometry Tool can be embedded natively in E-LAS without the need to open a separate tab. This feature allows the visual exploration process and student work to be stored in an integrated learning environment, while facilitating real-time monitoring of learning progress by teachers (Fawensi & Susanti, 2025).

Previous studies have explored the use of cultural context and digital technology separately. Research on ethnomathematics and PMRI, for example, shows that cultural artifacts such as traditional woven fabrics can increase learning engagement and serve as concrete starting points for learning geometric concepts, including transformations (F. A. Andriyani et al., 2024; Filda & Armiati, 2023; Sari, Putri, Zulkardi, et al., 2025b, 2025a). Meanwhile, other research confirms that using digital tools like Desmos significantly improves students' understanding of transformation geometry topics through the dynamic visualization features provided (Dani & Ashok, 2025; Hidayati & Murtiyasa, 2024; Luhukay et al., 2025). However, a literature review indicates that studies integrating cultural contexts and digital technology within a structured pedagogical framework remain scarce. More specifically, research explicitly employing Songket Mak Raje as a mathematics learning context is yet to be found, despite the motif's rich geometric patterns being highly relevant for teaching transformation geometry. Furthermore, prior research on technology in mathematics learning has tended to use commonly known platforms like GeoGebra (Azis & Rohaeti, 2025), whereas the integrated use of the Desmos Geometry Tool with its classroom feature for teaching transformation geometry is still rarely explored.

Based on the identification of these gaps, this study proposes a three-dimensional innovation through the synergistic integration of the PMRI framework as a pedagogical approach, the cultural context of Songket Mak Raje, which has not been explored in previous mathematics learning studies, and the development of E-LAS (*Lembar Aktivitas Siswa Elektronik* or Electronic Student Activity Sheet) based on the Desmos Geometry Tool. This innovation is accommodated within the framework of Hypothetical Learning Trajectory (HLT), which provides a systematic blueprint for designing and researching learning sequences. Thus, this research not only bridges the gap between cultural context and digital technology but also makes an original contribution through enriching the Indonesian ethnomathematics repertoire by highlighting specific and unmapped cultural artifacts, learning design innovation through the development of E-LAS on a digital platform that has not been widely explored, and the provision of empirically tested HLT for learning about rotation that is rooted in local wisdom but relevant to developments in educational technology.

Therefore, this study aims to design and evaluate Hypothetical Learning Trajectories to develop high school students' understanding of rotation concepts and spatial reasoning skills. The research focuses on answering the question: How can PMRI-based HLTs that integrate the cultural context of *Songket Mak Raje* and the Desmos platform be designed to develop students' understanding of rotation concepts and spatial reasoning skills?

Methods

This study applied a validation study type of design research method to develop a Hypothetical Learning Trajectory (HLT) for rotation-based material using the PMRI approach. This method was carried out in three stages, namely Preliminary Design (cycle 1), Design Experiment (cycles 2 and 3), and Retrospective Analysis (cycle 4) (Gravemeijer & Cobb, 2006). The Design Experiment stage itself consists of two phases, namely Pilot Experiment (cycle 2) and Teaching Experiment (cycle 3). This method was chosen for its ability to develop learning theory through repeated cycles of improvement carried out directly in the context of real learning, as shown in Figure 1.

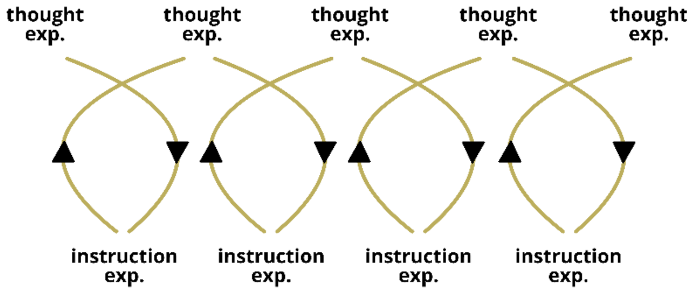


Figure 1. Research flowchart

In the Preliminary Design stage, a literature review was conducted to develop HLT and E-LAS that integrated the principles of RME and PMRI (Gravemeijer, 1994; Sembiring et al., 2008), ethnomathematics (D'ambrosio, 1985), Dynamic Geometry Pedagogy (Sinclair & Moss, 2012), and HLT constructs (Simon, 1995). The prototype was then validated through a Focus Group Discussion (FGD) with experts. The Design Experiment stage began with a Pilot Experiment in a small group to test and refine the design, followed by a Teaching Experiment in a full class. The Retrospective Analysis stage was conducted by analyzing all data to revise the HLT based on empirical findings.

The research was conducted in the 2025/2026 academic year at SMA Srijaya Negara Palembang. SMA Srijaya Negara Palembang is a laboratory school of Universitas Sriwijaya that has established a partnership with the Faculty of Teacher Training and Education (FKIP) Universitas Sriwijaya in supporting academic and research activities. To ensure variation in student abilities, subjects for the Pilot Experiment were selected through purposive sampling based on teacher recommendations, involving six 11th grade students representing high, medium, and low ability levels. Meanwhile, the Teaching Experiment was conducted in one whole class consisting of 36 students.

Data collection techniques included participatory observation, informal interviews during the learning process, and document analysis. The main research instrument was Desmos-based E-LAS. Data was collected comprehensively through (1) direct observation and recording of student behavior during activities, (2) documentation of student work on E-LAS, (3) structured field notes, and (4) transcripts of learning conversations that included group tutoring sessions and class Q&A. This multi-source approach allowed for data triangulation to validate the findings. Data analysis was conducted using a qualitative approach with thematic analysis techniques. The analysis process followed the constant comparative method model, which included data coding, pattern identification, data source triangulation, and analytical narrative construction. All data were analyzed retrospectively to reconstruct the students' learning process and evaluate the effectiveness of the developed HLT.

Results

Preliminary design

Realistic Mathematics Education (RME) as elaborated by K. P. E. Gravemeijer (1994), positions mathematics as a human activity through three main principles. The first principle is guided reinvention and progressive mathematization, which is the process of gradually rediscovering concepts from concrete contexts toward formal forms. The second principle is didactical phenomenology, namely the use of real phenomena as the starting point for learning. The third principle is self-developed models, which involves students developing models from models-of (context-based) to models-for (formal thinking tools). This approach has proven adaptable across various contexts, including in Indonesia through *Pendidikan Matematika Realistik Indonesia* (PMRI), which emphasizes the use of contexts closely aligned with students' experiences (Sembiring et al., 2008). In this regard, ethnomathematics provides the foundation that mathematics is a cultural practice, so that local contexts can serve as valid sources of mathematical knowledge and meaningful learning resources (D'ambrosio, 1985). One cultural context that can be utilized is the Songket Mak Raje.

Dynamic Geometry Pedagogy (DGE) functions as a dynamic visual tool that allows students to manipulate geometric objects through dragging (continuously moving points or shapes) and animation (automatic movement), thereby helping students understand transformability, the ability to observe changes in shape without losing its fundamental properties (Sinclair & Moss, 2012). Within the RME framework, DGE functions as a visual medium that helps students understand dynamic shape changes during the model-of stage. From an ethnomathematics perspective, DGE can be used to explore and formalize geometric patterns from cultural artifacts, thereby bridging informal and formal knowledge. One platform supporting this function is Desmos. To operationalize this framework, the Hypothetical Learning Trajectory (HLT) concept from Simon (1995) is employed, encompassing learning objectives, learning activities, and predictions of students' cognitive development. The HLT serves as an initial guide that provides a direction for learning while remaining open to revision based on students' cognitive development throughout the learning process.

Referring to this theoretical foundation, the learning trajectory in this study was designed with four main characteristics: rooted in authentic cultural contexts as the context of PMRI didactical phenomenology and ethnomathematics; promoting progressive mathematization through stages of activities with increasing levels of abstraction; providing space for self-developed models and guided reinvention through DGE-assisted exploration; and incorporating predictions of students' cognitive development at each stage. These characteristics are then realized in the five phases of the HLT outlined in Table 1, encompassing the introduction to concrete phenomena through Songket Mak Raje, the exploration of geometric patterns aided by Desmos, the discovery of concepts through coordinate representations, the generalization of rotation formulas, and the application of concepts in solving both contextual and non-contextual problems.

Table 1. HLT for rotation material based on the PMRI approach

Learning Activity	Learning Objective	Student Thinking Conjecture	Developed Abilities
Informal Stage (Concrete Phenomena)			
Activity 1: Observing the description and visualization of the Songket Mak Raje motif. **Activity 2:** Identifying the same motif with variations in position, direction, and orientation.	Students can identify recurring patterns in songket motifs and hypothesize their relationship with the concept of rotation.	1) Students recognize repeated patterns with different orientations. 2) Students hypothesize a connection between the songket motif and the concept of rotation.	**Conceptual Understanding:** The ability to recognize visual patterns and identify initial relationships with the concept of rotation.
Model Of (Geometric Pattern Exploration)			
Activity 3: 1) Choose one Songket Mak Raje motif. 2) Draw the base pattern using the Desmos polygon tool. 3) Rotate the base pattern with the geometry transformation tool to form the complete motif.	Students can explore the influence of varying rotation parameters (center, angle, direction) on the resulting image through digital experimentation.	1) Students conduct experiments by trying different centers and rotation angles. 2) Students realize the influence of center, rotation angle, and direction of rotation on the resulting image. 3) Students make conjectures about the relationship between object position, rotation center, angle size, and rotation result.	**Spatial Reasoning:** The ability to visualize, manipulate, and predict rotation outcomes through digital exploration.
Model For (Concept Discovery)			
Activity 4: Identifying factors influencing rotation, identifying rotation properties, and formulating a definition of rotation based on previous exploration. **Activity 5:** Guessing the rotation center that maps a shaded polygon to an unshaded one and stating findings based on observation. **Activity 6:** Utilizing trigonometry to find the rotation formula with center (a, b). **Activity 7:** Substituting center $(0,0)$ into the result from Activity 6.	Students can find informal definitions, identify properties, and apply trigonometric reasoning in coordinate systems to find rotation formulas.	1) Students begin composing their own operational definition of rotation. 2) Students identify the properties of rotation based on Desmos exploration and visual observation of changes in the position and orientation of polygons. 3) Students connect visual rotational motion with the concept of position change in coordinates. 4) Students derive the rotation formula through coordinate and trigonometric reasoning. 5) Students begin to understand the generalization of the formula from (a, b) to $(0,0)$.	**Conceptual Understanding:** Ability to formulate definitions, properties, and mathematical formulas for rotation. **Spatial Reasoning:** Ability to connect visual motion with coordinate representation and derive formulas through spatial reasoning.
Formal Knowledge (Generalization)			
Activity 8: Formulating conclusions about the result of rotating point (x, y) by	Students can formulate formal definitions, deduce abstract	1) Students conclude the formal definition of rotation based on consolidation from	**Conceptual Understanding:** The ability to construct formal definitions, general

Learning Activity	Learning Objective	Student Thinking Conjecture	Developed Abilities
angle θ about center (a, b) and $O(0,0)$. **Discussion Session:** Formulate definitive conclusions about the definition and properties of rotation.	properties, and discover general formulas for rotation.	various initial definitions in previous stages. 2) Students agree on the properties of rotation in standard mathematical language. 3) Students write the rotation formula as the final constructed knowledge.	formulas, and mathematical properties of rotation.
Formal Application (Problem Solving)			
Activity 9: Solving contextual problems about rotation. **Activity 10:** 1) Identifying the transformation that maps an unshaded polygon to a shaded polygon considering geometric constraints. 2) Sketching the transformation process. 3) Providing a written explanation.	Students can apply rotation concepts in contextual and geometric problems, and explain the transformation strategies used.	1) Students identify relevant information (center point, angle, direction) from the problem. 2) Students select and apply the correct rotation formula. 3) Students justify their solution based on learned rotation properties. 4) Students plan a transformation strategy that considers geometric constraints.	**Conceptual Understanding:** Ability to apply rotation formulas in real-world contexts and perform mathematical calculations. **Spatial Reasoning:** Ability to visualize, plan, and explain transformation strategies with geometric constraints.

The learning activities within the E-LAS were systematically designed by leveraging the Desmos platform to support the implementation of the Hypothetical Learning Trajectory (HLT). The sequence of E-LAS activities was structured in a progressive order as outlined in Table 1. During the phase of formalizing the definition and properties of rotation, a teacher-guided class discussion was conducted after the completion of the E-LAS to reach a consensus on the appropriate definitions and properties. The draft E-LAS can be accessed via the following link: https://classroom.amplify.com/activity/692b233498dc42152e5447d7. As a validation draft, this E-LAS includes an example in Activity 3, which features an illustration of a base pattern and its rotation results forming a complete motif. Several activities from this draft E-LAS can be observed in Figure 2.

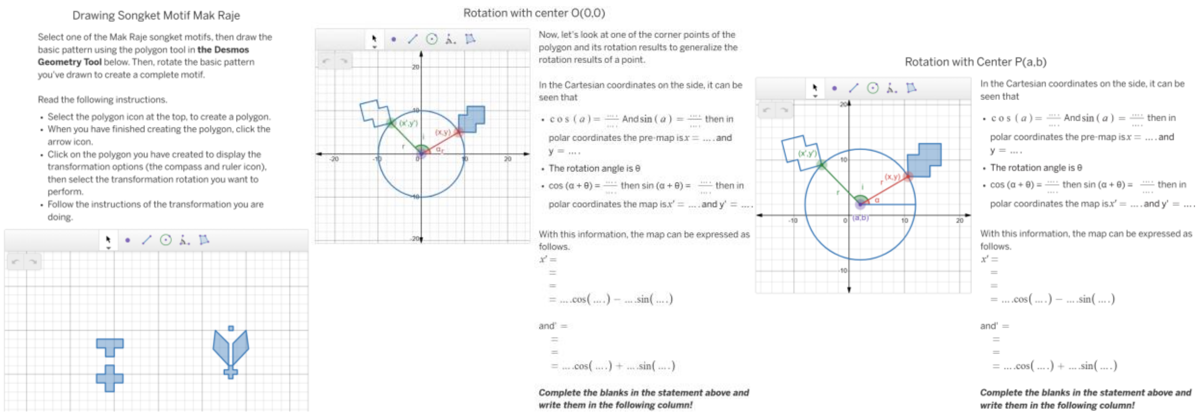


Figure 2. E-LAS draft

A Forum Group Discussion (FGD) with mathematics education experts confirmed the validity of the Hypothetical Learning Trajectory (HLT) and the E-LAS. The main findings state that both devices have fulfilled the principles and characteristics of PMRI and have strong potential in bridging cultural contexts with formal mathematical concepts, including the novelty and superiority of the solutions offered.

Based on input from the FGD, two major improvements were made. First, the lesson duration was reassessed, as the activity of discovering formulas for both rotation centers was predicted to require a longer time allocation. Consequently, the activity for discovering the rotation formula with center $(0,0)$ was changed from a coordinate and trigonometric reasoning method to direct substitution into the previously obtained formula for (a, b). This change in approach was considered still capable of achieving the objectives of developing conceptual understanding and spatial reasoning, which had been attained during the stage of discovering the formula for (a, b). Second, to support students' transition from visual patterns to mathematical abstraction, the FGD emphasized the need for more adequate scaffolding. As a follow-up, the dot-to-dot activity format in the model for stage was changed to a series of guiding questions designed to guide students' independent investigation in a more structured manner. The results of the revisions to these activities are presented in Figure 3.

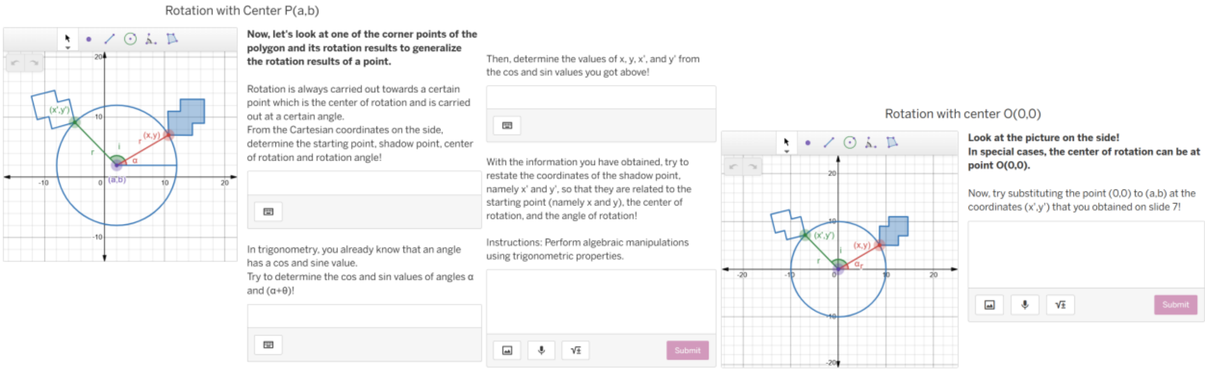


Figure 3. Revised E-LAS results after FGD

Design experiment

Stage 1: Pilot experiment

Based on the results of the Pilot Experiment involving six students, a number of critical findings were obtained for the improvement of learning activities and validation of the Hypothetical Learning Trajectory (HLT). Overall, the HLT structure was found to be appropriate and effective in guiding the learning process, thus requiring no substantive revisions. This suitability was particularly evident in the application of PMRI principles and characteristics, which were reflected in three main aspects. First, the use of the cultural context of Songket Mak Raje as a starting point (didactic phenomenology) proved successful in triggering student engagement and served as an effective bridge to the abstract concept of rotation. Second, the sequence of activities from the informal stage, model of, model for, to formal knowledge represented a clear principle of progressive mathematization, in which students' understanding developed gradually from concrete understanding to formal generalization. Third, interactivity and knowledge construction were evident when students actively explored the Desmos

Geometry Tool to independently construct visual models of rotation, which then became the basis for them to discover and formulate the definition and formula of rotation (self-developed models and guided reinvention). Thus, the findings of the pilot experiment not only validate the suitability of HLT with the PMRI philosophy, but also confirm that this learning design has facilitated a meaningful learning process through the use of appropriate contexts, models, and interactions.

In addition, several technical issues were found in E-LAS that need to be improved. In Activity 2, students were able to identify the patterns and orientation of Songket motifs. However, in Activity 3, obstacles arose, such as the example images being separated from the Desmos work area. A more serious problem arose in Activity 6. Students had difficulty deriving the rotation formula through the coordinate and trigonometry approaches. The main difficulty arose because the position of the rotation point was in quadrant II, which made it difficult for them to identify right-angled triangles as the basis for trigonometric reasoning and required a deeper conceptual understanding. Technical constraints also complicated the learning process, as the math response column did not allow for the creation of new lines, thus limiting the space for students to explore answers. Based on these findings, design revisions were made, including placing the songket image on the same page as Desmos, relocating the point to quadrant I to simplify analysis, adding auxiliary lines to guide trigonometric understanding, and changing the math response format to a more flexible free form. The results of the revisions to these activities are presented in Figure 4.

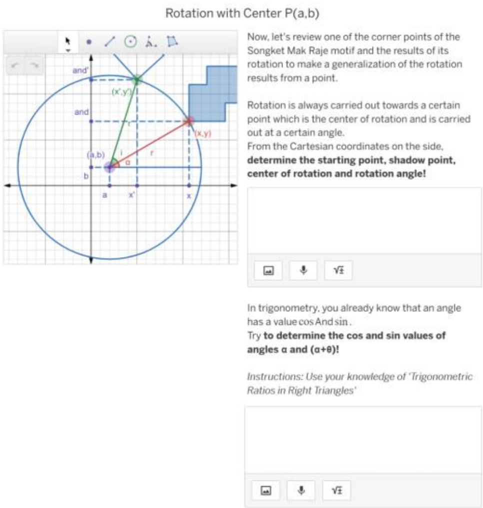


Figure 4. Revised E-LAS results after pilot experiment

Stage 2: Teaching experiment

The teaching experiment was conducted to examine the implementation of the revised Hypothetical Learning Trajectory (HLT) in a classroom setting and to analyze the alignment between the HLT and the Actual Learning Trajectory (ALT). This stage focused on observing how students' conceptual understanding and spatial reasoning developed throughout the learning activities. This study involved 36 students divided into six heterogeneous groups, each facilitated with one laptop to access E-LAS in Amplify Classroom via the link https://classroom.amplify.com/activity/6904b928cd906725cfe672c7.

Activity 1: Observing the Songket Mak Raje motif

Based on observations and interviews conducted during activity 1, students were able to understand the contextual information presented through the description, video, and visual representations of the Songket Mak Raje motif. Students showed interest in the cultural background of the motif, particularly its origin, historical value, and its association with the Palembang Sultanate. The visual presentation, including the use of songket in formal events, further strengthened students' engagement at the beginning of the lesson. This activity did not yet involve mathematical analysis; however, it played an important role in introducing a meaningful learning context. The use of cultural narratives and visual representations successfully attracted students' attention and prepared them for subsequent exploration of geometric concepts. Thus, Activity 1 functioned as an initial stage in connecting real-world context to mathematical learning in accordance with the PMRI approach.

Activity 2: Identifying motif variations in position, direction, and orientation

The students' answers to activity 2 show that they were able to identify variations in position, direction, and orientation in the same Mak Raje Songket motif. The students were also able to hypothesize that the motif composition could be formed through a process of rotation from one basic motif. One of the students' answers to this activity is presented in Figure 5.

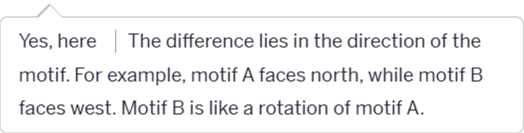
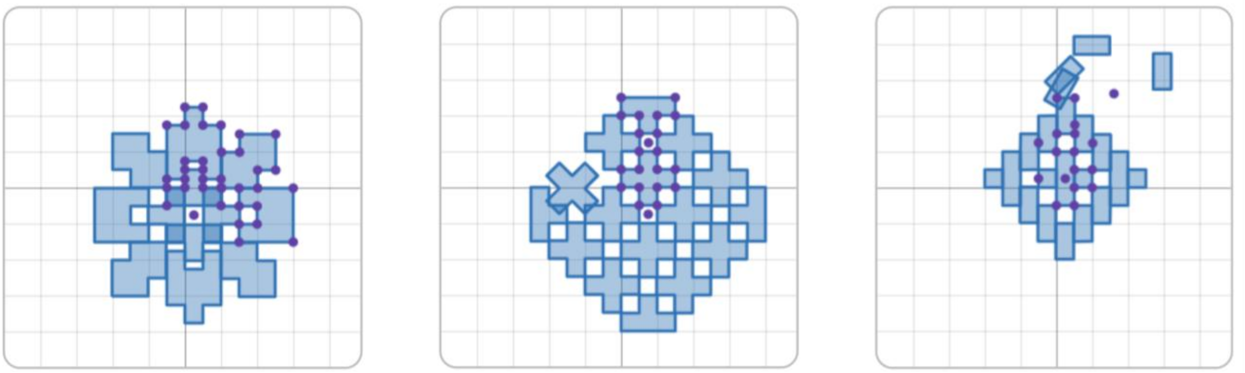

Yes, here | The difference lies in the direction of the motif. For example, motif A faces north, while motif B faces west. Motif B is like a rotation of motif A.

Figure 5. Group 3's answers to activity 2

Activity 3: Drawing a base motif in Desmos and rotating it to form the complete pattern

The students' answers in activity 3, as seen in Figure 6, show that the three groups chose different Songket motifs to explore. This variation in choice created different geometric challenges for each group, resulting in variations in the process of discovering the concept of rotation. In the figure, the polygon showing the vertices is the basic pattern, while the polygon without visible vertices is the result of the rotation transformation.



(a) (b) (c)

Figure 6. (a) Group 1's answer for activity 3; (b) Group 2's answer for activity 3; (c) Group 3's answer for activity 3

Based on observations and interviews during the learning process, there were variations in the level of difficulty in drawing Songket motifs between groups. Group 1 chose a motif with four basic patterns in the form of complex polygons. Although the drawing stage was quite challenging, the rotation process was relatively simple because all additional polygons could be produced using a single center point of rotation. In contrast, Group 2 chose a motif that had two simple basic patterns, namely a + shaped polygon and a truncated + shaped polygon. To compose a complete motif consisting of many polygons, this group needed to perform many rotations. However, the process was not too complicated because it did not require much exploration of the center point. Group 2 successfully identified the principle that the rotation result was highly dependent on the angle and direction of rotation. This identification arose when they tried to rotate the + polygon by 45° and -45°. Both angles changed the shape to an × with different positions, even though the target motif only contained the shape + or a truncated +. Through further exploration, they concluded that the correct rotation angles were ±90°, ±180°, or ±270°. Group 3 chose the motif with the highest level of difficulty. This motif consisted of three basic patterns, namely two rectangles of different sizes and one square. All rectangles in the motif were oriented vertically. When trying various rotation angles such as 30°, 45°, and 90°, the orientation of the rectangles changed to horizontal or slanted, so these angles were not suitable. After many attempts, group 3 concluded that only the 180° angle could be used. They also found that the position of the shadow was largely determined by the positive or negative sign of the angle, which affected the direction of rotation. The next challenge was to determine the correct center point of rotation, because almost every polygon resulting from rotation required a different center.

Overall, the results of observations and interviews during the learning process showed that students were very enthusiastic about digital exploration. Their curiosity was particularly triggered by finding rotations that could transform the basic pattern into a single complete motif.

Activity 4: Identifying rotation factors and properties, and formulating a definition of rotation

The students' geometric exploration process in Activity 3 greatly influenced their answers in Activity 4. Groups 2 and 3 conducted various rotation experiments by varying the center point and angle size (including negative angles), enabling them to successfully identify the three determining factors of rotation, its properties, and formulate a definition of rotation. Conversely, Group 1, which conducted fewer varied experiments, was only able to identify two factors, namely the angle size and rotation center. Informal interviews revealed that Group 2 did not identify rotation direction because they did not attempt to use negative angles. Consequently, this group also failed to identify rotation properties related to direction.

Activity 5: Determining the rotation center from polygon mapping and stating observations

Student answers in Activity 5 showed that students could estimate the location of the rotation center and analyze the movement of the shaded polygon toward the unshaded one. Group 2, in particular, successfully identified that each vertex of the initial polygon had a precise counterpart in the rotated polygon. For example, the vertex closest to the rotation center in the original polygon remained the closest point after rotation. However, no student was able to deduce the fundamental property of rotation concerning distance namely, that the distance from any point on the original object to the rotation center is always equal to the distance from its counterpart on the rotated object. Group 2's answer for this activity is presented in Figure 7.

> Each corner point of the original polygon always corresponds to a corner point of the resulting rotation. For example, if an original corner point is closest to the center of rotation (compared to the other corners), then the resulting corner point will also remain closest to the center of rotation.

Figure 7. Group 2's answer for activity 5

Activity 6: Deriving the rotation formula with center (a, b) using trigonometry

In Activity 6 for Question 1, Groups 1, 2, and 3 successfully determined the initial point, shadow point, center of rotation, and angle of rotation accurately. In question section 2, Groups 2 and 3 also successfully determined the values of $\cos\alpha$, $\sin\alpha$, $\cos(\alpha+\theta)$, and $\sin(\alpha+\theta)$. However, Group 3 did not successfully determine these values accurately because they did not take into account that the coordinates must be measured relative to the center (a, b). As a result, the values of $\cos\alpha$, $\sin\alpha$, $\cos(\alpha+\theta)$, and $\sin(\alpha+\theta)$. obtained by Group 3 were for the center $(0,0)$. The answers of Groups 1 and 2 in this second part are presented in Figure 8.

$$\cos\alpha = \frac{x}{r}$$
$$\sin\alpha = \frac{y}{r}$$
$$\cos(\alpha+\theta) = \frac{x'}{r}$$
$$\sin(\alpha+\theta) = \frac{y'}{r}$$

(a)

$$\cos\alpha = \frac{x-a}{r}$$
$$\sin\alpha = \frac{y-b}{r}$$
$$\cos(\alpha+\theta) = \frac{x'-a}{r}$$
$$\sin(\alpha+\theta) = \frac{y'-b}{r}$$

(b)

Figure 8. (a) Group 1's answers to activity 6 part 2; (b) Group 2's answers to activity 6 part 2

In question section 3, groups 2 and 3 successfully determined the coordinates x, y, x', and y' from the cos and sin values they had previously found. Conversely, due to the error made in section 2, group 1 was unable to accurately determine the coordinates x, y, x', and y'. In question 4, groups 2 and 3 successfully used trigonometric properties, performed algebraic manipulations, and made substitutions to determine the shadow coordinates x' and y' related to the initial coordinates x and y and the angle of rotation. Group 3 also successfully used trigonometric properties, performed algebraic manipulations, and made substitutions to determine the shadow coordinates x' and y'. However, because the coordinates x, y, x', and y' obtained by group 3 were for the center $(0,0)$, the coordinates x' and y' of the image related to the initial coordinates x and y and the angle of rotation obtained by group 3 were also for the center $(0,0)$. The answers from Groups 1 and 2 in this fourth section are presented in Figure 9.

$$
\begin{aligned}
x' &= r\ \cos(\alpha+\theta)\\
x' &= r\ (\cos\alpha\cos\theta-\sin\alpha\sin\theta)\\
x' &= r\cos\alpha\cos\theta-r\sin\alpha\sin\theta\\
x' &= x\cos\theta-y\sin\theta\\
\\
y' &= r\sin(\alpha+\theta)\\
y' &= r\ (\sin\alpha\cos\theta+\cos\alpha\sin\theta)\\
y' &= r\sin\alpha\cos\theta+r\cos\alpha\sin\theta\\
y' &= y\cos\theta+x\sin\theta
\end{aligned}
$$

(a)

$$
\begin{aligned}
x' &= r\ \cos(\alpha+\theta)+a\\
&= r\ (\cos\alpha\cos\theta-\sin\alpha\sin\theta)+a\\
&= r\cos\alpha\cos\theta-r\sin\alpha\sin\theta+a\\
&= (x-a)\cos\theta-(y-b)\sin\theta+a\\
\\
y' &= r\sin(\alpha+\theta)+b\\
&= r\ (\sin\alpha\cos\theta+\cos\alpha\sin\theta)+b\\
&= r\sin\alpha\cos\theta+r\cos\alpha\sin\theta+b\\
&= (y-b)\cos\theta+(x-a)\sin\theta+b
\end{aligned}
$$

(b)

Figure 9. (a) Group 1's answers to activity 6 part 4; (b) Group 2's answers to activity 6 part 4

Activity 7: Applying the formula for center $(0,0)$

The students' answers in Activity 7 show that Groups 2 and 3 were able to obtain the rotation formula for the center (0,0) by substituting values into the formula derived in Activity 6. This indicates that these groups understood the relationship between the general form of the rotation formula and its special case. In contrast, Group 1 did not perform the substitution because the formula they obtained in Activity 6 did not include the variables $a$and b. As a result, they assumed that the formulas for different centers were identical.

Activity 8: Formulating conclusions about rotation results for (x,y) *around* (a,b) *and* $(0,0)$

The students' answers in Activity 8 show that Groups 2 and 3 were able to formulate accurate conclusions about the rotation of a point (x,y) with respect to the centers (a,b) and $(0,0)$. They successfully generalized the rotation formulas and demonstrated an understanding of the relationship between different centers of rotation. In contrast, Group 1 was unable to formulate accurate conclusions due to errors in the previous activities, particularly in deriving and applying the rotation formula. The discussion session conducted after this activity played a crucial role in aligning students' understanding. Through teacher-guided discussion, students refined their conclusions and agreed on the formal definition and properties of rotation. This process helped align students' understanding and ensured that the constructed knowledge was mathematically accurate.

Activity 9: Solving contextual rotation problems

The students' answers in Activity 9 demonstrate their ability to translate story problems into mathematical form, indicating conceptual understanding and identification of the correct procedural steps. Specifically, Group 2 successfully determined all coordinate points on the damaged pattern accurately. Group 3 only managed to determine one point due to time constraints. Meanwhile, Group 1 produced an incorrect solution due to an error in the rotation formula for the center (a,b) that they had previously derived. They applied the rotation formula for center $(0,0)$ to a problem requiring an arbitrary center (a,b). Thus, although all points could be calculated, the results did not match the problem's context. The students' answers for point A in this activity are presented in Figure 10.

$$x' = x\cos\theta - y\sin\theta$$
$$x' = 1\cos 180° - 3\sin 180°$$
$$x' = 1(-1) - 3(0)$$
$$x' = -1$$

$$y' = y\cos\theta + x\sin\theta$$
$$y' = 3\cos 180° + 1\sin 180°$$
$$y' = (3)(-1) + (1)(0)$$
$$y' = -3$$

(a)

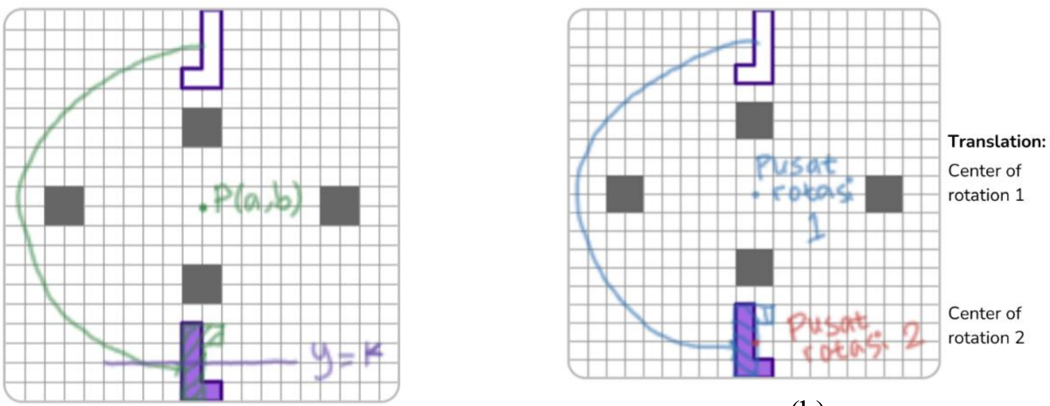
$$A(x,y) = A(1,3)$$
$$x' = (x-a)\cos\theta - (y-b)\sin\theta + a$$
$$= (1-1{,}8)\cos 180° - (3-3{,}6)\sin 180° + 1{,}8$$
$$= (-0{,}8)(-1) - (-0{,}6)(0) + 1{,}8$$
$$= 0{,}8 - 0 + 1{,}8$$
$$= 2{,}6$$
$$y' = (y-b)\cos\theta + (x-a)\sin\theta + b$$
$$= (3-3{,}6)\cos 180° + (1-1{,}8)\sin 180° + 3{,}6$$
$$= (-0{,}6)(-1) + (-0{,}8)(0) + 3{,}6$$
$$= 0{,}6 + 0 + 3{,}6$$
$$= 4{,}2$$
$$A'(x',y') = A'(2{,}6;4{,}2)$$

(b)

Figure 10. (a) Group 1's Answer for Activity 9; (b) Group 2's Answer for Activity 9

Activity 10: Identifying transformations under geometric constraints, sketching the process, and explaining the results

Student answers in Activity 10, indicate that Groups 1 and 2 successfully identified the correct transformation sequence, namely rotation followed by reflection. Group 2 sketched the transformation in detail, including the rotation center point, the resulting rotation image, and the reflection line with its equation. However, Group 1 did not sketch the transformation. On the other hand, Group 3 provided a less accurate answer by choosing two rotations. They argued that the first rotation changed the shape from '˩' to 'r', necessitating a second rotation to achieve the 'L' shape. Analysis shows this reasoning is flawed because a second rotation on a polygon shaped like 'r' would produce an orientation of '˩', not the desired 'L'. The sketches of Groups 2 and 3 are shown in Figure 11.



(a) (b)

Figure 11. (a) Group 2's answer for activity 10; (b) Group 3's answer for activity 10

However, none of the student answers were entirely correct. Although the sequence of rotation followed by reflection is conceptually accurate, all groups failed to consider the existing geometric constraints. This occurred because no student successfully identified the fundamental property of rotation in Activity 5: that the distance from any point on the original object to the rotation center must equal the distance from its counterpart on the rotated result. In this case, the distance from the student-chosen rotation center to the unshaded polygon is 6 grid units, the same as the distance to the constraints on its left and right. Consequently, the rotation would cause the polygon to collide with both constraints. Furthermore, there is no other center of rotation that can move the unshaded polygon directly to the position of the shaded polygon without passing through the obstacle. Thus, even though the transformation sequence is theoretically correct, the transformation cannot be applied because it violates the existing geometric obstacle.

Restropective analysis

Hypothetical Learning Trajectory (HLT) in this study was designed using the PMRI approach through five stages, starting from the informal stage, model of, model for, formal knowledge, to formal application. In general, these stages successfully guided the students' learning process from real observation to the point where they could conclude the concept of rotation abstractly and use it to solve problems. This learning structure proved to be effective in triggering interest in exploration and directing students' learning activities in a more structured manner.

However, Actual Learning Trajectory (ALT) showed that there were slight differences from what was predicted in the HLT. Although all groups followed the same stages, the depth of their understanding and their success in concluding the core concepts varied. Groups 2 and 3, which were more active in exploring parameter variations through Desmos, succeeded in building a complete understanding, from recognizing the factors that influence rotation to finding the correct mathematical formula. In contrast, Group 1 had difficulty understanding the key concepts because they did not explore parameter variations and did not take into account that coordinates must be measured relative to the center (a, b). Another important finding was that none of the groups concluded that the distance of a point to the center of rotation remains the same after rotation, even though the activity was designed to lead to that conclusion. In addition, when it came to the formal application stage, none of the groups took into account the existing geometric obstacles. This shows that there is still a gap between conceptual understanding and spatial reasoning skills.

The difference between HLT and ALT produces important teaching guidelines that shape Local Instructional Theory (LIT). The resulting LIT emphasizes that context-based rotation learning and digital exploration require a more structured and explicit learning scaffold, especially when students encounter difficult concepts, such as when students must determine the distance from point (a, b) to (x, y). This can be seen from students' mistakes in determining the values of sin and cos because they do not consider the center point of rotation (a, b) as a reference. In addition, the basic properties of rotation, such as the fixed distance of a point to the center of rotation, also need to be emphasized through measurement and comparison activities. Then, problems with geometric obstacles need to be introduced earlier in the exploration cycle so that students become accustomed to evaluating the solutions they propose. Thus, this empirically proven LIT not only refines HLT for the topic of rotation, but also offers principles that can be applied to other mathematical concepts that require conceptual understanding and spatial reasoning.

Discussion

Overall, the findings of this study support and enrich existing learning theories. First, the results confirm that the cultural context of Songket Mak Raje, as an authentic and previously unexplored concrete phenomenon, is effective in triggering initial interest, activating students' geometric intuition, and forming a strong bridge to formal mathematical concepts. The successful use of this motif in facilitating students' pattern observation, conjecture, and initial exploration is in line with the principles of didactic phenomenology in PMRI (K. P. E. Gravemeijer, 1994) and enriches the field of ethnomathematics research (Andriyani et al., 2024; Sari et al., 2025b), while also making a new contribution by highlighting local culture that has not been widely explored.

Second, ALT shows that guided digital exploration using the Desmos Geometry Tool plays an important role in encouraging students' progressive mathematization process. The interactivity and dynamic visualization on this platform facilitate independent experimentation with parameter variations (center, angle, direction), enabling students to construct visual models of rotation (model of) which then become the basis for inferring informal definitions, properties, and deriving formulas (model for). These findings support previous research on the effectiveness of dynamic geometry tools in developing conceptual understanding and spatial reasoning (Dani & Ashok, 2025; Luhukay et al., 2025; Nuralam et al., 2024). Furthermore, this study highlights the key role of exploration variation in building a complete and deep understanding, as seen in the difference in results between groups that actively explored (Groups 2 & 3) and those that were limited (Group 1).

The learning process in this study clearly reflects the iceberg model in PMRI, where formal understanding, namely formulas and standard definitions, is built on a strong foundation of informal activities and models developed by the students themselves, as shown in Figure 13.

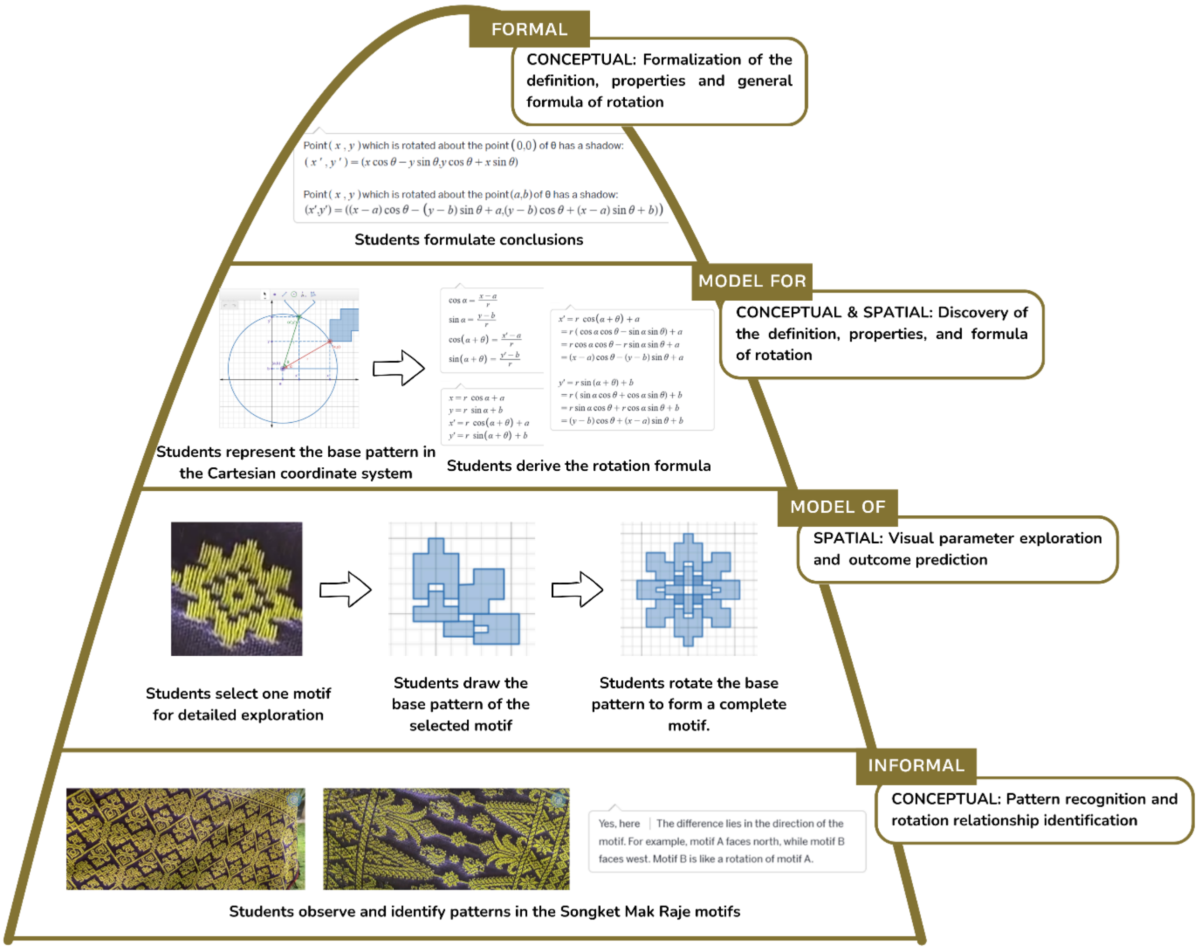


Figure 13. Iceberg model

Furthermore, retrospective analysis of the differences between HLT and ALT produced important teaching guidelines that formed LIT. This LIT positions context-based learning and digital exploration as an integrated instructional foundation, operationalized through three complementary learning strategies. First, gradual scaffolding is systematically emphasized to support students' transition from informal exploration to formal mathematical reasoning, thereby strengthening previous research findings that highlight the importance of scaffolding in discovery-based learning (Mamun, 2022). Second, the explication of rotation properties is designed through structured exploratory activities prior to formalization, ensuring that essential fixed properties such as the constant distance from the center of rotation are not only observed implicitly but also consciously constructed by students. Third, the early introduction of non-contextual geometric obstacles in the exploration phase extends PMRI practice by exposing students to constraint-based situations that require the evaluation of solution validity, thereby strengthening students' spatial reasoning and metacognitive control. Overall, these three strategies refine the HLT into a more robust instructional model that not only bridges conceptual understanding and spatial reasoning in a more systematic way, but also offers a more operational and transferable LIT framework for teaching transformation geometry in technology-enhanced learning environments.

Conclusion

The PMRI-based HLT that integrates the cultural context of Songket Mak Raje and the Desmos platform has been successfully designed and validated. This HLT is effective as a systematic roadmap for developing students' conceptual understanding of rotation and spatial reasoning, as evidenced by the progressive mathematization process and active knowledge construction by students. Retrospective analysis resulted in LIT, which is an improvement on the initial HLT. This LIT emphasizes context-based learning combined with guided digital exploration applied through three main strategies, namely the provision of gradual scaffolding, the explicitation of the properties of rotation through directed investigation activities prior to formal generalization, and the early introduction of non-contextual problems with geometric obstacles in the exploration cycle to train spatial reasoning and solution evaluation skills.

The implications of this study are to provide a concrete model of the integration of ethnomathematics and digital technology in geometry transformation learning, to produce LIT with PMRI principles as a guide for more effective learning design, and to offer Desmos-based E-LAS as a prototype for practical and tested teaching materials. The limitations of this study lie in the context of implementation, which was limited to two schools and the observation period. Further research is needed to test the application of LIT principles in a broader context, develop similar instruments for other transformation topics, and explore factors that influence the depth of student exploration in a digital environment.

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Declarations

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