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Developing a dynamic unitizing GeoGebra applet to support conceptual understanding of area and volume

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Abstract

This study was motivated by the limited conceptual understanding of area and volume among prospective mathematics teachers, who often rely on procedural strategies instead of conceptual reasoning. This study aimed to develop and evaluate an interactive GeoGebra applet based on dynamic unitizing, an approach that extends GeoGebra beyond dynamic visualization by supporting conceptual construction through the manipulation of unit squares and cubes. An explanatory sequential mixed-methods design was used. The applet was developed using the ADDIE model integrated with Tessmer's formative evaluation and implemented through one-on-one (three students), small-group (seven students), and field testing (28 students). The applet demonstrated very high validity (media: 99.48%; material: 99.31%) and high practicality, as evaluated by students (92%) and lecturers (89.93%). Students' conceptual understanding improved significantly after using the applet (Wilcoxon Signed-Rank Test: $Z = -4.541$, $p < 0.001$), with a large effect size ($r = 0.86$) and moderate normalized gain (N-gain = 0.542). Qualitative findings further indicated that the applet enabled students to move beyond procedural strategies by independently exploring and constructing the area and volume concepts. These findings suggest that the applet is a valid, practical, and promising learning medium for enhancing the conceptual understanding of area and volume.

Keywords: conceptual knowledge; dynamic unitizing; GeoGebra

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Introduction

Area and volume are key mathematical concepts (Huang & Wu, 2019; National Council of Teachers of Mathematics, 2006). These ideas are closely linked to other fields like science, technology, and engineering (Dorko & Speer, 2015; Huang & Witz, 2011). For future math teachers, understanding these concepts is essential because they will teach them to their students. If their grasp of area and volume is weak, they might pass that misunderstanding to the next generation. Therefore, it is vital that prospective math teachers fully understand the concepts of area and volume.

In learning about area and volume, two types of knowledge should develop together: procedural knowledge and conceptual knowledge (Kilpatrick et al., 2001). Procedural knowledge involves students' ability to correctly use formulas and calculations, while conceptual knowledge refers to understanding why a formula is used and what the meaning of the resulting numbers is (Canobi, 2009; Hiebert & Lefevre, 1986; Johnson et al., 2001; Rittle-Johnson & Schneider, 2014).

In the concept of area, understanding means students grasp that the area of a flat region is the number of unit squares needed to cover a plane without gaps or overlaps (Alexander & Koeberlein, 2011; Billstein et al., 2010; Long et al., 2012). Similarly, volume refers to the number of unit cubes that completely fill a space (Alexander & Koeberlein, 2011; Bennett et al., 2012). If someone has strong procedural and conceptual knowledge, they can not only calculate area and volume but also explain the meaning behind the values they obtain.

However, findings from a preliminary study indicate that this ideal condition has not yet been achieved. The study, involving 29 prospective mathematics teachers at a university, revealed that when asked to conceptually define area and volume, 86% of students could only answer using formulas, such as "area is length times width," without being able to explain the spatial meaning behind them. Only 14% of students could relate the concepts of area and volume to units of measure. This finding aligns with previous research that reported similar results (Baturo & Nason, 1996; Livy et al., 2012; Ma, 2010; McNeal et al., 2024). These findings suggest that students' understanding of the concepts of area and volume remains dominated by procedural knowledge rather than conceptual understanding. In other words, students know how to calculate, but they do not fully grasp what they are actually calculating.

This situation emphasizes the need for a learning approach that encourages students to discover the meaning behind the formula themselves. One alternative is to use interactive learning media that students can access independently. Interactive learning media allows students to actively engage, experiment, observe dynamic changes, and build their understanding through cycles of exploration, observation, and reflection without relying solely on lecturer explanations (Darmayanti et al., 2025; Saharani & Abadi, 2024; Yu et al., 2021). This approach is grounded in constructivist learning theory, which emphasizes that understanding is created when individuals actively build knowledge through direct experience rather than passively receiving information (Bruner, 1966; Piaget, 1970). Among various digital learning environments that support constructivist learning, dynamic mathematics software provides opportunities for students to actively construct mathematical concepts through the manipulation of multiple representations. GeoGebra is one of the most widely used applications for this purpose (Ocal, 2017).

Several previous studies have demonstrated that GeoGebra can improve conceptual understanding in geometry learning (Arbain & Shukor, 2015; Zulnaidi & Zamri, 2017). Other studies (Gurmu et al., 2024; Zhang et al., 2025; Ziatdinov & Valles, 2022) suggest that the dynamic visualization and manipulation features of GeoGebra can facilitate students' understanding of geometric concepts by enabling them to explore mathematical relationships interactively. By allowing learners to dynamically construct, manipulate, and observe geometric objects, GeoGebra promotes exploratory learning and helps students connect multiple mathematical representations, leading to deeper conceptual understanding rather than procedural memorization (Ziatdinov & Valles, 2022). Furthermore, a meta-analysis of two decades of research found that GeoGebra-based dynamic visualization has a significant positive effect on students' mathematics achievement, with particularly strong benefits in geometry learning (Zhang et al., 2025). The interactive nature of dynamic geometry environments also encourages students to investigate geometric invariants through dragging and experimentation, thereby fostering mathematical reasoning and conceptual understanding (Pin & Rosli, 2023).

Despite these well-documented benefits, previous studies have primarily focused on demonstrating the effectiveness of GeoGebra for enhancing students' conceptual understanding, while comparatively less attention has been given to the instructional design of GeoGebra applets themselves. Although GeoGebra has been widely used to visualize geometric objects and relationships, considerably less attention has been paid to how its interactive features can be intentionally designed to support the cognitive processes underlying measurement learning. In particular, the integration of dynamic unitizing as a design feature for supporting students' construction of area and volume concepts has received limited attention in the existing literature.

More specifically, most existing GeoGebra applets primarily emphasize the manipulation of completed geometric figures, dimensions, or geometric relationships. Consequently, measurement units often remain implicit, allowing students to observe how formulas change without explicitly constructing the underlying concepts through the accumulation and organization of equal-sized units. Therefore, the cognitive process of unitizing receives limited instructional support in many existing GeoGebra learning environments.

To address this limitation, the present study proposes a GeoGebra applet that incorporates a dynamic unitizing feature to explicitly support students' conceptual construction of area and volume. In this study, dynamic unitizing refers to an interactive feature embedded within a GeoGebra applet that enables learners to construct area and volume concepts through the dynamic manipulation of unit squares and unit cubes. Rather than simply displaying completed figures or formulas, the applet allows measurement units to be arranged dynamically, enabling students to observe how equal-sized units progressively cover a rectangle or fill a rectangular prism. This process provides opportunities for learners to infer area and volume formulas inductively through guided exploration instead of memorizing them. Unlike conventional GeoGebra applets, where geometric figures are the primary objects of manipulation, the proposed applet makes measurement units themselves the focus of interaction. This instructional design encourages learners to reason about the structure and accumulation of measurement units, thereby supporting the conceptual construction of area and volume rather than merely illustrating their formulas.

The developed applet enables the number of unit squares or unit cubes to be controlled interactively using sliders, allowing learners to observe how changes in dimensions influence the accumulation of measurement units. In addition, scaffolding questions embedded throughout the learning activities guide students to reflect on the relationships between measurement units, geometric dimensions, and the resulting area and volume formulas. By integrating these features into the applet's instructional design, the proposed learning environment extends GeoGebra beyond dynamic visualization toward actively supporting students' conceptual construction of measurement concepts.

This research aims to develop a learning media that is both valid and beneficial for students' independent learning. It also intends to provide a clear example of how technology like GeoGebra can be designed as a virtual laboratory for exploration activities to support conceptual understanding through the dynamic construction and manipulation of measurement units. Specifically, this research aims to: (1) develop a GeoGebra applet based on dynamic unitizing for the topic of area and volume that meets valid and practical standards, and (2) examine the potential of the applet to support prospective mathematics teachers' conceptual understanding of area and volume.

Methods

This research employed a mixed methods approach with an explanatory sequential design. This design was selected because the study had two complementary objectives. The quantitative phase was conducted to evaluate whether the developed GeoGebra applet met the criteria of validity and practicality and to examine its potential to support prospective mathematics teachers' conceptual understanding of area and volume. The subsequent qualitative phase was conducted to explain and enrich the quantitative findings by exploring participants' learning experiences and perceptions while using the applet. Accordingly, qualitative data were collected after the quantitative phase and integrated with the quantitative results during the interpretation stage using a joint display strategy, allowing qualitative evidence to explain and provide contextual meaning for the quantitative findings. The development of the learning media in this study followed the ADDIE model, which includes five stages: analyze, design, develop, implement, and evaluate (Branch, 2010). During the development and implementation phases, formative evaluation was conducted using Tessmer's procedures, including expert review, one-on-one sessions, and small group sessions before field testing.

The study involved three lecturers from the Mathematics Education Study Program as media and subject matter experts, and two additional lecturers from the same program to evaluate the practicality of the developed applet. All lecturers were selected using purposive sampling based on their expertise and experience in mathematics education, digital learning media development, instructional material development, and teaching practices. Furthermore, all expert participants had published scholarly works in the field of educational media and had experience in developing digital mathematics learning media. The study participants consisted of three students in the one-to-one evaluation phase, seven students in the small-group phase, and twenty-eight students in the field-testing phase. Student participants were also selected using purposive sampling based on the following criteria: they had previously studied area and volume concepts, they had no prior experience using the developed GeoGebra applet, they voluntarily agreed to participate in all stages of the study, including the pretest, learning activities, posttest, questionnaires, and interviews, and they completed the entire research procedure.

The research instruments included a media validation sheet, a material validation sheet, a practicality sheet, a student response questionnaire, pretest questions, posttest questions, and interview guidelines. The media validation sheet covered four assessment aspects: visual and interface design (UI), usability and user experience (Usability & UX), learning design for independent learning, and technical and functionality, with a total of 16 indicators. The material validation sheet addressed three aspects: content feasibility and accuracy, material depth and coverage, and material learning design, totaling 12 indicators. The practicality sheet evaluated three aspects: efficiency and ease of implementation, support for the teaching and learning process, and flexibility and relevance of content. The student response questionnaire consisted of 13 closed-ended statements using a four-level Likert scale: strongly agree, agree, disagree, and strongly disagree. These statements were grouped into three main areas: ease of use (usability), conceptual usefulness (utility), and motivation and attractiveness (interest). The questionnaire also included three open-ended questions to explore students' learning experiences, challenges faced when using the media, and suggestions for future improvements. Both the pretest and posttest included five questions designed to assess students' understanding of area and volume concepts. The questions were developed based on five indicators of conceptual understanding derived from the framework of Kilpatrick et al. (2001) and theories of measurement learning proposed by (Battista, 2004; Battista et al., 1998; Battista & Clements, 1996; Clements & Sarama, 2014; Hiebert & Carpenter, 1992; Lehrer, 2003), as presented in Table 1. Additionally, the interview guide contained several questions tailored to each implementation stage: one-on-one, small group, and field testing

Table 1. Indicators of the conceptual understanding test instrument

No	Conceptual Understanding Indicator	Question Indicator
1	Understanding area as unit measurement	Explaining area as the measure of a region covered by equal-sized square units rather than as the result of applying a formula.
2	Understanding volume as unit measurement	Explaining volume as the amount of three-dimensional space occupied by equal-sized unit cubes rather than as the result of applying a formula.
3	Explaining the mathematical reasoning underlying measurement formulas	Explaining why the formulas for the area of a rectangle and the volume of a rectangular prism emerge from organizing equal-sized units into rows, columns, and layers.

No	Conceptual Understanding Indicator	Question Indicator
4	Reasoning about the spatial structure of measurement units	Determining the total number of measurement units by reasoning about rows, columns, or layers as composite units rather than counting individual units.
5	Understanding the relationship between unit size and the number of units	Explaining the inverse relationship between the size of measurement units and the number of units required to measure or fill the same object

Before being used in the study, all research instruments, including the media validation form, the material validation form, the practicality evaluation form, the student response questionnaire, the pretest, the posttest, and the semi-structured interview guide, were validated through expert judgment by two lecturers from the Mathematics Education Study Program with expertise in digital learning media development and mathematics education. In addition, the internal consistency of the student response questionnaire was assessed using Cronbach's alpha coefficient. The results indicated good reliability for each dimension, with Cronbach's alpha values of $\alpha = 0.747$ for Ease of Use (Usability), $\alpha = 0.831$ for Conceptual Usefulness (Utility), and $\alpha = 0.783$ for Motivation and Attractiveness (Interest). The overall questionnaire achieved a Cronbach's alpha coefficient of $\alpha = 0.863$, indicating good internal consistency and confirming that the instrument was reliable for use in this study.

The pretest and posttest responses were independently scored by two raters using an analytic scoring rubric with scores ranging from 0 to 4 for each item, where a score of 0 indicated an incorrect or irrelevant response and a score of 4 indicated a complete and conceptually accurate response. Inter-rater reliability was assessed using the Intraclass Correlation Coefficient (ICC). The analysis showed excellent agreement between the two raters, with an ICC of 0.948 for the pretest and 0.967 for the posttest, indicating a high level of consistency in the scoring process. Any discrepancies between the raters were subsequently resolved through discussion, and the consensus scores were used as the final scores for subsequent analyses.

Data analysis in this study involved both quantitative and qualitative methods. Data from media and material validation were analyzed using percentage scores relative to the maximum score to assess the feasibility of the developed learning media. The resulting percentages were interpreted based on the media and material validity criteria adapted from Cheung et al. (2024) and Frialdo et al. (2025), with a validation percentage of at least 61% (valid category) considered acceptable.

Meanwhile, practicality data collected from questionnaires completed by lecturers and students were analyzed using percentage scores relative to the maximum score to evaluate how practical the media were in supporting the learning process. The practicality percentages were interpreted based on the practicality criteria adapted from Cheung et al. (2024) and Frialdo et al. (2025), with a practicality percentage of at least 61% (practical category) considered acceptable.

The effectiveness of the learning media was assessed by comparing students' pretest and posttest scores. Prior to hypothesis testing, the normality of the data was assessed using the Shapiro–Wilk test. The choice of the statistical test was based on the normality results, with the Wilcoxon Signed-Rank Test applied when the assumption of normality was violated. The Wilcoxon Signed-Rank Test was used to determine whether there were significant differences between students' pretest and posttest scores.

Additionally, student learning gains were measured using Normalized Gain (N-Gain) to evaluate the level of improvement in students' conceptual understanding after engaging with the developed media. The normalized gain values were interpreted according to Hake's (1998) criteria, with N-Gain values of at least 0.30 (medium category) considered acceptable.

Qualitative data obtained from the open-ended questionnaire responses and semi-structured interviews were analyzed using thematic analysis following the six-phase framework proposed by Braun & Clarke (2006). During the first phase, familiarization with the data, the interview recordings were transcribed verbatim. The interview transcripts and questionnaire responses were then read repeatedly to achieve a thorough understanding of the data. At this stage, the researchers made initial notes regarding students' experiences using the developed GeoGebra applet, particularly concerning dynamic visualization, unit manipulation, the use of sliders, and scaffolding questions.

The second phase, generating initial codes, involved identifying meaningful features of the data using an inductive coding approach. Initial codes included initial confusion with sliders, learning through exploration, dynamic visualization, unit manipulation, scaffolding questions, independent learning, and prior familiarity with GeoGebra. The coding process was conducted iteratively, allowing initial codes to be refined, merged, or expanded as additional transcripts were analyzed until a coherent set of codes that consistently represented the dataset was established.

During the third phase, searching for themes, conceptually related codes were grouped into broader categories and subsequently developed into preliminary themes that captured participants' experiences of using the GeoGebra applet. In the fourth phase, reviewing themes, these preliminary themes were systematically examined against the entire dataset to ensure that each theme accurately and consistently represented participants' experiences, was clearly distinguishable from other themes, and was aligned with the objectives of the study.

The fifth phase, defining and naming themes, involved refining the essence of each theme and assigning descriptive names that reflected their central meanings. This process ensured that each theme clearly explained how the applet's features supported conceptual understanding, independent learning, and students' overall learning experiences. Finally, in the sixth phase, producing the report, the identified themes were interpreted and used to explain and enrich the quantitative findings.

To enhance the credibility of the qualitative findings, data triangulation was conducted by comparing themes derived from the open-ended questionnaire responses with those obtained from the semi-structured interviews. In addition, member checking was performed by inviting participants to review whether the researchers' interpretations accurately reflected their learning experiences and perceptions. Peer debriefing was also carried out through discussions among the researchers to critically examine the coding process, theme development, and interpretation of the findings until consensus was reached. Finally, the quantitative and qualitative findings were integrated during the interpretation stage using a joint display strategy, enabling the qualitative evidence to explain, corroborate, and provide contextual meaning for the quantitative results.

In this study, the field-testing phase was conducted in a single three-hour (180-minute) session outside the regular class schedule. The session began with a pretest to assess students' prior understanding of area and volume concepts, followed by independent use of the developed GeoGebra applet. At the end of the session, students completed a posttest to measure changes in their conceptual understanding. After completing the posttest, all participants completed a questionnaire to provide feedback on the developed learning media. Subsequently, semi-structured interviews were conducted with selected students to obtain more in-depth insights into their learning experiences while using the applet. The interviews were carried out either immediately after the field-testing session or on the following day, depending on the participants' availability.

Results

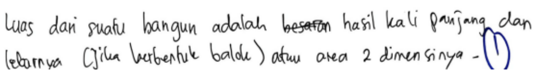
GeoGebra applet development based on dynamic unitizing

The GeoGebra applet was developed using the ADDIE model. The following outlines the steps involved.

Analysis stage

Preliminary studies indicate that students need learning tools to deepen their understanding of area and volume. Observations reveal that some students can solve problems related to area and volume procedurally, but they generally lack a conceptual understanding of the definitions.

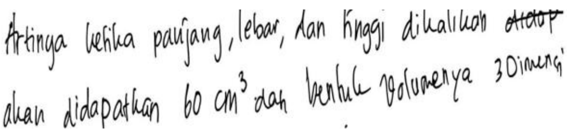
When asked to define the concept of area, most students had difficulty. They typically described area as the product of length and width, as shown in Figure 1.



The area of a shape is the product of its length and width (if it is a rectangular prism) or its two-dimensional surface area.

Figure 1. Example of student answers

A similar situation happened when students were asked to explain the meaning of the known volume of a cuboid. When asked the question “Physically, what happens inside a cuboid when we say that the volume is 60?”, most students answered that multiplying the length, width, and height of the cuboid gives a value of 60, as shown in Figure 2.



It means that when the length, width, and height are multipled, the result is 60 cm^3 and the measure represents a three-dimensional volume.

Figure 2. Example of student answers

Click or tap here to enter text.Click or tap here to enter text.Click or tap here to enter text.This situation shows that students still need learning experiences that help them develop a deeper understanding of concepts. Learning experiences that include visualization and manipulation of objects can help students understand concepts more clearly, especially for geometry topics with strong visual characteristics.

Based on these learning needs, the GeoGebra platform was chosen as the tool for teaching the concepts of area and volume. This is because GeoGebra can provide visual and interactive representations of geometric ideas, aiding students in understanding the connection between area and volume through the visualization of shapes and the arrangement of units that comprise them. Moreover, GeoGebra is easily accessible on various internet-connected digital devices without requiring special installation, offering flexibility in the learning process. Therefore, considering the results of the learning needs assessment and the characteristics of suitable media, it was decided to develop the learning media for this study using the GeoGebra platform.

Design stage

At this stage, a conceptual framework for the media was developed, including defining learning objectives, designing interaction flow, selecting visual elements, and choosing interactive components for the GeoGebra applet. The primary goal was to create media that not only offered an engaging visual representation but also promoted conceptual exploration, enabling students to discover the mathematical relationships behind the concepts of area and volume independently.

The design process began by establishing the structure of the learning flow, guiding students from concrete representations to more abstract understanding. This flow was based on constructivist principles, enabling students to observe, manipulate, and draw conclusions through direct interaction with mathematical objects. As a result, the media interface was designed with a focus on functionality, readability, ease of navigation, and consistency between visual elements and learning objectives. All visual components were kept simple yet informative to prevent distracting students from concentrating on the main concepts being developed.

In the first part of the media, the area applet, the design emphasizes the use of unit squares as a visual representation of the concept of area. Two sliders are provided to adjust the length and width of the rectangle. Each adjustment of the third slider causes a dynamic change in the shape's appearance and the number of unit squares covering its surface. This design allows students to directly see the relationship between the number of unit squares and the concept of area. Additionally, this setup helps students recognize that the area of a rectangle can be calculated by multiplying the number of unit squares in a row by the number in a column, thus developing a deeper understanding of rectangular area.

In the second part of the media, the volume applet uses a three-dimensional design to give a clearer visual representation of the concept of volume. Length, width, and height sliders allow users to adjust the dimensions of the cuboid, while unit cubes are shown as a layered arrangement filling the cuboid's space. The design is set up so that each change in dimension updates the number of unit cubes displayed on the screen, which can be moved using the slider. The numerical display of the unit cubes strengthens the connection between the physical model and the idea of volume.

In addition to designing the visual appearance and slider mechanism, the design phase also involved developing constructive questions and sequencing practice questions. These questions were carefully positioned to guide students in making detailed observations, understanding the mathematical reasons behind the changes, and deducing relationships between variables. They were arranged step-by-step, beginning with basic activities such as recognizing geometric shapes, exploring unit cubes, and calculating volumes manually or with formulas. At the end, a reflection column was added to assess students' understanding after engaging with the media. With this design, the media aims to serve as an exploratory tool that encourages students to build conceptual understanding independently.

Development stage

The development phase involves transforming the media design created during the design stage into a tangible product that can be used for learning. In this study, the final product was a GeoGebra-based interactive learning medium with two applets: one focusing on the concept of area and the other on the concept of volume. The process was conducted systematically, from media development to validation by experts.

GeoGebra media development

The media creation process was carried out using GeoGebra software, utilizing sliders, grids, dynamic objects, and automatic numeric displays. Each media element was developed through several components, as follows.

In creating the area concept media, it was designed to help students understand the concept of area and discover the formula for the area of a rectangle by examining unit squares. The components included: a) sliders for length and width to dynamically change the rectangle's size; b) a display of unit squares that can cover the rectangle and be adjusted with the sliders; c) a colored rectangle visualization representing the calculated area; d) an automatic display of area values based on the number of unit squares; and e) instructions to guide students during exploration. Students were prompted to observe the pattern of changes to discover the relationship between length, width, and area. Therefore, the area formula was not provided directly but was derived through observation and generalization. The applet display is shown in Figure 3.

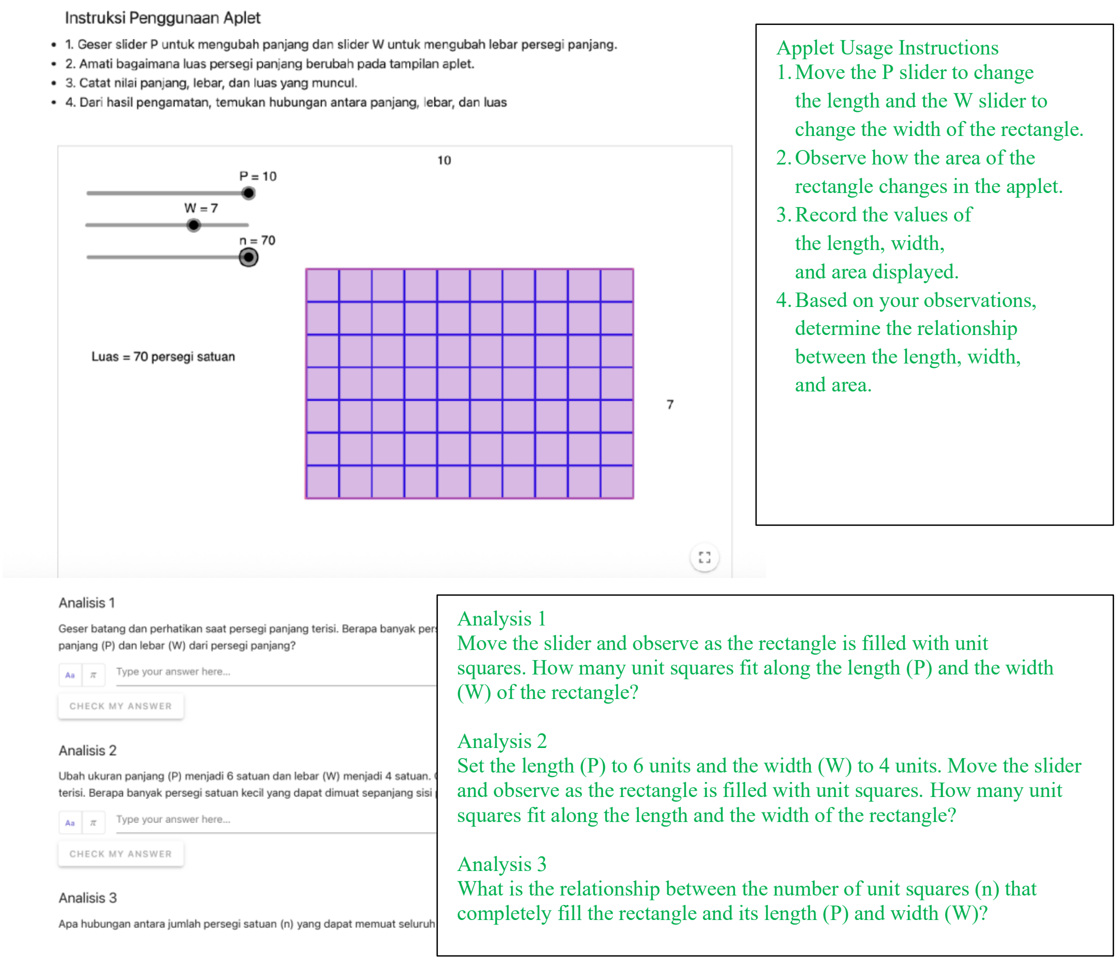


Figure 3. Prototype 1 broad concept applet and scaffolding questions

In creating the volume concept media, the media was developed as a three-dimensional model using unit cubes. The components include: a) sliders for length, width, and height to adjust the cuboid dimensions, b) a visualization showing how unit cubes fill the cuboid layer by layer, with the number of cubes adjustable via a slider, c) a display of the number of unit cubes numerically representing volume, and d) conceptual exploration instructions guiding students to find relationships between dimensions. Changes to one slider automatically update the number of unit cubes. Students can observe that the volume of a cuboid equals the number of unit cubes filling it. Through this, students are expected to understand why the volume formula is the product of length, width, and height.

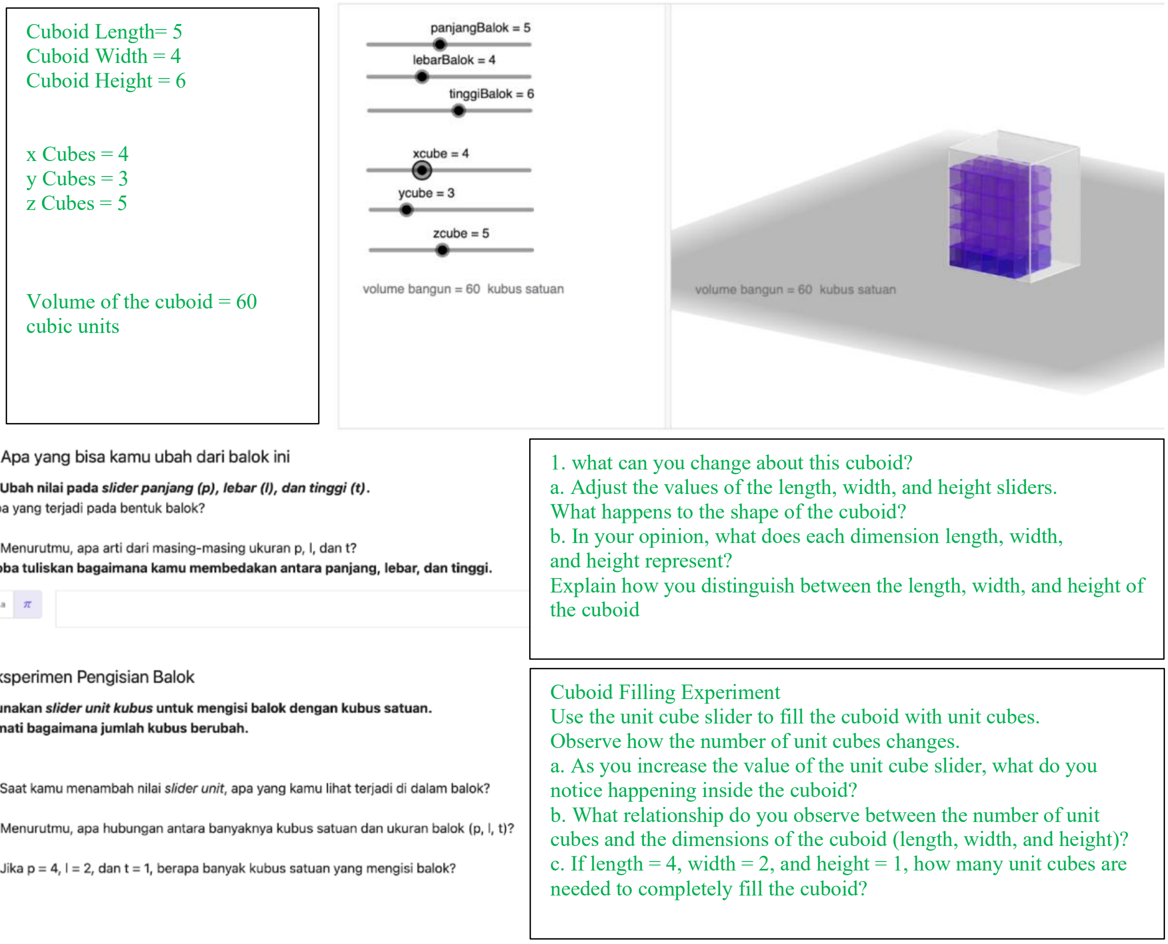


Figure 4. Prototype 1 volume concept applet and scaffolding questions

Both media are designed with interactive, dynamic principles and aim to build conceptual understanding, so students not only memorize formulas but also grasp their origins through concrete representations.

Expert validation stage (expert review)

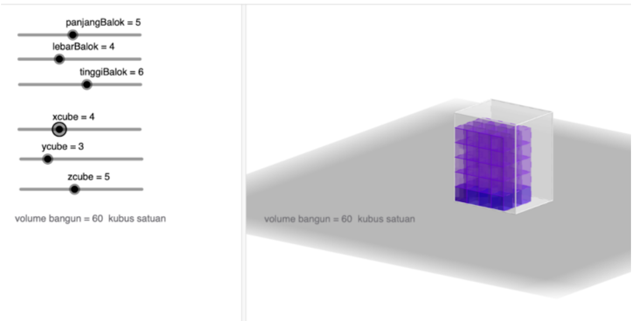
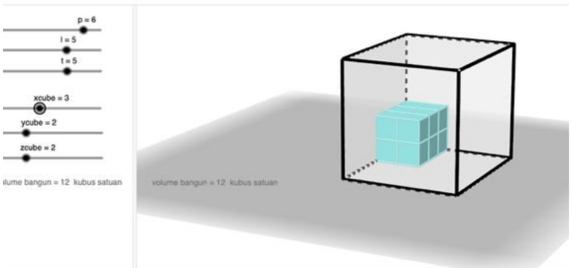
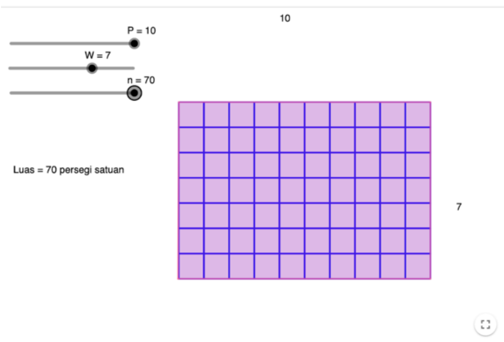
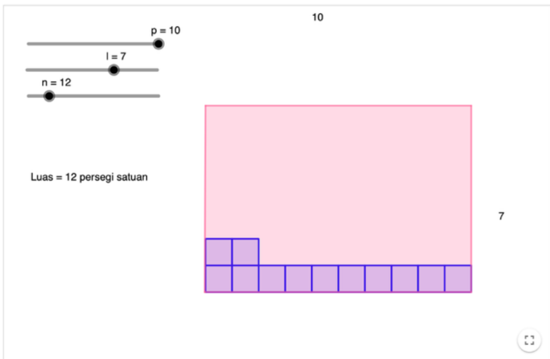
The expert review phase was conducted to verify the suitability of the media for learning from the perspectives of media experts and subject matter experts. During this stage, the applet was validated by three Mathematics Education lecturers and received the following feedback.

From the media experts' perspective, the main focus was on visual layout and technical performance. They recommended improvements to the color scheme used to represent the volume of a cuboid, specifically criticizing the use of dark gray and dark blue, which they found unclear. This color choice was believed to cause visual misinterpretation or difficulty for students in counting the unit cubes that make up the cuboid, potentially reducing the effectiveness of the visual in supporting conceptual understanding. Additionally, regarding functionality, the media experts noted that the 'zcube' slider could be moved accidentally. While they considered the overall functionality of the media to be good, this issue was seen as disruptive to students' learning process and needed improvement to provide a more seamless and distraction-free user experience.

Meanwhile, the material experts focused on visual representation and terminology consistency. Like the media experts' input, the material validators also pointed out the issue with the coloration of the volume concept, which was seen as too dark and unattractive, potentially lowering the material's appeal and clarity. Additionally, they emphasized the importance of consistently writing mathematical symbols and terms. The validators noted that in the applet, the symbols for length and width were sometimes written as "Length (P)" and "Width (W)" or simply as "length width" within a sentence, even though they are usually represented with lowercase letters ('p' and 'l'). This inconsistency needs correction to ensure standard notation and proper formatting throughout the applet, preventing confusion for students.

In response to these suggestions, changes were made to the applet. The color scheme for the cuboid volume visualization was adjusted to be more contrasting and visually appealing. The stability of the 'zcube' slider was also improved to prevent accidental shifts. Additionally, the writing of mathematical symbols and terms was aligned with standard conventions and refined throughout the applet. A comparison of the appearance before and after the revisions can be seen in Table 2.

Table 2. Comparison of media appearance before and after revision

Media view before revision	**Media view after revision**
	
	

After undergoing a revision process and being confirmed to meet the criteria set by the validators, the two applets were evaluated. The assessment results from the three validators are shown in Tables 3 and 4.

Table 3. Results of media validation score percentage analysis

Assessment Aspect	Number of Indicators	Average Score	Percentage (%)
Visual and Interface Design (UI)	3	4.00	100
Usability and User Experience (Usability & UX)	3	4.00	100
Instructional Design for Self-Learning	7	4.00	100
Technical and Functional	3	3.89	97.22
Overall	16	3.98	99.48

Table4. Results of the percentage analysis of material validation scores

Assessment Aspect	Number of Indicators	Average Score	Percentage (%)
Content Accuracy and Suitability	4	4.00	100
Content Depth & Scope	3	3.89	97.22
Instructional Design	5	4.00	100
Overall	12	3.97	99.31

Based on Tables 3 and 4 above, it is clear that the developed GeoGebra applet meets the eligibility criteria with a "Very Valid" rating. The average media validation percentage is 99.48%, and the material validation percentage is 99.31%. These results demonstrate excellent quality across all aspects of the assessment. Most indicators achieved a perfect score (100%), highlighting the media's strengths in visual design, functionality, ease of use, and material appropriateness.

Limited trial phase (one-to-one and small group)

After expert validation, the applet underwent limited testing to identify potential usability issues and collect direct feedback from users. In one-on-one sessions with three students, observations and interviews indicated that students generally used the applet successfully. They found it easy to operate because it required no downloads or installation. The slider feature for adjusting filler units was especially helpful in visualizing concepts.

Subsequently, during small group sessions with seven students, similar information was collected. All students reported no issues using the applet. The ability to adjust the unit square and unit cube independently made exploring the applet more engaging and motivated them to learn. Overall, students found both applets to be effective.

Effectiveness of learning media

The effectiveness of the developed applet was evaluated using a one-group pretest-posttest design. Twenty-eight students participated as research subjects. Before implementation, all students completed a pretest to measure their initial abilities. Afterward, they took a posttest to determine if there were any changes in their conceptual understanding. Both the pretest and posttest consisted of five questions, each with a maximum score of four. Therefore, the highest possible score for a student was 20.

Description of pretest and posttest results

The results of the descriptive analysis of students' pretest and posttest scores are presented in Table 5. According to this table, there was an increase in the average student scores from pretest to posttest.

Table 5. Descriptive statistics of students' pretest and posttest scores

	Pretest	**Posttest**
Valid	28	28
Missing	0	0
Mean	11.75	16.07
Std. Deviation	2.367	2.748
Minimum	9.000	11.00
Maximum	18.00	20.00

Table 5 shows that the students' average pretest score was 11.75, with a standard deviation of 2.367. After using the GeoGebra applet, the average posttest score increased to 16.07, with a standard deviation of 2.748. This rise in the average score indicates the potential for better student understanding after using the developed GeoGebra applet.

Effectiveness hypothesis test

Before conducting the comparative test, a Shapiro-Wilk normality test was performed on the pretest and posttest data. The results of the Shapiro–Wilk normality test, as shown in Table 6, indicate that the pretest data were not normally distributed ($p = 0.002$), whereas the posttest data were normally distributed ($p = 0.117$). Since the assumption of normality was not met, a non-parametric statistical test was selected for subsequent analysis.

Table 6. Results of normality test

	Pretest	**Posttest**
Shapiro-Wilk	0.867	0.941
P-value of Shapiro-Wilk	.002	.117

The results of the Wilcoxon Signed-Rank Test are presented in Table 7. The analysis revealed a statistically significant difference between the pretest and posttest scores ($Z = -4.541, p < 0.001$), indicating that students' conceptual understanding significantly improved after using the developed learning media. The corresponding effect size was large ($r = 0.86$), indicating a substantial practical impact (Cohen, 1988).

Table 7. Results of difference test

Measure 1		**Measure 2**	**W**	**z**	**df**	**p**
Pretest	-	Posttest	0.000	-4.541		$< .001$

Note. Wilcoxon signed-rank test

N-Gain score analysis

To measure the level of effectiveness of increasing understanding more quantitatively, a Normalized Gain (N-Gain) calculation is carried out.

Table 8. N-Gain test results

	N-Gain
Valid	28
Missing	0
Mean	0.542
Std. Deviation	0.298
Minimum	0.000
Maximum	1.000

The average N-Gain score for students, as shown in Table 9, is 0.542. According to the N-Gain criteria, this score falls within the medium category. This indicates that the GeoGebra applet is moderately effective at improving students' understanding of the area and volume of geometric shapes. Although a statistically significant improvement has been demonstrated, the level of progress suggests there is still room for further optimization to achieve even greater effectiveness.

Evaluation of learning media

The assessment of the GeoGebra applet involved using questionnaires and interviews. The student response questionnaire consisted of 13 closed-ended questions and 3 open-ended questions, while the lecturer response questionnaire included 10 closed-ended questions and 3 open-ended questions.

Evaluation based on student responses

The analysis results of student responses to the developed GeoGebra applet are shown in Table 9.

Table 9. Results of the Analysis of the Average Score of the Student Response Questionnaire on the Practicality of the GeoGebra Applet

Practicality Aspect	Number of Indicators	Average Score (0–4)	Percentage (%)
Ease of Use (Usability)	4	3.68	92
Conceptual Utility	5	3.68	92
Motivation and Interest	4	3.68	92
Overall Practicality	13	3.68	92

Based on Table 9 above, it is clear that all indicators in the Ease of Use (Usability), Conceptual Utility, and Motivation and Interest categories received the same average percentage score of 92%. This percentage score indicates that all indicators are in the very practical range. It demonstrates that students generally have a positive and consistent perception of all the assessed aspects.

Qualitative data from open-ended questions and interviews further clarified students' understanding of the practicality of GeoGebra applets. First, they stated that both applets they developed were very practical because they could simply click on the applet link without installing the application.

Furthermore, most students said that the visualization and manipulation features for unit squares and unit cubes using sliders were very helpful in understanding the concepts of area and volume. These features helped them see how unit squares or unit volumes fill rectangles and cuboids.

The ability to resize rectangles and cuboids with sliders was also considered very useful. This is because students could see the animated images change instantly. It helped them explore ideas independently and gain a better understanding of how the area and volume formulas work.

While generally easy to use, some students experienced technical difficulties or found certain parts of the applet confusing. Issues mentioned included performance and connection problems, with some students reporting slow or lagging responses to the volume concept slider and long loading times when their internet connection was unstable.

Practicality based on lecturer responses

The results of the two lecturers' evaluation of the developed applet are shown in Table 10.

Table 10. Recapitulation of GeoGebra Applet Practicality Assessment by Lecturers

Practicality Aspect	Number of Indicators	Average Score (0–4)	Percentage (%)
Efficiency and ease of implementation	3	3.5	87.5
Support for the teaching and learning process	4	3.625	90.625
Flexibility and relevance of content	3	3.667	91.667
Overall Practicality	10	3.597	89.931

According to Table 10, the lecturers' evaluation of the GeoGebra applet's practicality received an average score of 3.597, placing it in the Very Practical category. This indicates that lecturers consider this tool very easy to use and likely to have a positive impact on the learning process.

The results of the qualitative data analysis further support these quantitative findings. Lecturer 1 highlighted the tool's main strength as its ability to improve students' understanding of area and volume concepts, as well as enabling them to experiment and draw conclusions from the relationship between the square/unit cube and the entire plane/dimensional figure in question. Lecturer 2 also mentioned that the tool is very user-friendly.

However, several potential obstacles were identified. Lecturer 1 mentioned unstable internet connections and the availability of student devices as common issues. Lecturer 2 pointed out possible difficulties if lecturers are unfamiliar with GeoGebra, especially regarding its technical terminology.

Discussion

Enhancing conceptual understanding with dynamic unitizing features

The improvement in students' conceptual understanding of area and volume can be explained through the hands-on experience provided by the dynamic unitizing feature in the developed applet. As discussed earlier, students previously tended to define area and volume solely using formulas without understanding their spatial meaning. This applet is specifically created to fill this gap by allowing students to directly manipulate measurement units, discover patterns, formulate concepts inductively, and transition from concrete visualization to abstract thinking.

First, this applet provides a feature to manipulate units of measure, specifically unit squares or unit cubes, which can be adjusted using a slider. This feature allows students to see how unit squares cover a rectangle or how unit cubes fill a cuboid, aligning with the conceptual definitions of area and volume (Bennett et al., 2012; Billstein et al., 2010; Özdemir & Özçakır, 2026; Zazkis, 2025). By directly manipulating these units, students can understand that the results of area and volume calculations actually represent the number of measurement units that make up a shape.

...I understand the concept of area and volume better, which I can adjust to my preferences, the size I want to find, and also change how many squares can fill the area or volume I've used earlier, the size I had previously determined.

From a visual standpoint, yes, sir. Because it shows how the area is composed in terms of square units and how the volume is made up of cube units. What's most interesting about it is the visual aspect.

This instructional design differs from conventional GeoGebra visualizations, which typically allow learners to manipulate completed geometric figures while the underlying measurement units remain implicit. By making unit squares and unit cubes the primary objects of interaction, the applet encourages students to reason about how equal-sized measurement units are iterated and spatially organized to construct area and volume. Rather than simply manipulating completed shapes, students actively construct area and volume by organizing unit squares into rows and columns and unit cubes into rows, columns, and layers. This learning process supports unitizing by enabling learners to conceive equal-sized units as iterable measurement units and promotes spatial structuring by organizing these units into coherent spatial configurations. These cognitive processes have been identified as fundamental to developing conceptual understanding of measurement (Battista, 2004; Battista et al., 1998; Battista & Clements, 1996; Lehrer, 2003).

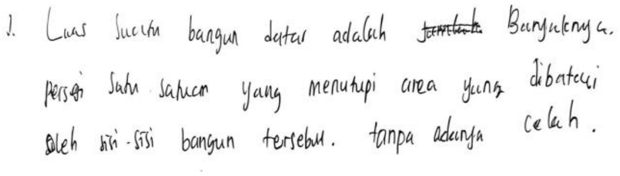
Second, through a series of systematically designed activities, students are invited to identify the relationship between length, width, height, or the number of filler units displayed numerically. This activity encourages students to inductively discover the relationship between the shape's dimensions and the number of units formed. Thus, the formula for the area of a rectangle $L = p \times l$ and the volume of a cuboid $V = p \times l \times t$ is not obtained through memorization, but through observation-based reasoning. Evidence of this process is seen in the results of student interviews which show that they are able to explain the reasoning behind the formula based on the number of filler units formed. Consequently, dynamic unitizing supports conceptual understanding by engaging learners in the cognitive processes underlying measurement rather than simply providing dynamic visual representations. This finding suggests that the educational value of the applet lies not in its visual appeal alone, but in the opportunities it provides for learners to repeatedly reorganize measurement units, test emerging hypotheses, and coordinate relationships among dimensions and units. Through these iterative actions, students gradually construct the multiplicative structure underlying the formulas for area and volume instead of treating the formulas as isolated symbolic expressions.

"*Well, then when it comes to the question below, I think it helps construct the definition of area and volume. ...*"

"*In the pre-test, I didn't know how to write the definition of area and volume, but after using that story, oh, the area can be written as how many small squares fill the entire shape and how many small cubes fill the entire shape....*"

"*Also, I didn't know that area, length times width, could be filled. That's really creative, sir. It can be filled with cubes and squares. Well, that really helps me visualize my understanding of it....*"

This claim is supported by students' responses when answering questions about the definition of area, as shown in Figure 5 below.


1. Luas suatu bangun datar adalah banyaknya persegi satu satuan yang menutupi area yang dibatasi oleh sisi-sisi bangun tersebut. tanpa adanya celah.

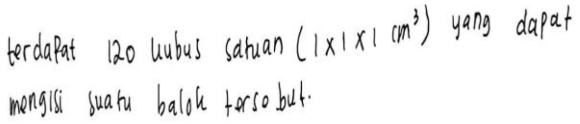
The area of a plane figure is the number of unit squares that completely cover the region bounded by its sides without any gaps.

Figure 5. Example of student answers

Third, applets offer tangible representations of previously abstract ideas. The concepts of unit squares and unit cubes, often just phrases in textbooks, become objects that can be visually manipulated. This helps bridge the gap between concrete representation and symbolic understanding of formulas, which is essential for developing conceptual knowledge (Hiebert & Lefevre, 1986), a finding that is further supported by recent studies on interactive visualizations and digital multi-representation environments (Merkel et al., 2026; Schröder et al., 2026).

"*For me, to understand area and volume, because area and volume are difficult, abstract subjects, sir. Well, that can be visualized, sir. You can imagine... Well, that can really visualize my understanding of it....*"

The findings of this study also show significant changes compared to the initial conditions of students. After using the applet, the results of interviews and observations showed that most students were able to explain the conceptual meaning of the value of a volume of a geometric figure. For example, when students were asked to explain the meaning of the value of 120 on a cuboid with a volume of $120\ cm^3$, students not only answered "length times width times height", but explained that the cuboid can be filled by 120-unit cubes measuring $1 \times 1 \times 1\ cm^3$ as seen in Figure 6. This shift indicates that students no longer interpreted volume merely as the result of multiplying three dimensions, but as a structured composition of unit cubes. As argued by Lehrer (2003), reasoning about volume is inherently more demanding because measurement units must be coordinated across three spatial dimensions simultaneously. The dynamic manipulation of unit cubes in the applet appears to support students in coordinating rows, columns, and layers, thereby facilitating the construction of volume as a structured arrangement of unit cubes rather than merely the product of three dimensions.



There are 120 unit cubes ($1 \times 1 \times 1\ cm^3$) that can fill the rectangular prism.

Figure 6. Example of student answers

These findings indicate that students developed a deeper conceptual understanding of volume. Rather than viewing the volume formula as a rule to be memorized, students came to understand it as a representation of the structured organization of unit cubes within a three-dimensional space. This interpretation is consistent with the literature highlighting the reciprocal relationship between conceptual knowledge and procedural knowledge, whereby conceptual understanding provides the foundation for meaningful procedural fluency (Kilpatrick et al., 2001; Rittle-Johnson & Schneider, 2014). This interpretation is further supported by another student's reflection comparing their understanding after using the applet with their previous learning experiences.

"*Of course, it's very different, sir, because at school I was taught directly the formula. So, when I went to the test yesterday, I was confused about how to explain the definition... so with this GeoGebra application, it can help me understand the actual definition of the area and volume of a shape, especially a cube or cuboid.*"

The above findings support previous research showing that GeoGebra can improve conceptual understanding of geometry (Ambarwati & Ambarwati, 2026; Anandita et al., 2026; Arbain & Shukor, 2015; Yahya & Yahya, 2026; Zulnaidi & Zamri, 2017). However, unlike previous studies that generally used GeoGebra as a visualization or demonstration tool, the applet developed in this study specifically emphasizes the exploration of dynamic unitizing, namely the process of forming units of measurement through direct manipulation of filler units. This design aligns with the constructivist approach, which emphasizes that knowledge is actively constructed through interaction with learning objects and direct experience (Bruner, 1966). Furthermore, the use of interactive media also provides opportunities for students to construct understanding independently through exploration, observation, and reflection (Darmayanti et al., 2025; Saharani & Abadi, 2024).

Motivation and self-confidence in independent learning

The results of the survey and interviews show that the developed GeoGebra applet can increase students' motivation and confidence in independent learning.

"*From the moment I was shown it, I was really interested, sir. It's interesting that you can adjust it to your abilities and desires. And the results also change. So, I'm more motivated. It's different from the others.*"

This increase in motivation leads to increased self-confidence, as expressed by the following student.

"*After studying the applet, I'm certainly more confident, sir. Because I can discover the concept myself. It's not just reading from a book. In books, we just read and memorize. Now, if we try it ourselves, we'll think, "Oh, I understand the concept, like this, like that." And maybe we can explain it to others.*"

This shows that the discovery-based approach and independent exploration through this applet are effective in building students' learning competencies.

The effectiveness of applets as a self-study tool outside the classroom was also a significant finding. This demonstrates their potential to support flexible, individual-based learning.

"*In my opinion, individual learning is extremely effective. Even without guidance, students can still use the application or website and experiment on their own, without requiring guidance from others. Because the complete procedures are already in place, using it is easier, simpler, and easier to understand, without requiring guidance from others*."

These results are consistent with several previous studies (Birgin & Yazıcı, 2026; Mawarni et al., 2024; Sagala & Sagala, 2023; Yorganci, 2026) which found that the use of GeoGebra can increase students' motivation to learn mathematics, promote their engagement in the learning process, and strengthen their confidence in understanding mathematical concepts. Pujianti (2025) further explained that the main factor influencing student learning motivation is the dynamic visualization provided by GeoGebra, an aspect also reflected in student responses regarding the interactive features of the developed applet.

Initial knowledge gap: Familiarity with GeoGebra

Although the applet is designed for ease of use, interviews revealed a gap in adaptation based on students' initial level of familiarity with the GeoGebra platform. Students who were already familiar with GeoGebra tended to adapt more quickly and understand the applet's interactive functions, such as using sliders and object manipulation.

"Maybe for those who are used to using GeoGebra, it's not a problem, it's easy. They already know the function of slides and so on. Because I'm used to using it, so it's like yesterday."

"When I first opened GeoGebra, it was because I'd used GeoGebra before, so I could immediately adjust to using the sliders."

In contrast, students with no prior background or experience using GeoGebra tended to face initial confusion.

"But for students who are unfamiliar with, or even unfamiliar with, GeoGebra, they might be a bit confused, sir, about what the slides are for, and so on."

This indicates that even though the applet is intuitive, prior experience with dynamic mathematics software like GeoGebra can be a determining factor in the speed of adaptation and efficiency of exploration. Therefore, in further implementation or development, it is necessary to consider having a short orientation or initial guide specifically for users who are less familiar with the GeoGebra interface to ensure that all students can make optimal use of the applet. Such an orientation can serve as a supplementary instructional resource that helps reduce initial barriers to using the software. This recommendation is consistent with previous literature highlighting the need for supplementary instructional materials to support the effective implementation of GeoGebra in mathematics learning (Azis & Rohaeti, 2025).

Study limitations and future research

Despite the positive findings reported in this study, several limitations should be acknowledged. First, the study employed a one-group pretest-posttest design without a comparison group. Therefore, although improvements in students' conceptual understanding were observed, alternative explanations such as testing effects or increased familiarity with the learning content cannot be entirely ruled out. Second, the relatively small sample size may limit the generalizability of the findings. Future studies involving larger samples and experimental or quasi-experimental designs are recommended to provide stronger evidence regarding the contribution of dynamic unitizing to students' conceptual understanding.

Conclusion

This study successfully developed an interactive GeoGebra applet based on the dynamic unitizing feature for the topic of area and volume that met the criteria of validity and practicality. Expert validation and participants' responses indicated that the developed applet is suitable for use as both an independent learning medium and a virtual laboratory that supports mathematical exploration. The findings suggest that the developed applet is a promising learning medium with the potential to enhance the conceptual understanding of prospective mathematics teacher. Its dynamic visualizations, intuitive interactive features, and scaffolding questions provided opportunities for students to actively explore and construct area and volume concepts independently. Furthermore, this study introduces dynamic unitizing as an instructional design principle for developing GeoGebra applets that foster conceptual understanding of area and volume. Rather than using GeoGebra solely as a dynamic visualization tool, the proposed approach emphasizes the manipulation of measurement units as the core learning activity, enabling students to construct measurement concepts through interactive unit-based exploration.

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