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Senior high school students' understanding of mathematical inequality

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Abstract

Mathematics inequality is an essential concept that students should fully understand since it is required in mathematical modeling and linear programming. However, students tend to perceive the solution of the inequalities problem without considering what the solution of inequality means. This study aims to describe students' mistakes variations in solving mathematical inequality. It is necessary since solving inequality is a necessity for students to solve everyday problems modeled in mathematics. Thirty-eight female and male students of 12th-grade who have studied inequalities are involved in this study. They are given three inequality problems which are designed to find out students' mistakes related to the change of inequality sign, determine the solution, and involve absolute value. All student work documents were analyzed for errors and misconceptions that emerged and then categorized based on the type of error, namely errors in applying inequality rules, errors in algebraic operations, or errors in determining the solution set, then described. The result shows that there were some errors and misconceptions that students made caused by still bringing the concept of equality when solving the inequalities problem. It made them did not aware of the inequality sign. Students are still less thorough in operating algebra and do not understand the number line concept in solving inequalities. The other factor was giving "fast strategy" to the students without considering the students' understanding.

Keywords: inequality; inequality error; students' mistake

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Introduction

Mathematics is always used to solve problems in other fields like biology, physics, chemistry (Li & Schoenfeld, 2019). We also often use mathematics to solve daily problems (Amalia et al., 2017; Widana, 2021). It can be said that mathematics is essential to understand. One of the mathematical concepts that students must understand is mathematical modeling. Mathematical modeling uses mathematics to illustrate and analyze real-world problems using the language of mathematics (Garfunkel et al., 2016).

Inequality is one of the requirements for students to model mathematics (Arseven, 2015). Mathematical inequalities are essential because they relate to other topics such as equations and their real-life applications (Moon, 2019). Solving equations and inequalities is essential in algebra, mainly its functions and applications (Mokh et al., 2019). It requires students to understand the method in finding the solution set for each inequality and equation (El-khateeb, 2016). Unfortunately, students tend to perceive the solution of the inequalities problem without considering what the solution of inequality means (Taqiyuddin et al., 2017). Hence, understanding and solving linear and quadratic equality and inequality are a necessity for all students.

There are three categories of conceptual errors in students' problem-solving work in inequalities based on Agung et al. (2021): error in algebraic operations, error in applying inequality rule, error in determining the solution set. Some mistakes that found in students grade 7,8,9,10 were combining unlike terms such as moving, deleting, or adding a variable, moving a term without changing its sign, changing/ not changing the direction of the inequality sign when inappropriate (Booth, Barbieri, Eyer, & Paré-Blagoev, 2014). First undergraduate students make errors in algebraic operations, determine the solution set, and apply the inequality rule (Agung et al., 2021). Most of the students incorrectly answer the question in conducting the algebraic operation (Daud & Ayub, 2019; Saputro et al., 2018; Taqiyuddin et al., 2017). However, there are still rare research results on student errors in solving inequalities in 12th grade. Based on this fact, we want to know what is also happening in students of 12th-grade high school. Moreover, we also want to know the factors that could make the mistakes and how they influence their understanding of the concept of inequalities itself.

This study aimed to know the students of 12th grade high school knowledge about the concept of inequalities, especially on the error made when solving the inequality problems and its reason. The result of this study can be used as the consideration for the teacher to teach the same materials for the next academic years. Moreover, it also can be used as preliminary guidelines for further research.

Methods

This study involved 38 students (30 female and eight male) of 12th-grade high school which already learned how to solve inequalities. The students are given three problems which have been said as valid by the validator, expertise in mathematics education. The three problems are about three different types of inequality and how to solve them. The question is aimed to

explore students' ability in solving questions related to inequalities and finding common mistake that occurs during the process of finding the solution to inequalities.
The given three problems about inequalities are,

1. *Find the value of* a *which satisfy:* $a - 3 \geq 3a - 9$.
2. *Find the solution of* $(x + 2)(x - 1)(x - 3) > 0$.
3. *Given,* $|a - 3| < |3a - 9|$. *Determine the solution of the given inequality.*

Problem 1 is given to find out students' behavior in solving linear inequality related to the number and or variable operations—especially division and multiplication by negative number and its relation with the inequality sign. Problem 2 is used to find out students' behavior in solving inequalities that involve polynomials. However, we do not focus on finding students' abilities or difficulties in factorizing the polynomial. We only want to know about students' thinking in determining the interval of possible solutions and determining the correct interval that satisfies the given inequality. While problem 3 is about solving inequality, which involves absolute value. Since the form is $|x| < |y|$ it will lead to quadratic inequality. We want to challenge students concept of determining the interval of solution which they had since secondary school, which is labelling the interval by "+" and "-" consecutively.

From the 38 students' answers, there are three error types: error in determining solution set, error in algebraic operation, and error in applying inequality rule (Agung et al., 2021). All the variations of error that happened in each category are qualitatively described. The students' reasons for their answers are dug through interviews. The interview is done for the students who represent each variation of error.

Results

The 38 students' answers to three given questions are analyzed, coded, and categorized into three categories of error: error in determining solution set, error in algebraic operation, and error in applying inequality rule (Agung et al., 2021). Each type of error is analyzed further and coded based on the error variation. We found two different errors for the first category, errors in applying the inequality rule. As for the second category, errors in algebraic operations, there are two errors. In the last category, the error in determining solution set, seven different errors occur. The error variations of each category are shown in Table 1.

Table 1. Error variation for each category

Question	Wrong Answer	Error Category	Number of students who make the error	Error variation
1	25	errors in applying the inequality rule	20	1) Students do not change the inequality sign as he divide the inequality with negative number

Question	Wrong Answer	Error Category	Number of students who make the error	Error variation
			5	2) Students does not understand the meaning of $6 \geq 2a$.
	2	errors in algebraic operations	1	3) Student' mistake in eliminating a constant in an inequality
			1	4) Does not know what to be done in the next step
2	10	error in determining solution set	3	5) Students mistake related to the procedure called finding the value of x which makes a zero.
			7	6) Students mistaken $(x+2)(x-1)(x-3) > 0$ as $(x+2)(x-1)(x-3) = 0$.
3	36	error in determining solution set case of absolute value inequality	26	7) Students get a wrong conclusion about $(a-3)(a-3) > 0$.
			5	8) Students only check one possible range of solution.
			3	9) Students wrong definition of $\|a-3\| < \|3a-9\|$
			1	10) Student conclude that the value of x which makes zero, as the solution of the inequality.
			1	11) Student thinking of $a-3 > \sqrt{0}$

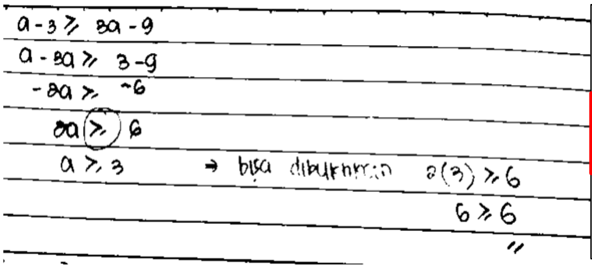
Even though students have already learned about solving inequality, several errors occurred in each question and category. The most error occurred in determining the solution set category, especially those involving absolute value. Students make a wrong conclusion about $(a-3)(a-3) > 0$. They believe that the solution is $(a-3) > 0$ so that $a > 3$. The second last common error happens is that students do not change the inequality sign as he divides the inequality with a negative number. It means that teacher needs to pay more attention to those errors when teaching mathematics inequality.

Another thing to be noted is that most students do not check each range of possible solutions. Instead, they check one possibility and then consecutively put "+, -"signs. It can be seen that while answering question 3, five students only check one possible range of solutions. The same tendency was also found out while looking at students' correct answers on question 2, in which they only checked one possible solution range.

In order to find out more about the students' errors, we chose a subject to represent each error variation, analyzed their works, and interviewed them regarding their thinking if needed. The description of each variation of students' error while solving three questions related to mathematics inequality and its reason follows.

Error in inequality rule

Given question 1: $\boldsymbol{a-3 \geq 3a-9}$ with aims to challenge students' mistake related to the change of inequality sign as it multiplied by or divided by negative number.



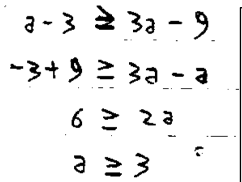
$a-3 \geq 3a-9$
$a-3a \geq 3-9$
$-2a \geq -6$
$2a \geq 6$
$a \geq 3$ can be proved $2a \geq 6$
$6 \geq 6$

Figure 1. Students does not change the inequality sign as he divide the inequality with negative number

Figure 1 tells that the students do not change the direction of the inequality sign after he multiplied or divided the inequality with -1 (row 4th). The properties of real numbers related to this were already given, yet they still make this mistake.

One student tried to check whether the obtained solution was correct or not by substituting one value of to the inequality. As we can see from Figure 1, he obtained the solution of $a-3 \geq 3a-9$ is $a \geq 3$ which is actually incorrect, the correct one is $a \leq 3$. However, when he checks his answer by only substituting the value of equals 3 to the inequality, the result shows that his answer is correct. It is happened since $a=3$ is a solution. However, the chosen value does not represent all the solution. He does not check for $a>3$, $a \in R$, yet conclude that his solution is satisfy the inequality. Based on the interview, it is known that he doesn't fully understand that $a \geq 3$ is $a=3$ or $a>3$.

Another mistake that occurs during the process of solving the given inequality is that five students indicate that they do not understand the meaning of $6 \geq 2a$ (Figure 2).



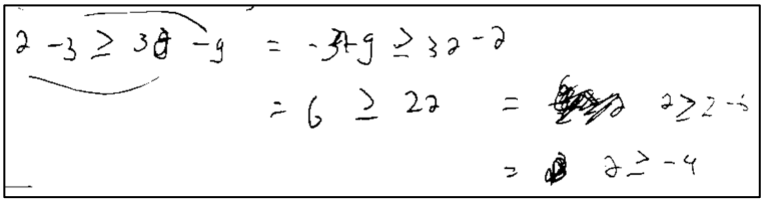
$a-3 \geq 3a-9$
$-3+9 \geq 3a-a$
$6 \geq 2a$
$a \geq 3$

Figure 2. Students do not understand the meaning of $6 \geq 2a$

In other words, we often write the solution with variables first. However, it is known that they only change the position of the variable and the scalar by ignoring the direction of the inequality sign. It means that they do not find the meaning of the inequality sign meaningful, or he does not understand what the inequality $(6 \geq 2a)$ means. What is essential for them is only the operation of the numbers. One of the reasons found out from the interview is that they only follow the procedure explained or given as an example by the teacher when studying the inequality.

Error in algebraic operation

There is a student who thinks that to solve $\boldsymbol{a - 3 \geq 3a - 9}$**,** just eliminate a constant or a coefficient by only adding or subtracting it with another number (Figure 3)



$$a - 3 \geq 3a - 9 = -3 + 9 \geq 3a - a$$
$$= 6 \geq 2a = 2 \geq 2 - 6$$
$$= a \geq -4$$

Figure 3. Student's mistake in eliminating a constant in an inequality

Based on Figure 3, we can see that the student first made the terms contain variables to be on the same side by changing the side, which is actually subtracting each side with a and then adding each side with 9. So that he get $-3 + 9 \geq 3a - a$ which is equal to $6 \geq 2a$. Then, to get the value of a he does the same step as before, that is by subtracting each side with 6 so that he get -4.

Actually, at first, he was doing right by dividing each side with 2 to get the value of a. However, he changed his mind and was taught that he could get the value by subtracting each side with 6 as he did in the previous step. Then, he gets $a \geq 2 - 6$ and $a \geq -4$. However, he does not realize that the procedure had done should get $0 \geq 2a - 6$. Beside those type of mistake, there is only a student who could not solve the inequality. he stops at $-2a \geq -6 \Leftrightarrow 2a \geq \cdots$. Yet, we do not know for sure why he stops at that step.

Error in determining the solution set

Given question: $(x + 2)(x - 1)(x - 3) > 0$, find the solution.This question is focused on how students determining the range of solution. No difficult calculation is needed. They only need to find the value of x so that $(x + 2)(x - 1)(x - 3) = 0$ and then find the possible range of solutions, and determining the right range of solution by doing a point test (substituting a value of x in the particular area or range to the inequality and see whether that value satisfy the inequality or not).

They conclude that $(x+2)>0,(x-1)>0, or\ (x-3)>0$. So that, $x>2, x>1, or\ x>3$. The strangest thing is that $x>2, x>1, or\ x>3$ do not have anything with the acquired solutions. They only used the number 2,1, and 3 to determine the interval of possible solution (Figure 4).

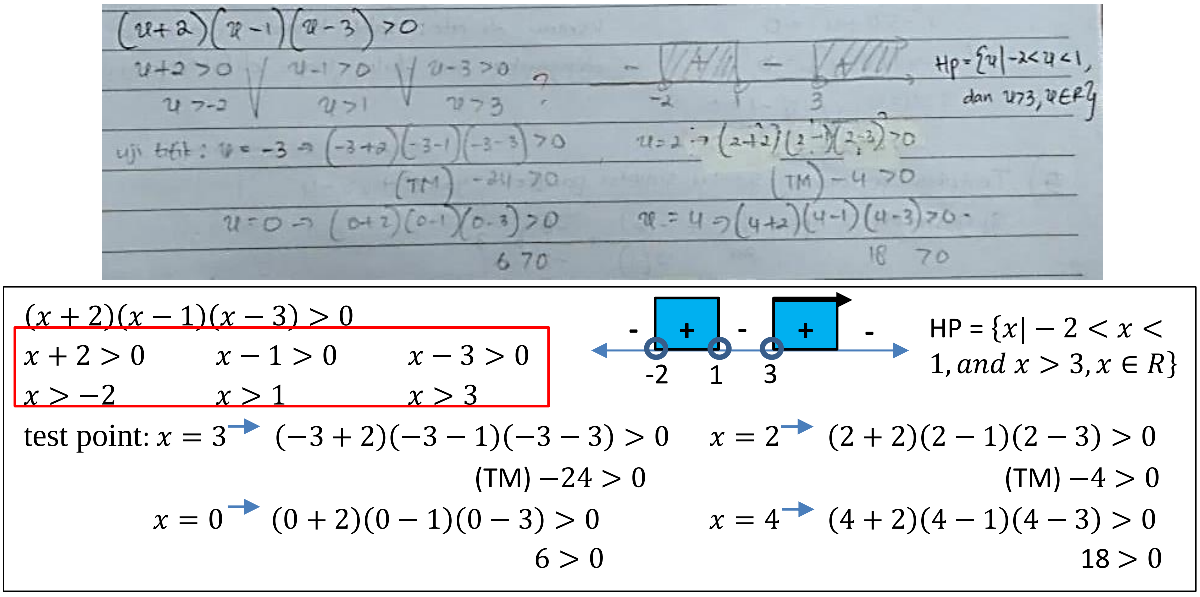


Figure 4. Students mistake related to the procedure called finding the value of x which makes a zero

This means that $(x+2)>0,(x-1)>0, or\ (x-3)>0$ do not have any meaning for them.

Failed to understand the meaning of $(x+2)(x-1)(x-3)>0$, yet make it the same as the meaning of $(x+2)(x-1)(x-3)=0$ makes the students wrongly decided on the solutions as shown at Figure 5.

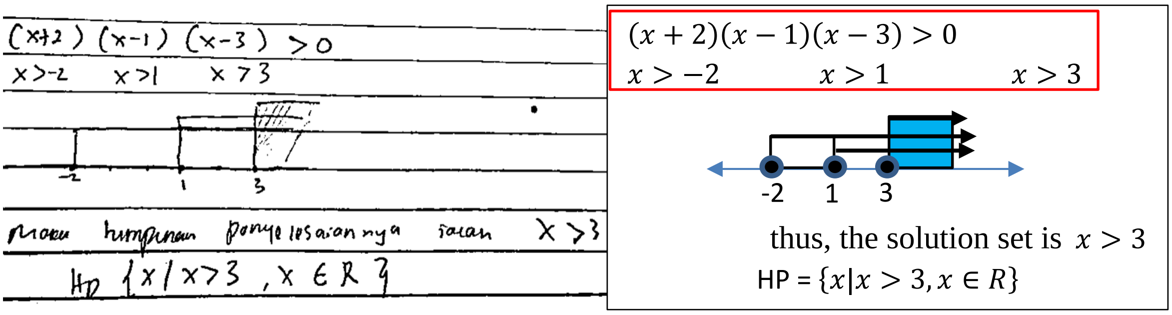
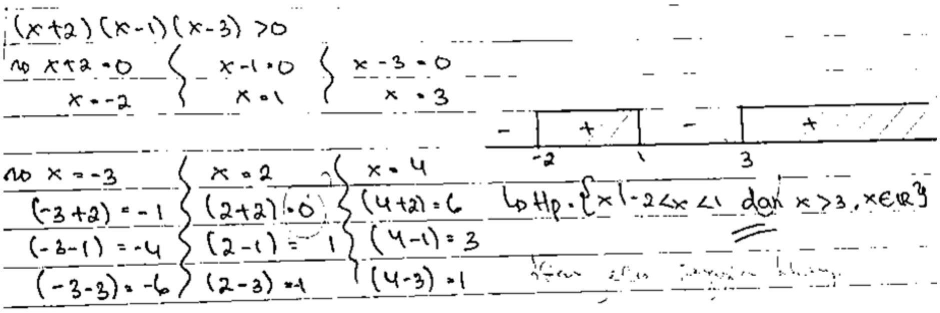


Figure 5. Students mistaken $(x+2)(x-1)(x-3)>0$ as $(x+2)(x-1)(x-3)=0$.

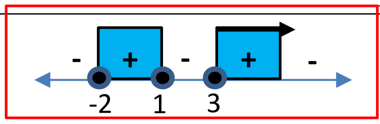
The Figure 5 tells that the students do not understand the meaning of $(x+2)(x-1)(x-3)>0$ which is there are possible values of $(x+2),(x-1),(x-3)$ so that $(x+2)(x-1)(x-3)>0$, they are $(x+2),(x-1),(x-3)$ should be positive and two of them are negative and the other one is positive. Yet, what the students do is make the meaning of $(x+2)(x-1)(x-3)>0$ analogue with $(x+2)(x-1)(x-3)=0$ which solution is required by finding the value of $(x+2)=0\ or\ (x-1)=0\ or\ (x-3)=0$. Hence, they conclude the possible solutions are $x>-2, x>1, and\ x>3$. So, they get the solution of the given inequality is $x>3$.

As we analyzed further about students' strategy in deciding the solution interval, it shows that students only check one possible interval and put sign "+", "-" consecutively (Figure 6). We do not put in on the misconception since it leads to the correct answer. However, this strategy needs to be considered for the student's answers to question 3 since this strategy will not lead to the correct answer for the inequality given in question 3.



$(x+2)(x-1)(x-3) > 0$

$x+2=0$	$x-1=0$	$x-3=0$
$x=-2$	$x=1$	$x=3$



HP = $\{x \mid -2 < x < 1, and\ x > 3, x \in R\}$

→ $x=-3$	$x=2$	$x=4$
$(-3+2)=-1$	$(2+2)=0$	$(4+2)=6$
$(-3-1)=-4$	$(2-1)=1$	$(4-1)=3$
$(-3-3)=-6$	$(1-3)=-1$	$(4-3)=1$

Figure 6. Students always put sign "+","-" consecutively

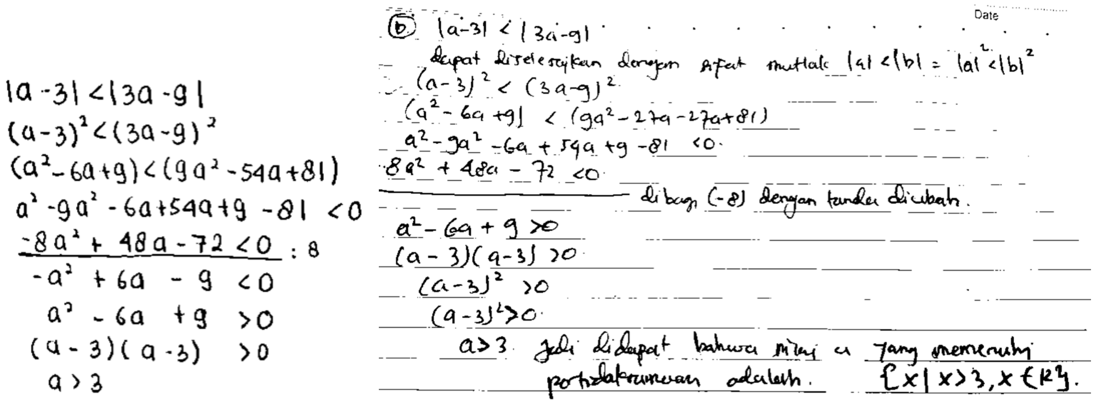
We can see from the figure 6 above that even though the students already find the value of x which makes zero and determining the possible interval of solutions correctly, however there is something strange when they check or test a value of x in each interval. Acquired solution of the given inequality is right, yet the process of testing a value of x in each interval can not justify their conclusion. First, the students only check the value of a x in three intervals and exclude $(-2,1)$ and suddenly conclude that the value of x in that interval are solutions or satisfy the inequality. Second, for interval $(1,3)$, they choose $x = 2$ to be checked, and they get and conclude that $(0)(1)(-1) < 0$. Yet, if calculate it carefully, $(0)(1)(-1) = 0$ not less than 0. This indicate that when doing the test, students do not really calculate the numbers but only pay attention to the positive and negative sign. So, for the case $(0)(1)(-1)$, they see that $(+)(+)(-) = (-)$, hence the value of x in interval $(1,3)$ do not satisfy the inequality.

For instance, if he finds that when $x < -2$, the inequality is unsatisfied or $(x+2)(x-1)(x-3) = (negative) < 0$ then for the next interval $-2 < x < 1$, the value of x will satisfied the inequality or $(x+2)(x-1)(x-3) = (positive) > 0$. And the next interval must be negative, then positive. This can be drawn as,

When we asked their reason during the interview, they answered that the teacher was teaching them. When we ask further whether they ever find the question in which the "-+" sign could not be put consecutively, they claim that the question has always satisfied the procedure.

Error in determining the solution set of absolute value

Given question below:
Find the solutions of $|a - 3| < |3a - 9|$
Common mistake that occurs (26 from 38) is they conclude that from $(a-3)(a-3) > 0$ the value of a is greater than zero (Picture 7).

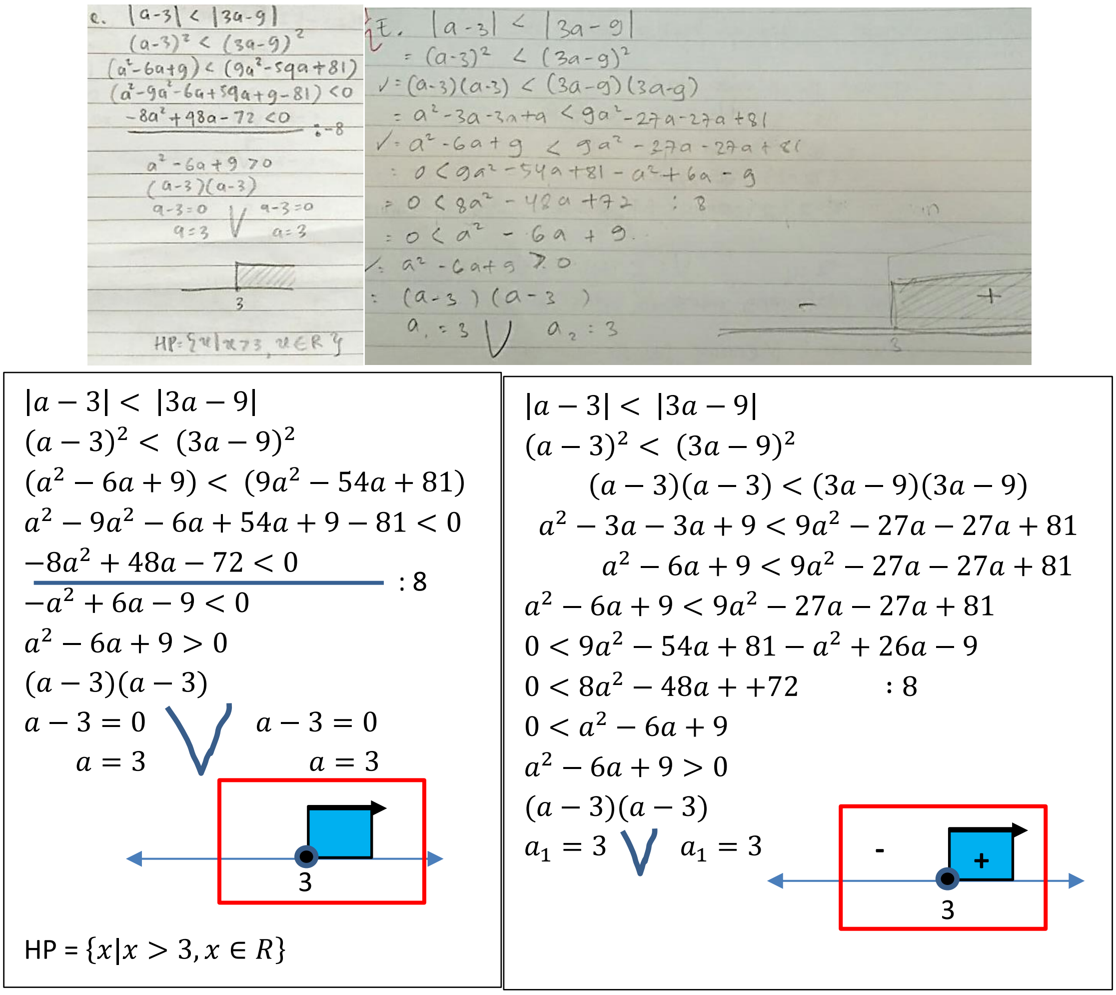


$\|a-3\| < \|3a-9\|$ $(a-3)^2 < (3a-9)^2$ $(a^2-6a+9) < (9a^2-54a+81)$ $a^2-9a^2-6a+54a+9-81<0$ $-8a^2+48a-72<0$: 8 $-a^2+6a-9<0$ $a^2-6a+9>0$ $(a-3)(a-3)>0$ $a>3$	$\|a-3\| < \|3a-9\|$ can be solved by absolute nature $\|a\| < \|b\| =$ $\|a\|^2 < \|b\|^2$ $(a-3)^2 < (3a-9)^2$ $(a^2-6a+9) < (9a^2-54a+81)$ $a^2-9a^2-6a+54a+9-81<0$ $-8a^2+48a-72<0$ divided by (-8) with changing the sign $a^2-6a+9>0$ $(a-3)(a-3)>0$ $(a-3)^2>0$ $a>3$ Thus, a value that fill up the inequality is $\{x\|x>3, x\{R\}\}$

Figure 7. Students wrong conclusion about $(a-3)(a-3) > 0$

Figure 7 shows that students solve $(a-3)(a-3) > 0$ as they solve $(a-3)^2 = 0$ which means that $(a-3) = 0$. So, they make an analogy that $(a-3)(a-3) > 0$ means $(a-3) > 0$. Hence, they get $a > 3$. In other word, they do not realize that $(a-3)(a-3) > 0$ means that $(a-3)$ could be greater than or less than zero.

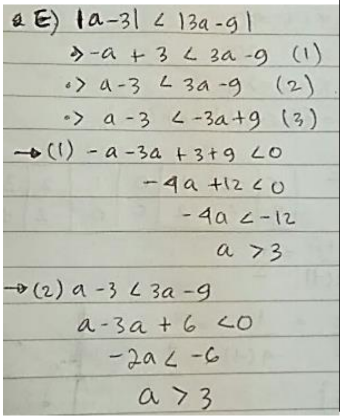
Second common mistake that happened is that even though they already determine the value of a which makes 0, they only check one value of a as a representative of the value of a in one possible range of solutions (Figure 8). They tend to ignore or do not check the other possible range or area of solutions $(a < 3)$ and just conclude that if one side or range which satisfy the inequality then the other side will not satisfy the inequality. This kind of thinking also shown in the task 2.



(a) (b)

Figure 8. Students only check one possible range of solution

Another mistake occurs while students are trying to solve inequality that involve absolute value is related to the properties of absolute value itself. For instance, instead of using the properties of absolute value in which $|a| > |b|$ can be written as $a^2 > b^2$, students try to define $|a - 3| < |3a - 9|$ as $|x| < scalar\ or\ scalar\ < |x|$ but they failed. They define $|a - 3| < |3a - 9|$ as: (i) $-(a - 3) < 3a - 9$ or (ii) $(a - 3) < (3a - 9)$ (Figure 9). Since the definition is wrong, the answer or solution is also wrong.



$|a-3| < |3a-9|$

- $-a+3 < 3a-9 \quad (1)$
- $a-3 < 3a-9 \quad (2)$
- $a-3 < 3a-9 \quad (3)$

$(1)\ -a-3a+3+9 < 0$

$-4a+12 < 0$

$-4a > -12$

$a > 3$

$(2)\ -a-3 < 3a-9$

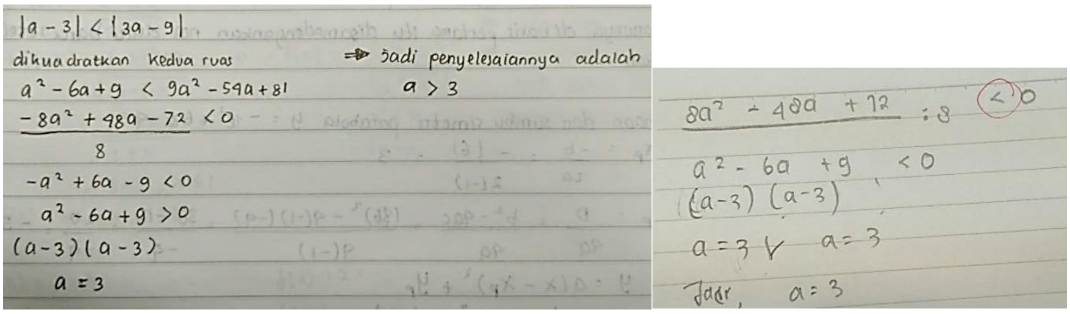
$a-3a+6 < 0$

$-2a > -6$

$a > 3$

Figure 9. Students wrong definition of $|a-3| < |3a-9|$

The last mistake is not about the properties of absolute value, but more about the calculation itself. As it happened in the task 2, in task 3, there is also student who conclude that the value of x which makes zero, in this case is $x = 3$ as the solution of the inequality (Figure 10).



(a)

$|a-3| < |3a-9|$

squaring both side thus, the solution is $a > 3$

$a^2-6a+9 < 9a^2-54a+81$

$\frac{-8a^2+48a-72}{8} < 0$

$-a^2+6a-9 < 0$

$a^2-6a+9 > 0$

$(a-3)(a-3)$

$a = 3$

(b)

$\frac{8a^2-48a+72}{:8} < 0$

$a^2-6a+9 < 0$

$(a-3)(a-3)$

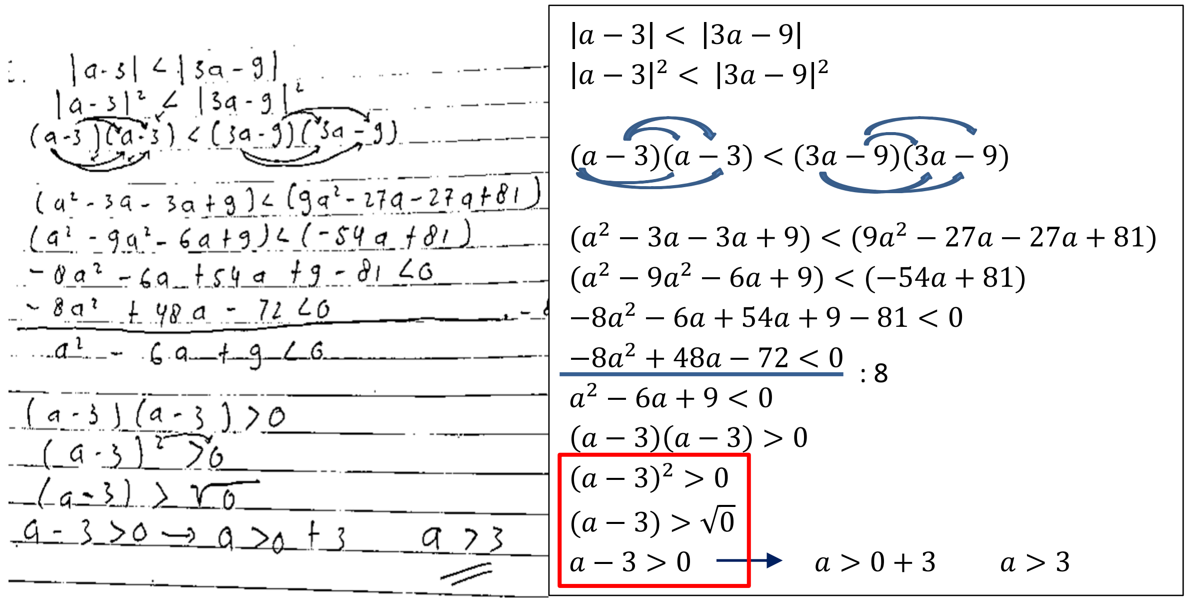
$a = 3 \ \lor \ a = 3$

thus, $a = 3$

Figure 10. Student's mistake in determining the solution of an inequality

In fact, $x = 3$ will result $(a-3)(a-3) = 0$. In other case, even though the student already find the value of x which makes zero, without doing point test the student conclude that $a > 3$. Doing point test or re-check whether the acquired solution will satisfy the inequality or not indeed will really helpful.

Beside mistaken the value of x which makes zero as the solution, there are also student who conclude that $a-3 > \sqrt{0}$ so $a-3 > 0$ and $a > 3$ as shown in the Figure 11.



Picture 11. Student's thinking of $a - 3 > \sqrt{0}$

Yet, $x^2 > a$ *means* $(x)(x) > a$ so there are some possibilities of x they are both x are positive $(x > 0)$ or both x are negative $(x < 0)$. if $(x - 3)(x - 3) > 0$ then $(x - 3) > 0$ or $(x - 3) < 0$. While what student do is only focus on $(x - 3) > 0$.

Discussion

For error in inequality rule at problem 1, the most common mistake by the students is that students tend not to change the direction of the inequality sign when the inequality is multiplied or divided by a negative number (Botty et al., 2015; El-khateeb, 2016; Taqiyuddin et al., 2017). More than 50% of the students who answer this question incorrectly do not change the direction of the inequality sign after multiplying or dividing $-2a \geq -6$ with -1 (Figure 1). This is because they only remember the rule or because they do not careful with the operation involving negative number in an inequality. Another error is student only checks solution just substitute a. If he check for other value of a, he will realize that his solution is incorrect (Figure 1). This may be because he doesn't fully understand that $a \geq 3$ is $a = 3$ or $a > 3$. We can see that it is important to explain the meaning of inequalities sign, what the meaning of solution in equalities (Almog & Ilany, 2012). Another error is student only changes the position of variable and the scalar by ignoring the direction of the inequality sign (Figure 2). Means that, he does not find the meaning of the inequality sign important or he does not understand what the inequality means (Almog & Ilany, 2012).

For error in algebraic operation at problem 1, in Figure 3, student miscalculated. He get $a \geq 2 - 6$ and $a \geq -4$, the procedure had done should get $0 \geq 2a - 6$. Errors that occur are usually students cannot perform the completion procedure correctly or students are wrong in doing calculations (Yuwono et al., 2021). A Student also does not know what to be done in the next step.

For the problem 2, almost all students can answer it correctly. However, if we look further their answer especially in the strategy they used in detemining the solution set, it shows that all the student who answer correctly (28 students) only check one range of possible solution and then put "+-" sign consecutively (Figure 6). As we interview them, they do at as the procedure taught by their teacher. Students do not really calculate the numbers but only pay attention to the positive and negative sign in number line (Figure 6). The way to solve that problem similar with research results by (Musafir & Susiswo, 2021) There is possibility that the students do not really check the value of x in each interval, instead they only checks for one interval and conclude that the next or previous interval would have the opposite conclusion. Even though many students answered correctly, there were still errors. The most common error is they failed to understand the meaning of $(x+2)(x-1)(x-3)>0$ (Figure 4, Figure 5). The way to solve that problem similar with research results by Anggoro and Prabawanto (2019) and Pratiwi and Rosjanuardi (2020). They only know the procedure that they need to determine the possible interval for solutions by doing calculation but without knowing the reason why do the procedure should be determining the value of x which makes zero (Jupri & Sispiyati, 2020).

Error in determining the solution set also happen in absolute value. From 38 students, there is no one who answer this question correctly. Few students that understand and can solve of the absolute value (Curtis, 2016; Panaoura, 2014). Common mistake is they conclude that the value of a is greater than zero (Figure 7). This happens because students just solved without check compatibility and use deeply understanding of the task (Almog & Ilany, 2012). Second common mistake,same with task 1, they only check one value of a as a representative of the value in range of solutions (Figure 8). Another mistake is the definition from absolute value itself is wrong (Figure 9). The teacher should embed the concept of absolute value (Almog & Ilany, 2012). The last mistake same with task 2, in task 3, there are also students who conclude that the value of x which makes zero is the solution of the inequality (Figure 10). Other mistake, students do is only focus on $(x-3)>0$ (Figure 11) similar with Figure 7.

Conclusion

Based on our conducted research, we found that most students did not fully understand the concept of inequalities. It was indicated by some mistakes that occurred in their work. The first factor that made some mistakes in solving inequalities occurred was because the students still bring out the characteristic of equalities while solving the inequalities problems. It makes them did not aware of inequalities signs. The suggestion for the next learning process was to introduce them to the number line for $x>a, x<a, x=a\ x\leq a, x\geq a$ without any inequalities problems. It was to make sure them really aware about the meaning of the inequalities sign.

The other factor which contributed to the students' mistake was the "fast strategy" for linear or polynomial inequalities that the students applied for the absolute inequalities. This strategy might help them solve some inequalities problems and obscure the concept of inequalities itself. It could happen if the understanding of the concept were immature. To avoid this mistake, the teacher should not introduce the "fast strategy" before confirming the students' comprehension of the inequalities.

In the following study, we can expand the other research to find the learning methods or strategies preventing students from avoiding mistakes. Considering that the undergraduate students involved in this study came from the science department during their senior high school, further research might also be applied in senior high school for the science department.

Conflicts of Interest

The authors declare that no conflict of interest regarding the publication of this manuscript. In addition, the ethical issues, including plagiarism, misconduct, data fabrication and/or falsification, double publication and/or submission, and redundancies have been completely by the authors.

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