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## Learning trajectory of quadrilaterals learning using the context of *Burongko Bugis* cake to improve students' critical thinking

## Andi Aras<sup>1\*</sup>, Fawziah Zahrawati<sup>2</sup>, Zulfiqar Busrah<sup>1</sup>, Claver Nzobonimpa<sup>3</sup>

<sup>1</sup> Mathematics Education Departement, Institut Agama Islam Negeri Parepare, South Sulawesi, Indonesia

<sup>2</sup> Social Science Education Departement, Institut Agama Islam Negeri Parepare, South Sulawesi, Indonesia

<sup>3</sup> Departement of Socio Anthropology, University of Burundi, Bujumbura, Burundi

\* Correspondence: andi.aras@iainpare.ac.id © The Authors 2022

### Abstract

One goal of learning mathematics is to foster students' critical thinking skills, but this has not been realized well in Indonesia. This study aims to produce a learning trajectory that can help students grow their critical thinking skills in studying quadrilaterals using the Burongko cake context. This study uses a design research method that comprises three stages, namely (1) preliminary design, (2) experimental design, and (3) retrospective analysis. This research was conducted at Madrasah Tsanawiyah As'adiyyah Bontouse with 25 students, nine males and 16 females of class VIII B. The results showed that this learning trajectory could improve students' understanding of learning quadrilaterals with ease and fun because mathematical concepts are associated with students' daily habits. The resulting learning trajectory is relevant to indicators of critical thinking skills, including interpretation, analysis, evaluation, and decision making. It was found that there was no significant difference in critical thinking skills between students who had high mathematical abilities and low mathematical abilities. Subjects with high mathematical abilities could carry out critical thinking processes to solve problems much better than subjects with low mathematical abilities. The results provide information about the use of local *Bugis* culture as a context of learning mathematics to improve students' critical thinking.

Keywords: critical thinking; design research; learning trajectory; traditional Bugis cakes

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## Introduction

Students' basic skills in the 21st century are critical thinking, communication skills, collaboration skills, and creativity (Aktaş & Ünlü, 2013). If students have critical thinking skills, they will try to find the truth by thinking systematically, interpreting problems, analyzing them well, and solving them using appropriate methods (Garisson et al., 2001). Thus, critical thinking is one of the basic skills that students must develop to become successful people in the 21st century.

The government has made various efforts to foster students' critical thinking skills through formal educational institutions. As written in the rationale for developing the 2013 curriculum, which mandates that learning be carried out oriented to developing students' critical thinking skills (Wanelly & Fitria, 2019; Zahrawati, 2020; Aras, 2020). Thus, each learning activity improves students' critical thinking skills. It includes learning mathematics.

Learning mathematics has the function of fostering critical thinking skills (Martyanti & Suhartini, 2018). The characteristics of mathematics subjects who study systematic thinking patterns consistent with the system, proof-based on definitions and axioms, clear, and accurate (Hendriana et al., 2019). So that when students learn mathematics, they learn to think critically.

Although it has been explained that mathematics learning activities can equip students with critical thinking skills, in reality, this has not been fully realized in Indonesia. The results of the International Trends in International Mathematics and Science Study (TIMSS) state that students in Indonesia are consistently ranked at the bottom. Indonesian students were in the bottom seven of 45 countries in 2015, and four years later, in 2019, Indonesian students were still in the bottom five of 50 countries (Rastuti & Prahmana, 2021). Students' critical thinking skills are essential to become the object of research on these problems.

Based on the observations at Madrasah Tsanawiyah As'adiyyah Bontouse, there are four indicators of critical thinking skills: interpretation, analysis, evaluation, and decisions. Students are only at the interpretation stage, which can describe the meaning of the geometric mathematical problem. However, they could not plan problem-solving (analysis) and determine methods for solving existing problems (evaluation). Students have difficulty solving problems such as story questions and pictorial questions, especially those that are a combination of 2D shapes. It is in line with research findings by Lestari et al. (2021), which concluded that critical thinking involves several activities, such as implementing information about new situations, analyzing the causes of phenomena, and evaluating opinions, to draw reliable conclusions. However, the learning process tends to be teacher-centered, thus preventing students from actively taking part in the learning process, which causes their critical thinking skills to be low.

Seeing these problems requires efforts to cultivate students' critical thinking skills in solving geometric problems. Based on the theory of Van Hiele, when a student studies geometry, five stages of thinking levels will be passed by the child, namely visualization, analysis, abstraction, deduction, and rigor (Crowley, 1987). Several researchers have studied this, including Pujawan et al. (2020), which conclude that a characteristic of Van Hiele's level of thinking is that activities more influence the speed of moving from one level to the next in learning. Thus, the organization of learning, content, and materials is an essential factor.

It aligns with Andila and Musdi's (2020) research, which concludes that teachers play an essential role in encouraging speed to move through a level. Higher levels of thinking can only be achieved through a learning process that is connected with things that students often encounter every day so that they can understand the properties of concepts and identify geometric shapes based on an informal analysis of their parts and attributes. The research results of Adhetia and Suhartini (2018) concluded that geometry learning that uses local culture as a learning medium could grow students' critical thinking skills.

Learning innovation using cultural contexts is very important to grow students' critical thinking skills. In the teaching process, the material will be linked to things that students often encounter every day to identify and relate parts of culture as a medium for learning geometry and mathematics. Thus, selecting learning activities under the students' thinking stage is necessary to help students reach higher thinking stages (Fitri & Prahmana, 2020).

Various previous studies have found that learning mathematics in a local context can make it easier for students to understand the material being taught and improve their critical thinking skills (Busrah & Pathuddin, 2021; Sutarto et al., 2021; Risdiyanti et al., 2019; Risdiyanti & Prahmana, 2020; Prahmana, 2015; Fitri & Prahmana, 2020; Prihastari, 2015; Supiyati et al., 2019; Pujawan et al., 2020). Pathuddin et al. (2021) have explored the *Burongko Bugis* cake as a source of learning mathematics and concluded that making *Burongko* involves mathematical concepts such as division, congruence, and similarity, as well as triangular prisms. However, there has been no research that aims to develop a trajectory of learning mathematics in a Burongko cake to help students understand the concept of quadrilaterals and develop their critical thinking skills.

Burongko cake is a traditional Bugis cake in the shape of a trapezoidal prism packaged traditionally using banana leaves. In ancient times, Burongko bananas were classified as luxury foods and were only especially served to the nobility of the Bugis kingdoms. Burongko contains philosophy and cultural values that have implied meanings. It is a traditional cake that is always served in every traditional event such as weddings, *mabarasanji*, *menretojang*, and *matampung*. Most Bugis people refer to Burongko as an honest cake. The main ingredient, which is made of banana and then re-wrapped with the same plant as the base material (banana leaf), represents honesty that it must be the same as what is seen on the outside with what is stored inside us (Pathuddin et al., 2021).

Because no research examines a learning trajectory that can help students grow their critical thinking in studying quadrilaterals using the Burongko cake context, the purpose of this research is to design local instruction theory of geometry learning to use Burongko Bugis cake media as a local context based on Van Hiele's theory to improve students' critical thinking. In addition, this study also aims to describe the profile of students' critical thinking according to Van Hiele's level of thinking after the learning process.

## Methods

The method used in this research was design research which aimed to build a learning trajectory of quadrilaterals to improve learning activities in the classroom and improve students' critical thinking skills by conducting an interactive analysis of the allegations of what is happening in the classroom and its implementation (Akker et al., 2006). There are three stages in this research, namely (1) preliminary design, (2) experimental design, and (3) retrospective analysis.

At the preliminary design of the Hypothetical Learning Trajectory (HLT), a literature review was conducted on learning models with a cultural context based on Van Hiele's theory. The results are as developing learning activity designs to achieve the learning objectives made at each stage, namely building a relationship between the objectives of each learning stage, indicators, and the conjecture of each activity. The resulting learning design is known as an HLT, which experts then validate. The conjecture of HLT is planned on learning objectives, learning activities, and tools to assist the learning process.

The next stage is the experimental design. At this stage, the Hypothetical Learning Trajectory that has been designed is tested. The researchers observe and analyze the learning activities that occur during the learning process in the classroom. This process aims to evaluate the conjectures in the learning activity. All teaching trials were recorded using photo documentation. The students' work was collected for analysis, and several students were selected to be interviewed.

The last stage is a retrospective analysis. After the experimental design, the data obtained from the learning activities and students' critical thinking skills were analyzed to evaluate the success of the learning activities that had been implemented. The results are a local instruction trajectory for learning to construct a quadrilateral using the context of the Bugis Burongko cake.

The subjects of this study were the class VIII B students of Madrasah Tsanawiyah As'adiyyah Bontouse, 25 students (9 males and 16 females). Then, two students were selected, each of whom had high mathematical and low mathematical abilities, which were obtained after the learning process. Each subject was asked to describe his mental activity in solving a problem. It is done to explore the subject's critical thinking process and other solutions that cannot be seen in the students' written work.

The instruments used in this study were (1) a learning implementation activity sheet that refers to the HLT quadrilaterals using the context of the Bugis Burongko cake, (2) a problemsolving ability test, and (3) an interview guide sheet. A team of experts has validated the instrument. The data got were analyzed retrospectively with HLT.

The data analysis carried out in this study was to compare the results of observations during the learning process with the HLT designed at the preliminary design stage. The stages of retrospective analysis are data analysis of critical thinking skills, reflection, interpretation of findings, and the formulation of recommendations for subsequent research.

## Results

## Stage 1: Preliminary design

The researchers conducted a literature review to design the Hypothetical Learning Trajectory (HLT) in a preliminary design. They implemented the initial idea of using the traditional Bugis Burongko cake context in learning the trapezoid shape. This Burongko cake was chosen based on preliminary research conducted by Pathuddin et al. (2021), which stated that the Burongko cake contained the application of the concept of a flat trapezoidal shape. Next, make observations at Madrasah Tsanawiyah As'Adiyyah Bontouse regarding the context used and end by designing a hypothetical learning trajectory (HLT), as shown in Figure 1.



Figure 1. Hypothetical Learning Trajectory (HLT) for learning to build a quadrilaterals

The development of HLT in every learning activity is an essential part of designing student learning activities. The design is inseparable from the learning trajectory, which contains a lesson plan for teaching the material. Here, the learning trajectory is a concept map

that students will pass during the learning process. The learning trajectory used in this study is an understanding of the concept of a flat trapezoid using the context of the Burongko cake. After that, students are expected to have a level 4 geometric thinking level at the rigor stage based on Van Hielle's level of thinking, namely being able to grow critical thinking skills by reflecting or evaluating the problem solving given.

A collection of trapezoidal learning activities based on the learning trajectory and students' thinking results are hypothesized in HLT. It aims to achieve students' understanding of the trapezoid concept and foster critical thinking skills in solving problems encountered in everyday life. The following is a trapezoidal learning conjecture using the Burongko cake context in Table 1.

Teacher Activities	Activity Description	Conjecture	Critical Thinking Ability
Phase I: Visualizatio n	Show students examples of trapezoidal shapes (traditional Bugis cakes that students often encounter, such as Burongko).	Pay attention to the objects shown by the teacher.	Interpretation
	Ask students a problem regarding traditional cakes.	With the learning experience students are expected to be able to understand or express ideas and meanings in the form of a given geometric problem.	
Phase II: Analysis	Asking students to analyze the elements in the traditional cakes shown	Students are asked to find relationships from data and information based on the problems given with the knowledge and experience they have to solve these problems.	Analysis
Phase III: Informal Deduction	Asking students to relate ideas, the properties of the spatial structure to the given problem.	Ask students to associate appropriate data and information. Carry out experiments. Solve problems according to plan	Evaluation

Table 1. Trape	zoidal conjecture	e learning using the	e context of Buron	gko cake

Teacher Activities	Activity Description	Conjecture	Critical Thinking Ability
Phase IV: Formal Deduction	Ask students to construct proofs, look for more than one method of proof, and explain the importance of proving theorems through deductive reasoning.	Students check the solutions provided. And trying to find other solutions in solving problems through deductive reasoning.	Decision
Phase V: Rigor	Proving the formula for the area of a trapezoid through deductive reasoning	With the knowledge and experience that students have, they prove the truth of the formula for the area of a trapezium	Decision and conclusion

## Stage 2: Experimental design

Researchers and a teacher collaborate to conduct learning trajectory experiments that have been previously designed for class VIII B Madrasah Tsanawiyah As'adiyyah Bontouse. The teacher acts as an implementer of learning, while researchers, as observers, observe student activities in the learning process and revise and develop subsequent learning activities. The development of HLT in every learning activity is essential in designing student learning activities. The design is inseparable from the learning trajectory, which contains a lesson plan for teaching the material. Then an analysis of the experimental results at the experimental design stage was carried out. There are five activities carried out in the experimental design stage classified into several stages, namely visualization, analysis, formal deduction, informal deduction, and rigor.

## Stage 1: Visualization

At this stage, the activity is to show students examples of flat shapes related to the trapezoid concept (traditional Bugis cakes such as Burongko). The teacher starts the lesson by asking students questions about the culture around them, one of which is traditional Bugis cakes. Then students mention the kinds of traditional cakes they know that are often served. Next, the teacher asked whether the students knew Burongko cake and asked about the meaning contained in the presentation of Burongko cake in traditional events such as weddings, *mabarasanji*, *menretojang*, *matampung*, and so on. Next, the teacher tells the philosophy and meaning contained in the presentation of Burongko cake in a traditional event to form a cultured student character. After that, to get a common perception of the students, the teacher asked the students to draw Burongko cakes freely according to the understanding in the students' minds, as shown in Figure 2.



Figure 2. Students draw Burongko cakes freely according to the understanding that is in the minds of students

After students finished drawing Burongko cakes freely, the teacher and students practiced again how to draw Burongko cake shapes to get the same perception. It is done to facilitate the implementation of activities in the next stage.

At this stage, students are familiar with the shape of the Burongko cake because it is often found in the presentation of traditional events such as weddings, *mabarasanji*, *menretojang*, *matampung*, and so on. So that students are easy to imagine and describe it. When drawing the shape of the Burongko cake, the students did not realize and understand that in drawing the shape the student had made a trapezoidal shape as a part of forming the image of the Burongko cake. This understanding is developed by discussing with students and posing a problem related to the shape of the Burongko cake, which is a trapezoid. At the end of the result, students can develop their critical thinking skills, namely the indicators of interpretation.

## Stage 2: Analysis

The activity carried out at this stage is asking students to analyze the elements in the Burongko cake that have been drawn based on the problems given in the previous stage. Students are asked to find relationships from data and information based on the given problem with the knowledge and experience they have to solve the problem.

## Stage 3: Informal deduction

In activities carried out at this stage, students connect ideas and properties of flat shapes with the problems given. Ask students to associate data and information and solve problems according to plan.

## Stage 4: Formal deduction

The activities carried out at this stage ask students to compile proofs, look for methods of proving over one way, and explain the importance of proving theorems through deductive reasoning. Students check the solutions provided and try to find other solutions to solving problems through deductive reasoning.

## Stage 5: Rigor

The activity carried out at this stage is asking students to prove the truth of the trapezoid area formula deductively based on students' knowledge and learning experiences.

## **Stage 3: Retrospective Analysis**

Learning to build a flat rectangular trapezoid in a Burongko cake has stages that students must go through during the learning process based on the previously designed HLT. These stages have been changed and adapted to the learning of trapezoidal quadrilaterals in growing students' critical thinking. Student achievement in meeting all indicators of critical thinking is explained:

a. Students can develop their critical thinking on indicators of interpretation.

All research subjects could draw a Burongko cake based on what was shown and then identify and analyze the image, which was a trapezoid shape.

b. Students can develop their critical thinking on indicators of analysis, evaluation, and decisions.

Students are given problems related to trapezoidal quadrilaterals. Based on the problem students express ideas and meaning as a geometric problem, look for relationships from data and information based on the problem, associate data and information, develop problem solving plans, solve problems according to plan, check the solutions provided, and trying to find another solution method in solving the problem.

## Description of students' critical thinking

Data on students' critical thinking were obtained by using a critical thinking ability test. This test is given before and after applying mathematics learning in an ethnomathematical context. The students' critical thinking results in class VIII B are described based on the pre-test and post-test results. The results of processing the data show a data recapitulation of students' critical thinking based on the absorption of each indicator of critical thinking, which is presented in Table 2.

Critical Thinking Ability	Pretest	Posttest	Enhancement
Interpretation	48.60%	100%	51.40%
Analysis	11.70%	99%	87.30%
Evaluation	1.44%	93%	91.56%
Decision	0.00%	35%	35%

Table 2. Recapitulation of the absorbability of students' critical thinking from each indicator

Based on Table 2, it can be seen that the absorption of students' critical thinking from the interpretation indicators on the pretest score is 48.60%, and the post-test score is 100%. Hence, there is an increase in absorption of 51.40% after the learning process. The absorption of students' critical thinking from the analysis indicators at the pretest score of 11.70% and the post-test score of 99% increased absorption to 87.30%. The absorption of students' critical thinking from the evaluation indicators at the pretest score was 1.44%, and the post-test score was 93%, increasing absorption capacity to 91.56%. The absorption of students' critical

thinking from the decision indicators at the pretest score is 0.00%, and the post-test score is 35%, increasing absorption by 35%. The four problem-solving indicators, namely interpretation, analysis, evaluation, and decisions, the highest absorption increase in the evaluation indicator is 91.56%.

The categories of students' pretest and post-test learning outcomes were grouped into five categories using a scale prepared by the Ministry of Education and Culture of Wajo Regency. The frequency distribution and presentation were obtained as shown in Table 3.

	Student A bility	Pretest		Posttest	
Interval	Category	Frequency	Percentage (%)	Frequency	Percentage (%)
90-100	Very high	0	0.0	6	24.00
80-89	High	0	0.0	9	36.00
70-79	Medium	0	0.0	7	28.00
55-69	Low	0	0.0	3	12.00
0-54	Very low	25	100	0	0
	Total	25	100	25	100

Table 3. Distribution of frequency and percentage of students' mathematical critical thinking

Table 3 shows that of the 25 students who were the research subjects, the pretest scores were in the very low category before learning, where all students scored in the 0-54 interval. It means that the initial ability of students' critical thinking is very low. Meanwhile, in the posttest of 25 students, six students were in the very high category, nine students were in the high category, seven students were in the medium category, and three students were in the very low category after learning to wake up in the Burongko cake context.

# Data exposure of students with high mathematics ability (LT) from critical thinking ability test results and interview results data according to Van Hiele's thinking level

#### Question

Take a look at the image below!



KLMN represents a square shape with KL = 4cm, KO = ON and MP = PN.

- a. Determine the three trapezoidal shapes shown in the figure!
- b. Calculate the area of LMNO!



(b)

Figure 3. Results of problem-solving test (TPMG 01) on students with high mathematics ability (LT)

From the results of problem-solving test (TPMG 01), the description of problem solving according to Polya's problem solving steps on students with high mathematics ability (LT) subjects according to Van Hiele's level of thinking is stated:

## Interpretation

In this indicator, the LT subject already understands the matter. This can be seen from the way the LT subjects write the things they know about the questions. It is known that KLMN is a square, with length KL = 4 cm, KO = ON and MP = PN. LT subjects can relate information that is interrelated to the question either from pictures or written information. The LT subject also clearly knows what is being asked from TPMG-01, namely (a) determining the 3 trapezoidal shapes stated in the KLMN image, and (b) calculating the area of the LMNO trapezoid.

## Analysis

In this indicator, the LT subject answers the question part a by paying attention to the trapezoidal pictures in the KLMN image, while for the question part b uses the trapezoid area formula to find the area of the LMNO. The formula is  $\frac{a+b}{2} \times t$ . He used this formula because, previously, the LT subject had studied the quadrilaterals and the use of the area formula.

## Evaluation

In this indicator, the LT subject solves the problem (question part b) by first looking for the parallel sides and the height of the LMNO trapezoid by linking the information in the problem. This can be seen clearly from the answer of TPMG-01. Subject LT wrote that the parallel sides of the LMNO trapezoid are 2 cm and 4 cm, respectively, while the height is 4 cm. Then put into the formula for the area of the trapezoid, so that the area of the trapezoid LMNO =  $12 \text{ cm}^2$ .

## Decision

In this indicator, the LT subject must retrace his answer by checking and rechecking every step he uses. The subject of LT is sure with the answers written. It can be seen from the answers that are reaffirmed that the area of the LMNO trapezoid is  $12 \text{ cm}^2$ .

The following explains test-based interview data for students with high mathematics ability (LT) on Problem-Solving Test (TPMG 01). In this interview, a brief description of solving student geometry problems based on Polya's steps is presented in Table 4.

Code	P/J	Interview Description
LT1-001	P	Have you ever got a question like this (question number 1) before?
LT1-001	J	It looks like it's about the olympics (while reading the questions)
LT1-002	P	What did Faqih think after reading question number 1?
LT1-002	J	I was asked to determine 3 trapezoidal shapes
LT1-003	P	Does Faqih have difficulty reading the questions?
LT1-003	J	Not too
LT1-004	Р	Does Faqih clearly know the known elements of the problem? Try to mention!
LT1-004	J	Yes KLMN is a square. $KL = 4 \text{ cm}$ , $KO = ON$ and $MP = PN$ because KO is half the length of the side of the square, then $KO = ON = MP = PN = 2 \text{ cm}$ .
LT1-005	Р	Are the questions given enough to find what is being asked? Try to mention!
LT1-005	J	Yes, that's enough. The first one is asked for 3 flat trapezoidal shapes and the second is the area of LMNO.
LT1-006	Р	Is Faqih able to recognize rectangular shapes based on the images/shapes he sees?
LT1-006	J	Yes
LT1-007	Р	Can Faqih give a name when faced with various kinds of rectangular shapes without realizing the properties of these flat shapes?
LT1-007	J	Yes, if the picture is clear

Table 4. Interview	description of LT	problem-solving test
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In the interpretation indicator, the LT subject can understand the problem because it can clearly state what is known, namely, KLMN is a square with a length of KL = 4 cm, KO = ON, and MP = PN because KO is half the length of the side of the square, then KO = ON = MP = PN = 2 cm (LT1-004). Then the LT subject also mentioned what was being asked in the question. First, three trapezoidal shapes were asked, and the second was the area of LMNO (LT1-005). The LT subject also mentioned that he could recognize rectangular shapes based on the images/shapes he saw and could give names when faced with various rectangular shapes without realizing the properties of the flat shapes. It can be seen in Table 5 below.

Code	P/J	Interview Description
<u>IT1-008</u>	 P	How does Fagih solve the problem?
LT1-008	Ĩ	For the question of part $A$ I first pay attention to the picture provided
L11-000	J	The first transzoid L saw was OPOP (pointing to the picture provided.
		The first unpedout I saw was OFQK (pointing to the picture in the
		problem), then I looked for other trapezoidal snapes expressed in the
		KLMN quadrilateral. As for part b, I used the trapezoid area formula to
		find the area of the LMNO.
LT1-009	P	What is the formula for the area of a trapezoid?
LT1-009	J	$\frac{a+b}{2} \times t$ (a and b are the parallel sides of the trapezoid)
LT1-010	P	Did Faqih have difficulty in formulating the alleged solution to the
		problem? (Try to express)
LT1-010	J	For problem part A, at first I had difficulty finding a trapezoidal image
		but after I looked at all pictures. I finally found all the trapezoid images
		expressed in the KLMN square, while for part b I don't think I found
		any difficultion
		any unificatives.

**Table 5.** Interview description of realizing the properties flat shapes

In the analysis indicators, the LT subject in solving part A questions first pays attention to the pictures provided. The first trapezoid he saw was OPQR (pointing to the picture in the problem), then he looked for other trapezoidal shapes expressed in the KLMN quadrilateral. As for part B, subject LT uses the trapezoid area formula to find the area of LMNO. The formula for the area of a trapezoid is  $\frac{a+b}{2} \times t$  (a and b are parallel sides of a trapezoid) (LT1-009). It can be seen in table 6 below.

Table 6. Interview description of formulating the area of trapezoid

Code	P/J	Interview Description
LT1-011	P	What are the steps to solve the problem? (Try to explain)
LT1-011	J	For part A, I think it's enough to look at the picture to get the answer.
		As for the problem part b, that is by first finding the parallel sides and
		the height of the LMNO. The base of the LMNO trapezoid is LM,
		because KLMN is a square, meaning all sides are the same, meaning
		LM = 4cm. Then the upper side of the trapezoid LMNO is NO, because
		KO=ON means NO = 2cm half of KN. The height of LMNO is MN,
		which is 4 cm. Continue to be entered the formula for the area of the
		trapezium.
LT1-012	P	Can Faqih prove every step used is correct?
LT1-012	J	Erm, yes (while looking at the answer sheet)
LT1-013	P	Is the formula used by the Faqih correct?
LT1-013	J	Yes, I think it's correct

In the evaluation indicators according to the plan, the LT subject in solving the problem first looks for sides that are parallel and high in LMNO. LT's subject said that the base of the LMNO trapezoid is LM, because KLMN is a square, meaning that all of its sides are the same length, meaning LM = 4cm. Then the upper side of the trapezoid LMNO is NO, because KO=ON means NO = 2 cm half of KN. The height of LMNO is MN, which is 4 cm. From the way of mentioning each side of the KLMN shape, it can be seen that the LT subject can relate every information in the question. In the next step, the subject of LT substituted the parallel sides and height of LMNO into the trapezoid formula (LT1-011) and got the area of the trapezoid LMNO =  $12 \text{ cm}^2$ . It can be seen in Table 7 below.

		<b>i i i i i i i i i i</b>
Code	P/J	Interview Description
LT1-014	Р	Is Faqih sure of the answer he got? Look again at the answer!
LT1-014	J	Yes, I'm sure (while checking answers)
LT1-015	P	If it's a question like this, is there any other way of solving it?
LT1-015	J	I don't think there is

Table 7. Interview	description	of checking	the answers
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In the decision indicator, the subject of LT has retraced the answer and is confident with the execution, he is sure that every step used is correct (LT1-014). The following contains information about the validity/consistency of data on solving geometry problems between the test method (TKBK) and the interview method (HW) in solving geometry problems according to Van Hiele's level of thinking. The full description can be seen in Table 8.

**Table 8.** Comparison of students' critical thinking abilities between test results data and interview results data for male subjects with high mathematical ability (LT) according to Van

ТКВК	HW
Interp	retation
The subject of LT already understands the matter. This can be seen from the way the LT subject wrote things that were known in the problem, namely that KLMN is a square, with a length of $KL = 4$ cm, $KO = ON$ and $MP =$ PN. The LT subject also wrote what was asked of the TPMG-01, namely (a) determining the 3 trapezoidal shapes stated in the KLMN image, and (b) calculating the area of the LMNO trapezoid.	LT subjects understand the question because they can clearly state what is known and what is being asked. What is being asked in this question is that KLMN is a square with $KL = 4$ cm, $KO = ON$ and $MP = PN$ , because KO is half the length of the side of the square, then $KO =$ ON = MP = PN = 2 cm. Then the LT subject also mentioned the things that were asked in the question, namely the first to be asked for 3 flat trapezoidal shapes and the second the area of LMNO (LT1-004).
An	alysis
LT subjects use the formula for the area of a trapezoid $\frac{a+b}{2} \times t$ to find the area of LMNO. He used this formula because, previously, the subject of LT had studied the quadrilaterals and the use of the area formula.	LT subjects used the trapezoid area formula to find the area of LMNO (LT1-008). The formula for the area of a trapezoid is $\frac{a+b}{2} \times t$ (a and b are parallel sides of a trapezoid).
Eval	uation
LT subjects look for the parallel sides and the height of the LMNO trapezoid. Subject LT wrote that the parallel sides of the trapezoid LMNO were 2 cm and 4 cm, respectively, while the height was 4 cm. Then put into the formula for the area of the trapezoid, so that the area of the trapezoid LMNO = 12 cm <sup>2</sup> .	LT subjects first look for the parallel sides and the height of LMNO. LT's subject said that the base of the LMNO trapezoid is LM, because KLMN is a square, meaning that all of its sides are the same length, meaning LM = 4cm. Then the upper side of the trapezoid LMNO is NO, because KO=ON means NO = 2cm half of KN. The height of LMNO is MN, which is 4 cm. Continue to be entered the formula (LT1-011) and the area of the trapezoid LMNO = 12 cm <sup>2</sup> is got.
Dee	cision
The LT subject has retired the answer and is confident in the workmanship. It can be seen from the answer that reaffirmed that the area of the LMNO trapezoid is $12 \text{ cm}^2$ .	LT subject has traced back the answer and is confident with the workmanship. He is sure that every step used is correct (LT1-014).

Hiele's Thinking Level

Data exposure of students with low mathematics ability (PR) from critical thinking ability test results and interview results data according to Van Hiele's thinking level

## Question

Take a look at the image below!



KLMN expresses a square shape with KL = 4cm, KO = ON and MP = PN

- a. Determine the three trapezoidal shapes shown in the figure!
- b. Calculate the area of LMNO!

Subject's answer:



(b)

Figure 4. Results of problem-solving test (TPMG 01) on students with low mathematics ability (PR)

From the results of Problem Solving Test (TPMG 01) the description of problem solving according to Polya's problem-solving steps about Students with Low Mathematics Ability (PR), according to Van Hiele's level of thinking, is stated:

## Interpretation

In this indicator, the subject of PR has understood the matter. This can be seen from the way the subject of PR writes the information that is known about the problem. The things that are known are that KLMN expresses a rectangular shape with length KL = 4 cm, KO = ON and

MP = PN. The PR subject also clearly knows what is being asked from TPMG-01, namely (a) determining the 3 trapezoidal shapes stated in the KLMN image, and (b) calculating the area of the LMNO trapezoid.

## Analysis

In this indicator, the PR subject determines 3 trapezoidal shapes by analyzing the images stated in the KLMN (part a) and using the trapezoid area formula to find the area of the LMNO (part b). PR subjects can plan problem solving by writing the formula for the area of the trapezoid LMNO is  $\frac{LM+NO}{2} \times t$ .

## Evaluation

In this indicator, the PR subject solves the problem (question part b) by first looking for the parallel sides and the height of the LMNO trapezoid by linking the information in the problem. This can be seen clearly from the answer to TPMG-01. The PR subject wrote that the parallel sides of the LMNO trapezoid are 2 cm and 4 cm, respectively, while the height is 4 cm. Then put into the formula for the area of the trapezoid, so that the area of the trapezoid LMNO = 12 cm<sup>2</sup>.

## Decision

In this indicator, the subject of PR has retraced his answer by re-checking every step he uses. The subject of PR is sure with the answer written, that the area of the LMNO trapezoid is  $12 \text{ cm}^2$ .

The following is an explanation of test-based interview data for students with low mathematics ability (PR) on problem-solving test (TPMG 01). In this interview, a brief description of solving student geometry problems based on Polya's steps is presented in Table 9 below.

Code	P/J	Interview Description
PR1-001	Р	What did Tarizha think after reading question number 1?
PR1-001	J	I'm surprised because I've never had a question like this before.
PR1-002	P	Does Tarizha have difficulty reading the questions?
PR1-002	J	Ehm, I have no difficulty in reading the question.
PR1-003	Р	Does Tarizha know the known elements of the problem? Try to mention!
PR1-003	J	He knows. KLMN expresses a rectangular shape with length $KL = 4$ cm, $KO = ON$ and $MP = PN$ .
PR1-004	P	Are the questions given enough to find what is being asked?
PR1-004	J	That's enough
PR1-005	P	Try to mention
PR1-005	J	Part A determines the 3 flat shapes of the trapezoid as shown in the KLMN drawing, while part b calculates the area of the LMNO trapezoid.
PR1-006	Р	Is Tarizha able to recognize quadrilaterals based on the pictures/shapes he sees?
PR1-006	J	Yes I can

**Table 9.** Interview description of PR on problem-solving test

Code	P/J	Interview Description	
PR1-007	P	Can Tarizha give a name if he is faced with various kinds of	
		quadrilaterals without realizing the properties of the flat shapes?	
PR1-007	J	Yes I can	

In the interpretation indicator, the PR subject understands the problem because it can clearly state what is known, namely KLMN states a rectangular shape with a length of KL = 4 cm, KO = ON and MP = PN (PR1-003). Then the PR subject also mentioned that what was asked in the question was first to determine the 3 flat shapes of the trapezoid as stated in the KLMN picture and the second was the area of LMNO (PT1-004). She can give names when faced with various kinds of quadrilaterals without realizing the properties of these flat shapes. It can be seen in Table 10 below.

Table 10. Interview description of PR in solving the flat shapes

Code	P/J	Interview Description
PR1-008	Р	How does Tarizha formulate the solution to this problem?
PR1-008	J	For part A questions, by looking at the shape, if part A uses the trapezoid area formula
PR1-009	Р	Did Tarizha have difficulty in formulating the alleged solution to the problem? (Try to express)
PR1-009	J	Yes, when looking for a trapezoidal shape, it's not too difficult to find the area because it's enough to use the formula
PR1-010	Р	Where is it difficult to find a trapezoidal shape?
PR1-010	J	It's been a long time looking for a trapezoid

In the analysis indicators, the subject of PR can formulate problem solving well. For part A, the PR subject determined 3 trapezoidal shapes by looking at the pictures stated in the KLMN, although initially they had difficulty finding the trapezoidal shapes stated in the KLMN. Meanwhile, for part b, the formula for the area of the trapezoid is used to find the area of LMNO (PR1-008). It can be seen in Table 11 below.

**Table 11.** Interview description of PR in finding the area of trapezoid

Code	P/J	Interview Description
PR1-011	Р	What are the steps to solve this problem? Explain!
PR1-011	J	For part B, that is, by using the formula for the area of a trapezoid
		$\frac{LM+NO}{2} \times t$ . Then enter the numbers and get the area of LMNO 12 cm <sup>2</sup>
PR1-012	P	Can Tarizha prove every step used is correct?
PR1-012	J	Ehm, yes (while looking at the answer)
PR1-013	P	Is Tarizha sure that the formula used is correct?
PR1-013	J	Yes

In the evaluation indicator, the PR subject solves the problem by substituting the parallel sides and the height of the LMNO trapezoid into the formula, namely  $\frac{LM+NO}{2} \times t$ . So that the area of LMNO = 12 cm<sup>2</sup> (PR1-011). It can be seen in Table 12 below.

Table 12. Interview descrip	tion of PR in substitutin	ng the parallel side	of trapezoid
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Code	P/J	Interview Description
PR1-014	Р	Is Tarizha sure about the answer he gets?
PR1-014	J	Yes, I am sure
PR1-015	P	Is there any other way to solve a problem like this?
PR1-015	J	There is no

The decision indicator, the subject of Students with Low Mathematics Ability (PR), has re-examined the answer and is confident with the work. She is sure that every step used is correct and there is no other way to solve the problem, as the students answered in question 15 (PR1- 015).

## Discussion

The preliminary design stage resulted in the instruments used in this study, such as critical thinking tests, learning implementation plans, and Hypothetical Learning Trajectory (HLT). The learning trajectory determines the stage of students' understanding and development of critical thinking in the material of trapezoidal quadrilateral using the context of Burongko cake. In the first stage (visualization), students can analyze the shape of a trapezoidal quadrilateral by asking students to draw a Burongko cake freely according to the understanding in the students' minds. When drawing the shape of the Burongko cake, students do not realize and understand that in drawing the shape, the student has made a trapezoid shape as part of the formation of the Burongko cake image. This understanding was developed by discussing with students and posing problems related to the shape of the Burongko cake, which is a trapezoid. As a result, students can understand the concept of a trapezoidal quadrilateral and develop their critical thinking, namely on indicators of interpretation, so students think that the context of Burongko cake can be applied in learning mathematics. The second stage analyzes the elements that exist as Burongko cakes drawn based on the problems given in the previous stage.

Students look for relationships from data and information based on problems with their knowledge and experience to solve problems in this activity. Students can find the formula for the area of a trapezoidal quadrilateral and develop their critical thinking on analytical indicators. The third stage is the informal deduction of students connecting ideas and properties of rectangles with problems. Ask students to associate data and information and solve problems according to a plan. The teacher guides students to find effective strategies for solving problems in this activity. Students can solve problems given based on previous knowledge and develop their critical thinking on evaluation indicators. The fourth stage is formal deduction; students construct a proof, look for proof methods in one way, and explain the importance of proving theorems through deductive reasoning. Students check the solution.

Furthermore, by trying to find other solutions to solving problems through deductive reasoning in this activity, students can solve problems and reflect on or evaluate problemsolving and develop critical thinking on decision indicators. The fifth stage is rigor. The activity carried out at this stage asks students to prove the truth of the trapezoid area formula deductively based on students' knowledge and learning experiences.

Students' understanding of the material of quadrilaterals can be supported by the learning trajectory based on the use of the Burongko cake context. Students can quickly analyze the quadrilaterals. It is in line with the results of research by Lestari et al. (2021), which reveals that the use of context has a positive impact on the learning process to be more enjoyable so that students are more active and make students not think that mathematics is abstract. In addition, learning to build a quadrilateral with the context of Burongko cake can also improve

students' critical thinking and shape students' character. For example, self-confidence, sympathy, empathy, respect for others, awareness of social problems and social spirit, and responsibility.

Students' understanding of the concept of quadrilaterals and critical thinking became better after quadrilaterals in Burongko Bugis cake. Students can understand mathematical concepts easily, be fun, close to students' daily activities, and can be imagined. It also makes it easier for students to solve given problems related to daily activities. In addition, the application of the Local Instruction Trajectory in a Burongko cake can shape students' character, such as self-confidence, sympathy, empathy, respect for others, awareness of social problems and a social spirit, and responsibility. These results support several previous research results that state that learning activities related to daily activities, namely ethnomathematics, can be a starting point in learning mathematics (Risdiyanti et al., 2019; Risdiyanti & Indra Prahmana, 2020; Prahmana, 2015; Fitri & Prahmana, 2020; Prihastari, 2015; Supiyati et al., 2019; Pujawan et al., 2020; Aras & Zahrawati, 2021)

Students' critical thinking in solving quadrilaterals problems after the learning process using the ethnomathematical context showed no significant difference in the geometric problem-solving process between students with high mathematical and low mathematical abilities. It is shown based on the results of the problem-solving ability test where students who have the high mathematical ability get a score of 90 and students with the low mathematical ability get a score of 79, and based on the results of interviews, students with high mathematical ability and low mathematical ability are both able to make an interpretation, analysis, evaluation, and good decision.

However, subjects with high mathematical abilities could develop critical thinking processes to solve problems much better than subjects with low mathematical abilities. In the learning process, mathematical concepts are associated with students' daily habits so that they are enthusiastic, active in the learning process, and critical in responding to the problems given. This finding strengthens the results of previous research conducted by Adhetia and Suhartini (2018), which concluded that ethnomathematical-based mathematics learning has relevance to indicators of critical thinking skills, which include interpretation, analysis, evaluation, and decision making. It is in line with several previous research findings, which concluded that students' critical thinking skills could be improved by using ethnomathematical-based geometry materials in the learning process (Sumiyati et al., 2018; Mirnawati et al., 2020; Suherman et al., 2021). Thus, ethnomathematics-based mathematics learning can be used as an alternative to learning mathematics to develop students' critical thinking.

Teachers in learning mathematics, especially in teaching geometry material, try not only to understand concepts but also need to develop students' critical thinking skills. Because in studying geometry, geometry skills are needed and several abilities, such as the ability to interpret, analyze, evaluate, and make decisions. This ability is an indicator of critical thinking.

## Conclusion

The learning trajectory of quadrilaterals with the context of Burongko cake which comprises five learning activities, namely visualization, analysis, informal deduction, formal deduction, and rigor, can develop students' critical thinking, including interpretation, analysis, evaluation, and decision making. The profile of students' critical thinking in solving rectangular problems after the learning process using an ethnomathematical context. From the results of qualitative analysis, it was found that there was no significant difference in the geometric problem-solving process between students who had the high mathematical ability and low mathematical ability.

Both research subjects have good interpretive and problem analysis skills and can evaluate and decide to get the correct answer. However, subjects with high mathematical abilities could develop critical thinking processes to solve problems much better than subjects with low mathematical abilities. With the Local Instruction Trajectory, the teacher can optimize learning to build a quadrilateral while equipping students with critical thinking. This learning trajectory is only for quadrilaterals, so developing a broader mathematics learning trajectory within a local context is necessary.

## **Conflicts of Interest**

The authors declare that no conflict of interest regarding the publication of this manuscript. In addition, the ethical issues, including plagiarism, misconduct, data fabrication and/or falsification, double publication and/or submission, and redundancies have been completely by the authors.

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