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Prospective teachers' thinking through realistic mathematics education based emergent modeling in fractions

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Abstract

The unconsciousness of a teacher in obtaining knowledge due to students can be known if the teacher was notified when he was a student. A student has an essential role in learning, and the teacher is responsible for supporting smooth learning. Students' problem-solving processes need to be found because each student's reasoning and ideas in solving problems are different. This study focuses on students' thinking processes using realistic mathematics education based on emergent modeling. In this study, the researcher is the teacher, and the student is the prospective teacher. The prospective teacher involved were students of the Institut Pendidikan Indonesia mathematics study program. Prospective teachers were selected as research subjects for as many as 74 people (11 males and 63 females) consisting of 3 classes. The research method uses descriptive qualitative. As learning activities progress, students get a kind of model that seems as solutions to solving problems given by the teacher. Various kinds of models emerged from various student ideas and ended with a mutually agreed for a model for. Through this study, a teacher can learn about student models in the learning process.

Keywords: descriptive qualitative; emergent modeling; model of; model for; prospective teachers; realistic mathematics education

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Introduction

The progressive concept of a model is a core element in realistic mathematics education (Sumirattana et al., 2017). The key to using this progressive concept involves the relationship between the development of the use of symbols and the development of the meaning of mathematics itself. Vygotsky (Oers, 2002; Walshaw, 2017) has advised that the development of student knowledge can be seen from different levels. Wertsch (Wertsch, 1985; Oers, 2002; Kaiser, 2020) named this level difference as 'genetic domains' and divided it into four levels related to 'modeling', namely: 1) the phylogenetic level, can be seen as the level of development of a person's thought representation function during phylogenesis; 2) the sociogenetic level, can be seen as a process of developing modeling as a cultural tool in history or natural science; 3) the ontogenetic level, trying to describe and explain the process of development of a person related to symbol functions and modeling; and 4) the micro genetic level refers to the deciding a model to be used in a particular situation.

Any students, in solving mathematical problems, often use words or movements that are difficult for teachers to understand (O'Connor & Michaels, 2019; Nurfadillah & Afriansyah, 2022). It happens in children aged 4-5 years, mathematical ideas that arise at this age we name the knowledge or activity 'pseudomathematical' (Oers, 2002; Danesi, 2018). These pseudomathematical pseudoconcepts in mathematics explain that a child can predict the situation precisely through the terminology he did himself but cannot give arguments to influence others why we can be sure that the answer is correct (Vinner, 2018).

Children aged 8-9 years start to progress in making their mathematical system based on the object being seen and its relation to the surrounding circumstances (Lehrer & Pritchard, 2002; Abdulhameed & Rashid, 2022). Therefore, the learning process in the form of activities can be a suitable starting point to begin the development of children's mathematical characteristics. A child will gain broad practical experience, mathematical and modeling provide a new perspective on symbolization, not just activity. The teacher is not as simple as simply focusing on his activities, a teacher supports students to consider how students' mathematical thinking can generate along with the learning activities. Things like this are an important role of a teacher in helping students build their identity as someone who thinks mathematically (Mead, 1934, 1938; Lehrer & Pritchard, 2002; Kieren, 2020).

Students can think mathematically in the learning process with support from the teacher, not a refusal, in promoting the representations/models they perform (Meira, 2002; Hefendehl-Hebeker, et al., 2019). The situation requires to be made as if students only have a focus on aspects of developing knowledge. Students' knowledge develops along with the development of the representations they do. A rich and meaningful number of ideas is needed in promoting representation. DiSessa (2002) revealed three quality criteria for representation, i.e.: 1) rich and generative of meaning, 2) reactive and unclear, and 3) connected with design; not all can relate in all contexts.

The development of representation/models of a mathematical concept does not develop fully directly but develops slowly through social interaction in discussions about concepts/problems encountered during learning activities take place, and are related to each other (Huang, et al., 2021; Cao, et al., 2021). These activities can be linked through pseudo-concepts and experiments in using and evaluating notations used as communication media with other students or teachers (Presmeg, 2002; Carroll et al., 2018). In Oers' research (Oers, 2002), a child is in the transition zone between playing and learning mathematics, so Oers takes a transition activity that children can use to play chili learning. This transition zone happens at the ontogenetic level and produces new psychological phenomena that are integrated and exceed previous knowledge.

Gravemeijer (1999, 2007) explained the new mathematical concept as the main goal of mathematics learning activities through modeling. Thus, in the 'emergent' perspective, modeling activities can be understood as a mathematical process where the situation appears during the transition zone of mathematical relations. Situations and models emerge simultaneously and form one another (Gravemeijer, 2002b; Afriansyah, 2021). In this way, students acquire high levels of 'common sense' (Freudenthal, 1991; Fredriksen, 2021) for reasoning about new phenomena in their reality, and formal mathematics emerges naturally from student activities. The term 'emergent' refers to two different but interrelated processes, i.e., the emergence of a model for mathematical reasoning; and, at the same time, the emergence of more formal mathematical knowledge (Gravemeijer, 1999).

In the beginning, the model that emerged was a particular situation model and earned its meaning from the activity in assigning tasks. It is a model of a particular process (Gravemeijer, 1999). Through extensive activities in assignments, students eventually begin to be able to conclude. The model does not change, but the meaning changes. No longer meaning certain circumstances, it has become a model-for way of thinking that is more formal. The learning process that begins to a shift from model-of to model-for states itself as a sequence of sub-models (Gravemeijer, 1999, 2007; Gravemeijer & Bakker, 2006].

However, to highlight more the emergence of concepts required for modeling, Gravemeijer (2002a, 2007) offered an alternative view where the model is the outcome of arranging activities. In the process of structuring the problem situation, the model emerges (Gravemeijer, 2002b). From this perspective, the situation model functions to earn insight into phenomena and create simple generalizations. At the same time, concepts must not remain situationally specific. However, they can also function in creating models for designs about different situations, thus allowing for deeper insights into the concepts themselves.

Gravemeijer (2007) put careful consideration into the teaching and learning process to support the emergent modeling process in learning. Two questions appear in the design of assignments like this: What new concepts should students create, and what are the overall models? Gravemeijer (2007) offered the concept of "distribution as an entity" for data analysis. For him, the idea of form is the core of this concept and can be obtained through a series of sub-models that change.

New ideas or concepts can be obtained from the perspective of 'emergent modeling' as the transfer of the role of a model, particularly: a model of to a model for (Buscher & Schnell, 2017). The situation model is context-specific and acts as an assistant used to talk about a specific phenomenon. This model aims to understand the given situation and allow informal deducing beyond the data provided. Model-for reasons in some situations and allows the organization to set various similar and new contexts.

Buscher and Schnell (2017) revealed that this 'emergent modeling' could be seen as an approach. The 'emergent modeling' approach is applied (1) as a theoretical framework to describe students' conceptual progress by providing model-of and model-for as a means to follow student development, and (2) as a design heuristic for developing teaching and learning settings intended at developing insight, one of which is the choice of contexts that are 'meaningful'. Therefore, two important roles can be played by 'emergent modeling,' i.e., to design and explain student learning.

Meanwhile, in creating a theoretical framework based on 'emergent modeling,' Doorman and Gravemeijer (Doorman & Gravemeijer (2008) provides an overview of (1) how students are expected to act and reason with 'tools', (2) how activities are related to previous activities and (3) conceptual development aimed at the activity. This tool emphasizes the context used at the situational level and the models that emerge at the next level. This theoretical framework based on 'emergent modeling' cannot be separated from learning realistic mathematics education. In Indonesia, we name realistic mathematics education with Pendidikan Matematika Realistik Indonesia (PMRI). PMRI still uses realistic mathematics education theories and adds to the Indonesian context (Afriansyah et al., 2021). During the transition zone, a child experiences a progressive process of mathematics on a large scale (Lehrer & Pritchard, 2002; Silinskas & Kikas, 2019). It can indicate that pseudo-concepts and the role of social interaction in the learning process activities are going correctly. Meanwhile, Meira (2002) figured that the system of notation (writing or representation) built by students as a learning media tool could be a trigger for the sustainability of mathematics learning activities. Not just writing or representation as to the final result of learning, but as an essential transition, component to be seen as a student's process of making a solution to a mathematical problem situation.

Through this study, different studies on applying the realistic mathematics education approach have been carried out. However, nothing has focused on the models developed by students. This study intends to see realistic mathematics education based on emergent modeling applied in the classroom. Researchers hope that through this learning, students get more meaningful learning based on the context of daily problems and solution models.

Methods

This qualitative study proposes analyzing and explaining students' answers to solving the problems given (Afriansyah et al., 2019). By using qualitative research methods, researchers expect that students' work can be analyzed in depth. The data analyzed is the students' written answers in groups, each group of five people. Obtained from working on written problems with research subjects in as many as three classes, totaling 74 people, 11 males and 63 females. The criteria for the subject of this research are a prospective teacher who comes from the mathematics education study program. So, the subjects of this study were the student from the Mathematics Education Study Program of the Institut Pendidikan Indonesia. The idea of this study was carried out from Gravemeijer's mathematical concept.

The test instrument used is in the form of test questions on the material. Four validators have thoroughly validated the test instrument, advanced validation, and content validation. These four validators are experts in their fields. Three validators are experts in Mathematics Education, and one is a lecturer who teaches subjects related to this research: *Kapita Selekta Matematika Pendidikan Dasar*. The data collection technique used a test instrument for one of the subjects discussed in the *Kapita Selekta Matematika Pendidikan Dasar*. The data analysis technique applied in this research analyzes what has been done.

In the learning process, Zulkardi (Riyanto & Putri, 2017) states the stages of learning realistic mathematics education: 1) What is done at the beginning is to prepare realistic problems. The teacher must understand the problem and have different strategies that students might practice to solve it; 2) Students are introduced to learning strategies that are used and introduced to realistic problems; 3) Then students are asked to solve the problem in their way; 4) Students try various strategies to solve these problems according to their experiences, the problems can be done individually or in groups; 5) Then each student or group present their work in front of the class, students or other groups respond to the presenter's work; 6) The teacher observes the course of class discussion and responds while directing students to get the best strategy and find rules or principles that are more general, and 7) After reaching agreement on the best strategy through class discussion, students are invited to conclude the lesson at that time. At the end of learning, students must go on evaluation problems in the form of formal mathematics.

Meanwhile, Wijaya (2012) stated the stages of the mathematical process in learning realistic mathematics education, namely: 1) Starting with real-world problems; 2) Identifying mathematical concepts that are relevant to the problem and then organizing problems according to mathematical concepts; 3) Gradually leave the real-world situation through the process of formulating assumptions, generalizations, and formalization. This process tries to translate real-world problems into representative mathematical problems; 4) Solve mathematical problems (this process happens in the world of mathematics); and 5) Re-translating mathematical solutions into real solutions, including recognizing the limitations of the solution.

Based on the numerous theories discussed earlier, the researcher arranges the stages of learning realistic mathematics education based on emergent modeling, shown in Table 1.

Table 1. The stages of realistic mathematics education based on emergent modeling.

No	**Stages**	**Activity**	
		Student	**Teacher**
1.	Situational	Recognizing together about numerous real-world problems	Communicate on various problems in the real-world
		Identify mathematical concepts that are related to the problem	Request about mathematical concepts that are related to the problem
		Understand the problem presented	Present mathematical problems related to the material
		Solve mathematical problems	Go around observing student work

No	Stages	Activity	
		Student	**Teacher**
2.	Referential	Study for associations between problem languages with symbols, images, or other mathematical representation models	Observe, motivate, and provide restricted direction to students so they can find their solutions
		Look for associations between problem languages with symbols, images, or other mathematical representation models	
3.	General	Understand different mathematical representations	Ask different groups that have different answers to explain the results of their work
		Perform mathematical arguments about different ways	Acting as a control of the direction of the discussion
		Generalize (model for) the correct way to solve	Collectively concludes the most meaningful and easy to understand solution
4.	Formal	Repeating and reflecting mathematical arguments	Ask back to students, what concepts are needed in solving the problem
		Apply formal mathematical language and symbols	Reply to different student answers and conclude together formal solutions from previous problems
		Formulate a formal solution	

Results

The results of written questions from all groups of students were analyzed based on the stages of learning realistic mathematics education based on emergent modeling. This study proposes to manage the application of realistic mathematics education learning that can focus on students' answer models. The following analysis is conducted by researchers based on the stages of realistic mathematics education based on emergent modeling, i.e.:

Situational stages

At this level, lecturers talk about various contexts of daily problems related to the division of fraction material with the hope that students can realize the problems around their lives. General discussion is held to draw student motivation in the context of daily problems. After that, students are invited to think a little about the mathematical concepts involved in daily problems. Students need to know for themselves, what mathematical material is used for each problem that has been discussed. The activities continued with group activities, students were divided into groups, each group consisting of five students.

Teachers present mathematical problems (Figure 1), students try to understand these problems in groups.

Seorang Ibu memasak $\frac{3}{5}$ kg beras per hari. Sementara di dapurnya terdapat 22 kg beras. Setelah berapa harikah Ibu itu harus membeli beras lagi ?

A mother cooks $\frac{3}{5}$ kg of rice per day. While in the kitchen there is 22 kg of rice. After how many days did the mother have to buy rice again?

Figure 1. Mathematical problem

Around 60 minutes, students must be able to solve these problems in the best way they can think of. The teacher went around to see how the mathematical problems of each group were being worked on.

Referential stages

Each student group needs to first interpret the mathematical problem by understanding the language of the given problem. After that, they need to represent the problem in other models, such as pictures, symbols, short sentences (Figure 2), and so on. They can use this representation as a tool to understand more than the problems given. Teachers keep going around and are always ready if there is a group that is having difficulty in the process of working on the question.

Diketahui

Beras yang tersedia : 22 kg

Beras yang dipakai per hari : $\frac{3}{5}$ kg atau 0,6 kg

Ditanyakan :

Berapa hari beras itu habis ?

Known

Available rice = 22 kg

Rice used per day = $\frac{3}{5}$ kg or 0.6 kg

Asked:

How many days did the rice run out?

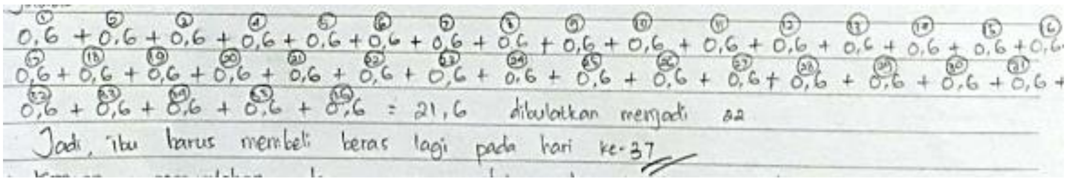
Figure 2. Representation of the problem in the form of short sentences

The teacher observes, motivates, and gives guidance to each group so that each student in each group can find their solution. Different ways/models (models of) emerge, the answer models given by each group display their thought processes and their discussion processes. Following are the models of each group and their analysis:

$$22 - \frac{3}{5} = \frac{110}{5} - \frac{3}{5} = \frac{107}{5} - \frac{3}{5} = \frac{104}{5} - \frac{3}{5} = \frac{101}{5} - \frac{3}{5} = \frac{98}{5} - \frac{3}{5} = \frac{95}{5} - \frac{3}{5} = \frac{92}{5} - \frac{3}{5}$$
$$= \frac{89}{5} - \frac{3}{5} = \frac{86}{5} - \frac{3}{5} = \frac{83}{5} - \frac{3}{5} = \frac{80}{5} - \frac{3}{5} = \frac{77}{5} - \frac{3}{5} = \frac{74}{5} - \frac{3}{5} = \frac{71}{5} - \frac{3}{5} = \frac{68}{5}$$
$$- \frac{3}{5} = \frac{65}{5} - \frac{3}{5} = \frac{62}{5} - \frac{3}{5} = \frac{59}{5} - \frac{3}{5} = \frac{56}{5} - \frac{3}{5} = \frac{53}{5} - \frac{3}{5} = \frac{50}{5} - \frac{3}{5} = \frac{47}{5} - \frac{3}{5} = \frac{44}{5} -$$
$$\frac{3}{5} = \frac{41}{5} - \frac{3}{5} = \frac{38}{5} - \frac{3}{5} = \frac{35}{5} - \frac{3}{5} = \frac{32}{5} - \frac{3}{5} = \frac{29}{5} - \frac{3}{5} = \frac{26}{5} - \frac{3}{5} = \frac{23}{5} - \frac{3}{5} = \frac{20}{5}$$
$$- \frac{3}{5} = \frac{17}{5} - \frac{3}{5} = \frac{14}{5} - \frac{3}{5} = \frac{11}{5} - \frac{3}{5} = \frac{8}{5} - \frac{3}{5} = \frac{5}{5} - \frac{3}{5} = \frac{2}{5}$$

Figure 3. Model of reduction of repeated fractions

This reduction model is a fairly easy method because it only needs to use the concept of reducing fractions. This method uses repeated fraction reduction (Figure 3), but this method is considered to be at great risk of making miscalculations. The method of adding together can also be said to be similar because it is an easy way to do, but it is risky for errors and requires a considerable amount of time.



0.6 + 0.6 = 21.6 rounded up to 22
So, a mother has to buy rice again on the 37th day

Figure 4. Model of the addition of decimals

This model of addition is a fairly easy method. This method uses the concept of changing fractions into decimals and adding decimals (Figure 4). This method is better than reducing fractions repeatedly because it can minimize errors in making calculation errors and also the processing time is not too long. How to reduce repeating decimals can also be said to be similar, because it is an easy way to solve.

General stages

At this level, the teacher questions many groups with different answers to explain the results of their work. Each different model of the answer is presented in front of the class so that each student can understand more than the work of his peers.

Other students can argue for the work of students who have different ways. The lecturer acts as a regulator of the course of the discussion and must not blame one of the parties, and he only needs to return it to the student if students make an error during the presentation. After that, the activity was continued by concluding the most appropriate and meaningful solution so that it could be easily understood and did not take up much time, namely the model for answers.

The model-for is not obtained from the lecturer, but the model-for is obtained from the work of the students themselves. These answers become answers that lecturers must display in front of the class. The answers were accepted and recognized by other friends as models for answers to the most suitable problems (Figure 5).

The model for grouping is a fairly effective way. Ideas in this work need a variety of mathematical concepts and critical abilities are quite good. This method uses a repeated grouping of fractions (Figure 5). The method of adding together repeated fractions can be said to be the best way that can be given.

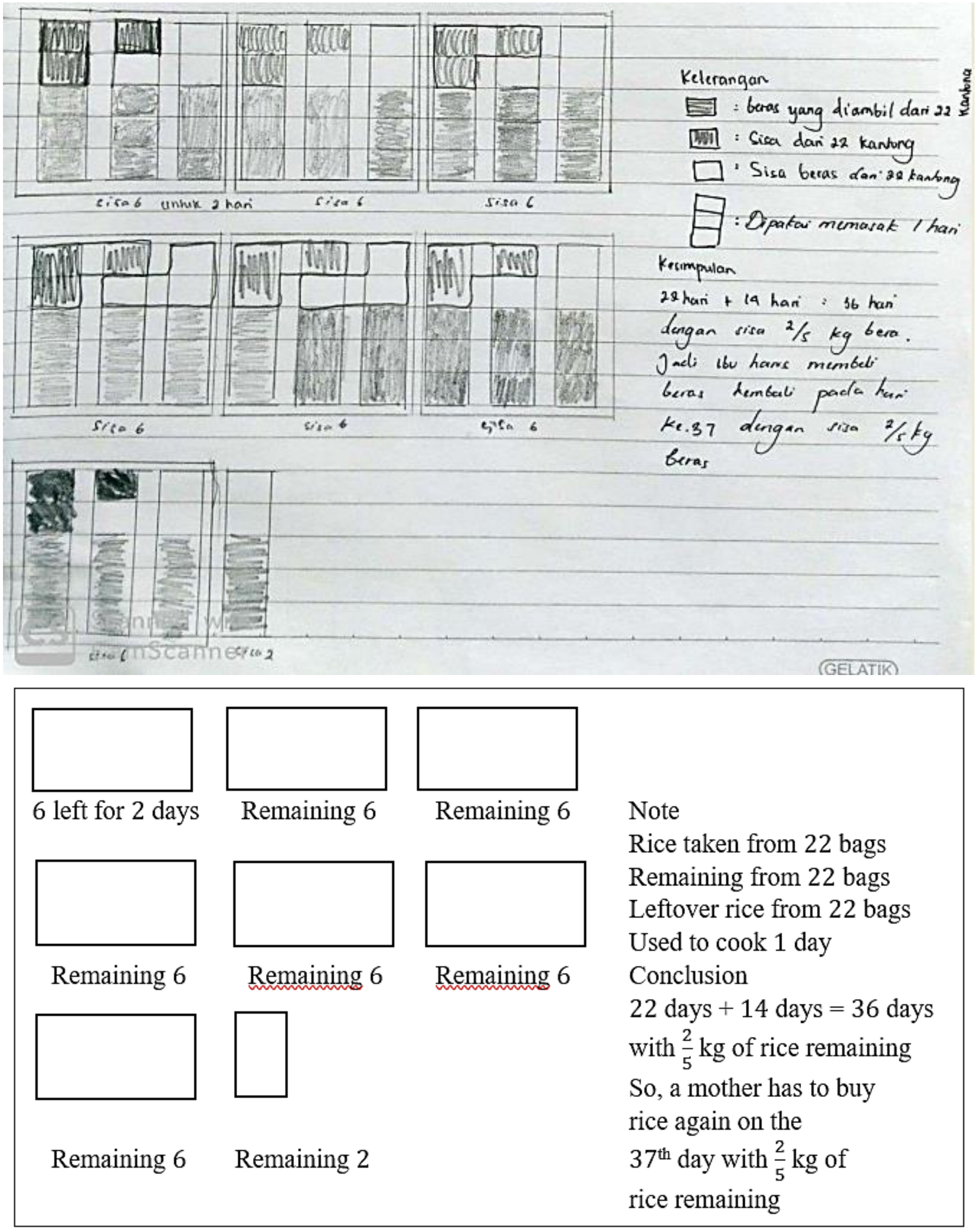


Figure 5. Model for fractional grouping

Formal stages

At this level, all answers are collected. Then, the teacher invites the students again to what concepts are needed in solving these mathematical problems. The discussion activity was opened, and the teacher replied to many students' answers and concluded together as a formal solution to the previous problem, namely the model for answers.

Discussion

The research results that need to be discussed further are the answer models students make at each stage. In situational stages, students need to know for themselves what mathematical material is used. It is the initial stage of learning; in line with the research of Baier et al. (2019), early learning students need to be allowed to explore their knowledge first, of course with guidance from the teacher. In the referential stages, students must represent the problem in other models, such as pictures, symbols, and short sentences. In the next stage of learning, the same thing was done by Awidi and Paynter (2019); students were given the freedom to create problem-solving models so that students could be trained to look for ideas in solving problems. At the general stages, the model-for is obtained from the work of the students themselves. It is an essential stage because each student has an agreement on ideas. In line with the words of Kitchen (2019) that there is no compulsion in learning; the only agreement occurs when finding the suitable model. In the formal stages, many students answered and concluded together as a formal solution. Formal solutions are understood meaningfully, in line with Marino (2020) said that formal solutions are not only in the form of numbers or symbols; they must be meaningful for students.

The explanation of these stages can accommodate a clear understanding of realistic mathematics education based on emergent modeling. In line with several other studies that chose realistic mathematics education as the core of their research, such as research by Octizasari and Haji (2018), they examine the problem-solving abilities of prospective mathematics education teachers by applying realistic mathematics education based on ethnomathematics. A similar study was carried out by Ardianingsih, Lusiyana, and Rahmatudin (2019); they examined students' High Order Thinking Mathematical Skills by providing realistic mathematics education based on ethnomathematics as well. Meanwhile, a study looking at the effect of realistic mathematics education based on ethnomathematics on students' literacy skills was carried out by Witha, Karjiyati, and Tarmizi (2020).

From different perspectives, when looking at realistic mathematics education based on learning trajectory, Sarvita and Syarifuddin (2020) make a series of lessons on integral topics. Meanwhile, Mardiah, Permana, and Arnawa (2021) designed a series of activities for high school students on function material. Afriansyah and Arwadi (2021) designed a series of activities on rectangular material. Then, another perspective, realistic mathematics education based on progressive mathematization, Johnson (2013) develops a strong mathematical theoretical foundation to support student analysis in the learning process, individually and in groups. Meanwhile, Herman (2018) relates improving students' mathematical representation abilities to the treatment of realistic mathematics education based on progressive mathematization. Various perspectives on the study of realistic mathematics education, seen from the perspective of ethnomathematics, learning trajectory, progressive mathematization, and so on. Therefore, this study exists to complement other current studies.

Conclusion

The stages of realistic mathematics education based on emergent modeling have been successfully implemented. Students can learn a lot. The knowledge earned is not limited to teachers but can be received from other friends. Students are also required to change ideas with their friends to learn the most suitable answers together. It can motivate students to be active and participate in the learning process. This learning phase can be applied if the center of learning is on the diversity of students' answers so that students can interpret the problem from various perspectives.

Through this study, a teacher can learn about student models in the learning process. These models are proof of students' thinking in solving problems. Students must think about their model first, then be conducted slowly through group activities, and conclude together with the model used. The results of this study have several limitations, such as the research subjects taken are mostly females, the material taken is fractions only, and so on.

Conflicts of Interest

The authors declare that no conflict of interest regarding the publication of this manuscript. In addition, the ethical issues, including plagiarism, misconduct, data fabrication and/or falsification, double publication and/or submission, and redundancies have been completely by the authors.

References

Abdulhameed, S., & Rashid, T. A. (2022). Child drawing development optimization algorithm based on child's cognitive development. *Arabian Journal for Science and Engineering, 47*, 1337-1351. https://doi.org/10.1007/s13369-021-05928-6

Afriansyah, E. A., Puspitasari, N., Luritawaty, I. P., Mardiani, D., & Sundayana, R. (2019). The analysis of mathematics with ATLAS. ti. *Journal of Physics: Conference Series, 1402*(7), 077097. https://doi.org/10.1088/1742-6596/1402/7/077097

Afriansyah, E. A. (2021). *Realistic mathematics education berbasis emergent modeling untuk meningkatkan kemampuan berpikir kritis dan kreatif matematis serta curiosity mahasiswa calon guru* [*Realistic mathematics education based on emergent modeling to improve mathematical critical and creative thinking skills and the curiosity of prospective teacher students*], [Unpublished doctoral dissertation]. Universitas Pendidikan Indonesia.

Afriansyah, E. A., & Arwadi, F. (2021). Learning trajectory of quadrilateral applying realistic mathematics education: Origami-based tasks. *Mathematics Teaching Research Journal, 13*(4), 42-78. https://commons.hostos.cuny.edu/mtrj/wp-content/uploads/sites/30/2022/01/v13n4-Learning-Trajectory-of-Quadrilateral.pdf

Afriansyah, E. A., Herman, T., Turmudi, & Dahlan, J. A. (2021, February). Critical thinking skills in mathematics. *Journal of Physics: Conference Series, 1778*(1), 012013. https://doi.org/10.1088/1742-6596/1778/1/012013

Ardianingsih, A., Lusiyana, D., & Rahmatudin, J. (2019). Penerapan pembelajaran realistic mathematic education berbasis etnomatematika untuk meningkatkan HOTS matematik siswa [Application of realistic mathematics education based on ethnomathematics to increase students' math HOTS]. *Mathline: Jurnal Matematika dan Pendidikan Matematika*, *4*(2), 148-161. https://doi.org/10.31943/mathline.v4i2.117

Awidi, I. T., & Paynter, M. (2019). The impact of a flipped classroom approach on student learning experience. *Computers & Education*, *128*, 269-283. https://doi.org/10.1016/j.compedu.2018.09.013

Baier, F., Decker, A. T., Voss, T., Kleickmann, T., Klusmann, U., & Kunter, M. (2019). What makes a good teacher? The relative importance of mathematics teachers' cognitive ability, personality, knowledge, beliefs, and motivation for instructional quality. *British Journal of Educational Psychology*, *89*(4), 767-786. https://doi.org/10.1111/bjep.12256

Büscher, C., & Schnell, S. (2017). Students' emergent modelling of statistical measures–a case study. *Statistics Education Research Journal*, *16*(2), 144-162. https://doi.org/10.52041/serj.v16i2.188

Cao, Y., Zhang, S., Chan, M. C. E., & Kang, Y. (2021). Post-pandemic reflections: lessons from Chinese mathematics teachers about online mathematics instruction. *Asia Pacific Education Review*, *22*(2), 157-168. https://doi.org/10.1007/s12564-021-09694-w

Carroll, J. A., Wilson, E. E., Klimow, N., & Hill, K. (2018). *Acts of teaching: How to teach writing: A text, a reader, a narrative*. ABC-CLIO.

Danesi, M. (2018). The mathematical mind. In *Ahmes' Legacy* (pp. 127-149). Springer. https://doi.org/10.1007/978-3-319-93254-5_5

diSessa, A. A. (2002). Students' criteria for representational adequacy. In Gravemeijer, K., Lehrer, R., Oers, B., & Verschaffel, L., *Symbolizing, modeling and tool use in mathematics education*, (pp. 105-129). Springer., Dordrecht. https://doi.org/10.1007/978-94-017-3194-2_7

Doorman, L. M., & Gravemeijer, K. P. E. (2009). Emergent modeling: discrete graphs to support the understanding of change and velocity. *ZDM*, *41*(1-2), 199-211. https://doi.org/10.1007/s11858-008-0130-z

Fredriksen, H. (2021). Exploring realistic mathematics education in a flipped classroom context at the tertiary level. *International Journal of Science and Mathematics Education*, *19*(2), 377-396. https://doi.org/10.1007/s10763-020-10053-1

Freudenthal, H. (2006). *Revisiting mathematics education: China lectures* (Vol. 9). Springer Science & Business Media.

Gravemeijer, K. (1999). How emergent models may foster the constitution of formal mathematics. *Mathematical thinking and learning*, *1*(2), 155-177. https://doi.org/10.1207/s15327833mtl0102_4

Gravemeijer, K. (2002a). Preamble: From models to modeling. In Gravemeijer, K., Lehrer, R., Oers, B., & Verschaffel, L. (Eds.), *Symbolizing, modeling and tool use in mathematics education* (pp. 7-22). Springer, Dordrecht. https://doi.org/10.1007/978-94-017-3194-2_2

Gravemeijer, K. (2002b, July). Emergent modeling as the basis for an instructional sequence on data analysis. In *Proceedings of the 6th International Conference on Teaching Statistics*. https://www.fisme.science.uu.nl/publicaties/literatuur/4795.pdf

Gravemeijer, K., & Bakker, A. (2006, July). Design research and design heuristics in statistics education. In *Proceedings of the Seventh International Conference on Teaching Statistics* (pp. 1-6).

Gravemeijer, K. (2007). Emergent modeling and iterative processes of design and improvement in mathematics education. *Recuperado enero*, *30*, 2009. https://www.criced.tsukuba.ac.jp/math/apec/apec2008/papers/PDF/3.Keynote(Dec.9)_Koeno_Gravemeijer_Netherlands.pdf

Hefendehl-Hebeker, L., vom Hofe, R., Büchter, A., Humenberger, H., Schulz, A., & Wartha, S. (2019). Subject-matter didactics. In *Traditions in German-speaking mathematics education research* (pp. 25-59). Springer. https://doi.org/10.1007/978-3-030-11069-7_2

Herman, T. (2018). Improving students' mathematical representational ability through RME-based progressive mathematization. *Journal of Physics: Conference Series*, *948*(1), 012038). https://doi.org/10.1088/1742-6596/948/1/012038

Huang, J., Liu, Q., Zheng, Y., Wu, L., Ding, Y., & Huang, L. (2021, August). Research on machine understanding math word problems: From the perspective of discourse comprehension models. In *2021 International Symposium on Educational Technology (ISET)* (pp. 139-144). IEEE. https://doi.org/10.1109/ISET52350.2021.00037

Johnson, E. (2013). Implications of realistic mathematics education for analyzing student learning. In *Conference on research in undergraduate mathematics education, Denver,*

CO. http://pzacad.pitzer.edu/~dbachman/RUME_XVI_Linked_Schedule/rume16_submission_106. pdf
Kaiser, G. (2020). Mathematical modelling and applications in education. *Encyclopedia of mathematics education*, 553-561. https://doi.org/10.1007/978-3-030-15789-0_101
Kieren, T. E. (2020). Rational and fractional numbers as mathematical and personal knowledge: Implications for curriculum and instruction. In *Analysis of arithmetic for mathematics teaching* (pp. 323-371). Routledge. https://doi.org/10.4324/9781315044606-6
Kitchen, W. H. (2019). *Wittgensteinian pedagogy for mathematics: Rule-following, and why it matters for mathematics teaching and learning*. Exeter University.
Lehrer, R., & Pritchard, C. (2002). Symbolizing space into being. In Gravemeijer, K., Lehrer, R., Oers, B., & Verschaffel, L. (Eds.), *Symbolizing, modeling and tool use in mathematics education* (pp. 59-86). Springer, Dordrecht. https://doi.org/10.1007/978-94-017-3194-2_5
Mardiah, N., Permana, D., & Arnawa, I. M. (2021). The validity of hypothetical learning trajectory based on realistic mathematic education on function topics for grade X senior high school. *Journal of Physics: Conference Series*, *1742*(1), 012005. https://doi.org/10.1088/1742-6596/1742/1/012005
Marino, M. C. (2020). *Critical code studies*. MIT Press. https://doi.org/10.7551/mitpress/12122.001.0001
Mead, G. H. (1934). *Mind, self and society* (Vol. 111). University of Chicago Press.
Mead, G. H. (1938). The philosophy of the act, edited by Charles W. *Morris et al. University of Chicago*.
Meira, L. R. D. L. (2002). Mathematical representations as systems of notations-in-use. In *Symbolizing, modeling and tool use in mathematics education* (pp. 87-103). Springer, Dordrecht. https://doi.org/10.1007/978-94-017-3194-2_6
Nurfadilah, P., & Afriansyah, E. A. (2022). Analisis gesture matematis siswa dalam menyelesaikan soal open-ended [Analysis of students' mathematical gestures in solving open-ended problems]. *Journal of Authentic Research on Mathematics Education (JARME)*, *4*(1), 14-29.
O'Connor, C., & Michaels, S. (2019). Supporting teachers in taking up productive talk moves: The long road to professional learning at scale. *International Journal of Educational Research*, *97*, 166-175. https://doi.org/10.1016/j.ijer.2017.11.003
Octizasari, G., & Haji, S. (2018). Penerapan model pembelajaran RME berbasis ethnomatematika untuk meningkatkan kemampuan pemecahan masalah mahasiswa calon guru pendidikan matematika FKIP universitas bengkulu [Application of the ethnomathematical-based RME learning model to improve problem solving ability of prospective students of mathematics education teachers FKIP university of bengkulu]. *Jurnal Pendidikan Matematika (Jupitek)*, *1*(1), 1-7. https://doi.org/10.30598/jupitekvol1iss1pp1-7
Presmeg, N. (2002). Transitions in emergent modelling. In Gravemeijer, K., Lehrer, R., Oers, B., & Verschaffel, L. (Eds.), *Symbolizing, Modelling, and Tool Use in Mathematics Education* (pp 131-137). Kluwer Academic Publishers. https://doi.org/10.1007/978-94-017-3194-2_8
Riyanto, B., & Putri, R. I. I. (2017, December). Mathematical modeling in realistic mathematics education. *Journal of Physics: Conference Series*, *943*(1), 012049. https://doi.org/10.1088/1742-6596/943/1/012049
Sarvita, L., & Syarifuddin, H. (2020, May). The developed hypothetical learning trajectory for integral topic based on realistic mathematics education. *Journal of Physics: Conference Series*, *1554*(1), 012032. https://doi.org/10.1088/1742-6596/1554/1/012032

Silinskas, G., & Kikas, E. (2019). Math homework: Parental help and children's academic outcomes. *Contemporary Educational Psychology*, *59*, 101784. https://doi.org/10.1016/j.cedpsych.2019.101784

Sumirattana, S., Makanong, A., & Thipkong, S. (2017). Using realistic mathematics education and the DAPIC problem-solving process to enhance secondary school students' mathematical literacy. *Kasetsart Journal of Social Sciences*, *38*(3), 307-315. https://doi.org/10.1016/j.kjss.2016.06.001

van Oers, B. (2002). Informal representations and their improvements. In Gravemeijer, K., Lehrer, R., Oers, B., & Verschaffel, L. (Eds.), *Symbolizing, Modelling, and Tool Use in Mathematics Education* (pp. 25-28). Springer, Dordrecht. https://doi.org/10.1007/978-94-017-3194-2_3

Vinner, S. (2018). The misconception fallacy, the pseudo-conceptual and the pseudo-analytical behaviors in mathematical contexts. In *Mathematics, Education, and Other Endangered Species* (pp. 23-52). Springer, Cham. https://doi.org/10.1007/978-3-319-90035-3_5

Walshaw, M. (2017). Understanding mathematical development through vygotsky. *Research in Mathematics Education*, *19*(3), 293-309. https://doi.org/10.1080/14794802.2017.1379728

Wertsch, J. V. (1985). *Vygotsky and the social formation of mind*. Harvard University Press.

Wijaya, A. (2012). *Pendidikan matematika realistik suatu alternatif pendekatan pembelajaran matematika* [*Realistic mathematics education is an alternative approach to learning mathematics*]. Graha Ilmu.

Witha, T. S., Karjiyati, V., & Tarmizi, P. (2020). Pengaruh model RME berbasis etnomatematika terhadap kemampuan literasi matematika siswa kelas IV SD gugus 17 kota bengkulu [The influence of the ethnomathematical-based RME model on the mathematical literacy ability of fourth grade elementary school cluster 17 bengkulu city]. *JURIDIKDAS: Jurnal Riset Pendidikan Dasar*, *3*(2), 136-143. https://doi.org/10.33369/juridikdas.3.2.136-143